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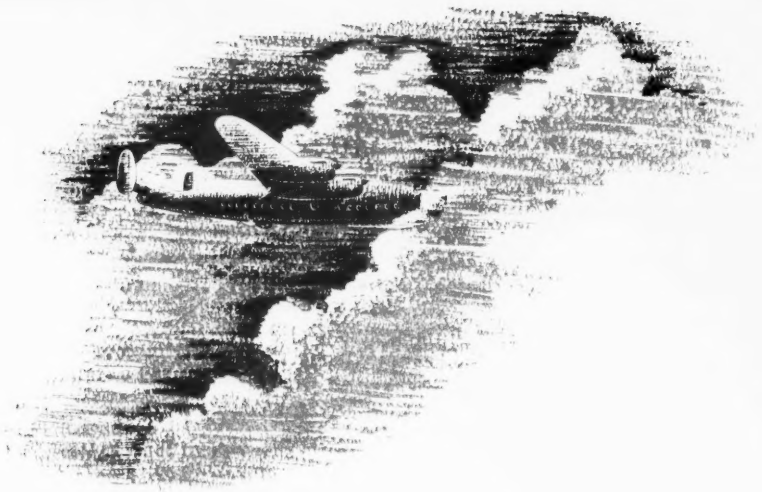
The Journal of

THE ROYAL AERONAUTICAL SOCIETY

FEBRUARY 1947 · NO 434 · VOL. 51



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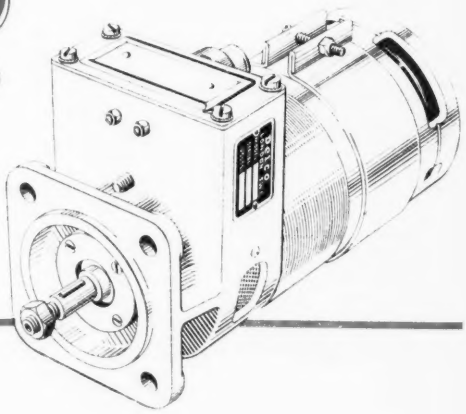

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THE ROYAL AERONAUTICAL SOCIETY

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FEBRUARY NOTICES 1947

JOURNAL PREMIUM AWARDS

The Council have set aside an annual sum of £250 for the award of premiums for papers published in the *Journal* and the Council hope that members (or non-members) will contribute papers on their own special subjects.

CONTENTS OF THE FEBRUARY JOURNAL

The Full-Day Discussion on Engineering Problems of Future Aircraft. Papers by Mr. F. M. Owner, F.R.Ae.S., " Power Installations "; Mr. G. H. Lee, A.F.R.Ae.S., " Tailless Aircraft "; Mr. C. P. T. Lipscomb, F.R.Ae.S., " Flying Boat Problems "; and Mr. A. E. Russell, F.R.Ae.S., " Large Aircraft."

Model Aircraft Illustrating Sign Convention Used in Applied Aerodynamics, by F. W. Free, A.F.R.Ae.S.

The General Theory of Cylindrical and Conical Tubes under Torsion and Bending Loads, by J. Hadji-Argyris, D.E., and P. C. Dunne, B.Sc.

ELECTION OF COUNCIL

Attention is drawn to the following Rules for election to Council. Nominations should be received by 1st March, 1947. Nomination forms may be obtained from the Secretary. Nomination forms may be signed by any members of the Society excepting Honorary Fellows, Graduates and Students.

Rule 71.—One-half of the Council (excluding the President and the immediate Past-President if a Member of the Council) shall retire annually. The members who shall retire shall be those longest in office, except as provided in Rule 77. Retiring Members of Council who have served for two terms in succession (four years) shall not be eligible for re-election until the next annual election, when they will be eligible.

Rule 72.—The composition of the Council shall be as follows:—

(*a*) Not less than sixteen shall be elected from the technical or scientific grades of the R.Ae.S.—that is to say, from among the Fellows, Associate Fellows and Associates.

(*b*) Of these sixteen, four at least shall be Fellows.

Rule 73.—Nominations of candidates for election to the Council must be received by the Secretary not less than twenty-one days before the Annual General Meeting, with an intimation in writing by the candidates that they are willing to serve. Nominations must be signed by one supporter and two seconders, who must each be entitled to vote in the Royal Aeronautical Society.

NEW YEAR'S HONOURS

The following members have received awards in the New Year's Honours Lists:—

K.C.I.E.

Sir F. Tymms, K.B., A.C.G.I., M.C., Fellow.

K.C.B. (Military Division)

Air Vice-Marshal A. C. Collier, C.B., C.B.E., Associate Fellow.

C.B. (Military Division)
Air Commodore F. Whittle, C.B.E., B.A., Fellow.

C.B.E. (Military Division)
Air Commodore E. J. D. Townsend, B.A., M.Inst.Met., Fellow.

C.B.E.
W. G. Carter, Fellow.

O.B.E.
A. Hillier, Companion.

M.B.E. (Military Division)
F/Lt. Ralph Brown, D.F.M., Associate.

Air Force Cross
P/O. J. W. Stone, Student.
K. J. Sewell, D.F.M., Associate.
S/Ldr. E. J. Spencer, Associate.

King's Commendations for Valuable Service in the Air
E. O. Draper, Associate Fellow.

35th WILBUR WRIGHT MEMORIAL LECTURE

Mr. John K. Northrop, President of Northrop Aircraft Inc., will read the 35th Wilbur Wright Memorial Lecture on the 29th May, 1947. A possible title may be " The Development of All-Wing Aircraft."

INSTITUTE OF THE AERONAUTICAL SCIENCES

The Council of the Institute announce the following elections:—

To Honorary Fellowship: Sir Geoffrey de Havilland, F.R.Ae.S.

To Fellowship: Dr. H. Roxbee Cox, F.R.Ae.S., Vice-President of the Society; Captain J. Laurence Pritchard, Hon.F.R.Ae.S.

Under the rules of the Institute not more than one Honorary Fellow, residing outside the United States, nor more than ten Fellows, residing outside the United States, may be elected in one year.

Dr. H. Roxbee Cox and Captain J. Laurence Pritchard are the only two foreign Fellows elected by the Institute since 1939.

THE HELICOPTER PRIZE

The closing date for papers entered for the Helicopter Association Prize Essay for 1946-7 has been extended to the **31st March, 1947.** The prize is to be known as the Juan de la Cierva Memorial Prize Essay and the subject for competition in 1946-7 is " The Technical and Economic Problems of Rotary-Wing as opposed to Fixed-Wing Aircraft and their application to the future of Civil Aviation."

Any enquiries should be made to the Helicopter Association of Great Britain, Finsbury Circus House, Blomfield Street, London, E.C.2.

MISS HILDA LYON

The late Miss Hilda Lyon was awarded the R.38 Memorial Prize in 1930, not the R.101 Prize as stated last month.

POWER UNITS FOR FUTURE AIRCRAFT—A CORRECTION

In Fig. 17 of the paper " Power Units for Future Aircraft " by Air Commodore Banks, on p. 44 of the January Journal, the altitudes shown on the curves should be 20,000 ft. and 40,000 ft. not 2,000 and 4,000 ft.

MEETINGS AT THE HEADQUARTERS OF THE SOCIETY

During the past month meetings have been held at the offices of The Society, 4 Hamilton Place, by the following organisations:—

Aeronautical Research Council (on various occasions).
Aircraft Recognition Society.
British Standards Institution.
Guild of Air Pilots and Air Navigators.
Light Aeroplane Technical Committee.
Society of British Aircraft Constructors.

ANNUAL SUBSCRIPTIONS OF MEMBERS

Members are reminded that their subscriptions became due on January 1st, 1947.

The present subscriptions are:—

	Home. £ s. d.	Abroad. £ s. d.		Home. £ s. d.	Abroad. £ s. d.
Founder Members ..	2 2 0	2 2 0	Graduates (age 21-25)	2 2 0	2 2 0
Fellows	4 4 0	3 3 0	Graduates (age 26-28)	2 12 6	2 12 6
Associate Fellows ..	3 3 0	2 2 0	Students	1 1 0	1 1 0
Associates	2 2 0*	2 2 0*	Companions ..	2 2 0	2 2 0

* £1 1s. 0d. without Journal.

It will avoid delay and confusion if members, when sending remittances for their subscriptions, state their name clearly and give their grade of membership and address.

ASSOCIATE FELLOWSHIP EXAMINATION—MAY 1947

Candidates for the Associate Fellowship Examination are reminded that entries for the May 1947 sitting should be received by 28th February, 1947. Entry forms may be obtained from the Secretary.

A new syllabus for the examination comes into force in January 1948, but those candidates studying under the old syllabus will be catered for in the initial period.

LECTURE PROGRAMME—SPRING SESSION 1947

Except where otherwise stated, the lectures will be held at 6 p.m. in the Lecture Hall of the Institution of Civil Engineers, Great George Street, S.W.1 (by permission of the Council of the Institution). Tea will be served at 5.30.

Visitors are welcome, but should obtain tickets through a member of the Society.

Wednesday, 12th February, 1947—Joint Lecture with the Institute of Petroleum. " Fuels and Lubricants for Aero Gas Turbines " by Dr. C. G. Williams, D.Sc., M.I.Mech.E., A.F.R.Ae.S., F.Inst.Pet.

This Lecture will be held at the Institute of Petroleum, Manson House, 26 Portland Place, London, W.1, at 5.30 p.m. (Tea at 5 p.m.)

Thursday, 27th March, 1947—" The War in the Ether " by Air Vice-Marshal E. B. Addison, C.B., C.B.E.

Thursday, 10th April, 1947—" A Review of Production Difficulties in Relation to Aircraft Design " by Mr. C. E. Fielding.

Thursday, 17th April, 1947—" Testing Civil Aircraft " by Mr. P. A. Hufton, M.Sc.

Thursday, 24th April, 1947—" The Risk of Fire and Fire Prevention Methods " by Dr. J. W. Drinkwater, B.Sc., D.Phil., and Mr. W. G. Glendening.

Thursday, 1st May, 1947—" The Development of the A.S. type Gas Turbine " by Mr. W. H. Lindsey, M.A., A.F.R.Ae.S.

Thursday, 8th May, 1947—" The Development of the Goblin Engine " by Mr. E. S. Moult, B.Sc., A.M.I.Mech.E., A.F.R.Ae.S.

Thursday, 29th May, 1947—The Thirty-Fifth Wilbur Wright Memorial Lecture will be read by Mr. J. K. Northrop.

GRADUATES' AND STUDENTS' SECTION

Tuesday, 25th February, 1947—" The Role of the Experimental Department " by Mr. G. R. Edwards, F.R.Ae.S.

Friday, 14th March, 1947—" Developments in Propeller Turbine Engines " by Mr. G. J. C. Davies, A.F.R.Ae.S.

Friday, 2nd May, 1947—" Flight Testing of Aircraft " by Mr. W. Humble.

Tuesday, 20th May, 1947—" Operation of Civil Aircraft in the Tropics " by Captain G. U. Allen, A.F.R.Ae.S.

All meetings will be held at 7.30 p.m. in the Library of the Society, 4, Hamilton Place, W.1.

BRANCH LECTURES AND NOTICES

Belfast Branch

11th February, 1947—Electrics in Aircraft, by Mr. C. G. A. Woodford, A.F.R.Ae.S.

11th March, 1947—Gas Turbines and Jets, by Mr. J. Hodge, A.F.R.Ae.S.

All lectures will be held in the Central Hall, Belfast College of Technology.

Birmingham Branch

19th February, 1947—Debate with Coventry Branch. At the Birmingham Chamber of Commerce at 7 p.m.

27th March, 1947—An historical survey of the aeroplane, by Lord Brabazon of Tara. At Queen's College, Paradise Street, Birmingham, at 7 p.m.

Bristol Branch

18th February, 1947—Radar—Its Uses and Possibilities in Civil Aviation, by F. R. Willis, Esq., B.Sc., A.M.I.E.E., A.F.R.Ae.S., Radio Development Section, B.O.A.C.

4th March, 1947—Some Aspects of Stability and Control, by Prof. A. R. Collar, M.A., B.Sc., F.R.Ae.S., Professor of Aeronautical Engineering, University of Bristol.

25th March, 1947—Competition for the best paper read by a Junior Member.

15th April, 1947—Annual general meeting and film show.

All meetings will be held in the Conference Room, Filton House, Bristol Aeroplane Co., Ltd., Filton, at 5.30 p.m.

DERBY BRANCH

17th February, 1947—Turbine Compounding of Piston Engines, by Dr. H. Ricardo, F.R.S., F.R.Ae.S.

24th March, 1947—Mr. P. G. Masefield, F.R.Ae.S., Director-General Long-Range Planning and Projects, Ministry of Civil Aviation.

Lectures are held at the Rolls-Royce Welfare Hall at 6.15 p.m.

GLOUCESTER AND CHELTENHAM BRANCH

6th February, 1947—A film, The Production of High Quality Steel introduced by Mr. W. H. Salmon, Assoc.Met., F.I.M.

27th February, 1947—Test Flying, by Mr. P. Stanbury, Chief Test Pilot, Gloster Aircraft Co. Ltd.

In the Staff Canteen, Gloster Aircraft Co. Ltd., Hucclecote, at 7.30 p.m.

MANCHESTER BRANCH

27th February, 1947—The Flying Wing, by Mr. E. D. Keen, B.Sc., F.R.Ae.S. At Reynolds Hall, College of Technology, at 7.30 p.m.

SOUTHAMPTON BRANCH

19th February, 1947—Problems in Fighter Design, by R. L. Lickley, B.Sc., D.I.C., F.R.Ae.S.

19th March, 1947—High Speed Flight, by R. Smelt, M.A., A.F.R.Ae.S.

9th April, 1947—An Address by E. F. Relf, C.B.E., F.R.S., F.R.Ae.S.

All lectures will be held at 7 p.m. in the Physics Lecture Theatre, University College, Southampton.

JOURNAL BINDING

Owing to increased costs, the prices of binding of Journals will be as follows:—

1946 Volume 12/6.
Previous Volumes 13/6.

Journals should be sent direct to The Lewes Press, Friars Walk, Lewes, Sussex, and the remittance to the Secretary at the office of the Society.

CHANGES OF ADDRESS

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).
Grade of membership.
New address (in block letters).
Old address.

NEW MEMBERS

Associate Fellows

Thomas Henry Cain (from Student), Edward John Cairns (from Graduate), Christopher Clarkson, Leonard Edward Christmas, Hugh bond Cundall, Guy Du Merle (from Associate), Eric John Farrow, William Victor Philip Ferguson, Naftali Francois Frenkiel, Kenneth George Hancock (from Graduate), Arthur St. George Harris, Derrick Arthur Higgs, Reginald Jackson (from Graduate), Raymond Lloyd Jefferies (from Graduate), Czeslaw Kapica, Roy Kendall (from Student), John Maynard (from Associate), Harold Clifford Olding, Bernard

Sutton Owen, Tom Ostle Peel, Kuchi Srihari Muralidhara Rao, Ralph Harry Price (from Graduate), George Simm (from Graduate), Fayez Ebtenago Sahyoun, Alan Ritson Scott.

Associates

Roy Frederick Birt, Edward Charles Body, Pei-Lin Chang, Kenneth William Course (from Student), Thomas Lloyd Davies, John Antony Dollahan, William Sidney John Drew, Robert William Everall, Gerald Hartnett, Kenneth Roy Hickson, Ralph Hutchinson, Walter Henry Kuhl, Frederick George Lewis, Peter John Ross (from Student), James Alwyn Bowman Sharp, Gordon Stanley Smyth, Lionel Stern, David Stewart, Reginald Robert Williams.

Graduates

Dennis Ashton, Christopher Richard Baines (from Student), Kenneth Nairn Beatts, Horace Tom Elliott Hone, Gerald Arthur Hughes (from Student), John Harcourt Leech, Peter Adrian Liley (from Student), John Graham George Marshall, Sydney Thomas Mathers, Ronald James Moppett, Kenneth Raymond Nuttall, Charles John Street (from Student), Albert Tootill.

Students

Joseph George Vibert Bland, Eric John Catchpole, Frank Edward Collins, Eric Neville Crabbe, George Ellar, Hugh Graham Gillespie, William Manley Gray, Ronald George Hagan, Gerald Harrison, Rodney Graham Hinkley, Peter George Jefferies, Peter Robert Jempson, Donald Allan Kirby, Ernest Fintan Lawlor, Robert John McQuillan, John Francis Mew, Donald Blan Minterne, Leslie Sydney Dennis Morley, Philip Audley Mucklow, Kenneth Tudor Owen, Thomas Shaw, Raymond Frederick Smith, Douglas Ramiro Walls, Peter Ernest Watts, Bruce William White.

Companions

Peter Wright Brooks (from Student), Dennis Bruce, William Percy Nash.

ACKNOWLEDGMENTS

The Council acknowledge with grateful thanks the following gifts:—Nieuport Catalogue, presented by Flight Lieut. E. M. Goodger; " Akrobatyka Lotnicza " by Roland Kalpas and Bolestaw Solak, presented by Mr. R. Kalpas; and back numbers of the Journal from Major O. G. C. Drury, Associate Fellow, Mr. S. W. G. Foster, Associate Fellow, Mr. J. N. Richmond, Associate Fellow and Mr. T. H. Turner, Associate.

ADDITIONS TO THE LIBRARY

Pamphlets in italics with location reference following in brackets. Books marked * or ** may not be taken out on loan.

A.a.329.—Explosion and Combustion Processes in Gases. Wilhelm Jost. McGraw Hill. 1946.

B.g.149.—Aircraft of the Fighting Powers. Volume VII. O. G. Thetford and E. J. Riding. Harborough Publishing Co. 1946.

BB.b.121.—Construction des Avions. Guy du Merle. Dunod, Paris. 1947.

I.d.8.—Mathematical Theory of Elasticity. Sokolnikoff and Specht. McGraw Hill. 1946.

L.c.29.—Akrobatyka Lotnicza. R. Kalpas and B. Solak. Ksiaznica Polska, Glasgow. 1945.

L.d.115.—*Notices to airmen*. Nos. 209, 210, 211, 212, 213, 214, 215, 218, 219, 220.

L.d.116.—*Notices to airmen*. Nos. 1, 2, 3, 6.

S.c.128.—Bomber Offensive. Air Vice-Marshal Sir Arthur Harris. Collins. 1947.

S.c.129.—Special Duties. Air Vice-Marshal A. S. G. Lee. Sampson Low. 1947.

S.c.130.—Near the Sun. V. Tempest. Crabtree. 1946.

T.a.117.—Blue is the Sky. G. D. Fleming. William Earl, Bournemouth.

X.c.F.8.—Dictionnaire technique de l'aviation. French-English. J. Lycett. Dunod, Paris. 1943.

X.c.G.7.—Dictionnaire technique de l'aviation. French-German. J. Lycett. Dunod, Paris. 1943.

A.R.C. Reports and Memoranda

2030.—*Theory of the propeller fin effect, including a review of existing theories. E. Priestley.*

N.A.C.A. Technical Notes

1130.—*Evaluation of two high carbon precision cast alloys at* 1,700 *and* 1,800 *degrees fahrenheit by the rupture test. E. E. Reynolds, J. W. Freeman and A. E. White.*

1131.—*The effect of engine variables on the pre-ignition limited performance of three fuels. D. W. Male.*

1134.—*Development of a protected air scoop for the reduction of induction-system icing. Uwe von Glahn and C. E. Renner.*

1135.—*Application of the method of characteristics to supersonic rotational flow. Antonio Ferrari.*

1138.—*Standard procedures for rating and testing multi-stage axial-flow compressors. N.A.C.A. Sub-Committee on Compressors.*

1142.—*An analysis of the airspeeds and normal accelerations of Douglas D.C.*3 *airplanes in commercial transport operation. A. M. Peiser.*

1147.—*Flight investigation of the cooling characteristics of a two-row radial engine installation. III. Engine temperature distribution. R. M. Rennak, W. E. Messing and J. E. Morgan.*

1148.—*Flow tests of an N.A.C.A. designed supercharger inlet elbow and the effects of various components on the flow characteristic at the elbow outlet. D. C. Guentert, D. J. Todd and W. P. Simmons.*

Thornton Aero Engine Research Laboratory.

Technical Memorandum No. T-76. Thermal properties of various organic compounds. F. L. Garton.

The following reports have also been received:—

Flygtekniska Forsoksanstalten. Tekniska Hogskolans Rotaprinttryckeri. Stockholm. Nos. 2-19 *inclusive.*

Institut für Aerodynamik. Zurich. Nos. 9, 10, 11, 12.

Monografie Scientifiche di Aeronautica. Associazione Culturale Aeronautica. Rome. Nos. I and II.

British Intelligence Objectives Sub-Committee.

B.I.O.S. Final Reports, Nos. 884, 899, 584, 532.

F.I.A.T. Final Reports, Nos. 862, 202.

J.I.O.A. Final Report, No. 48.

Combined Intelligence Objectives Sub-Committee.

Item Nos. 5, 21, 25 and 27. File No. XXXII-26.

J. Laurence Pritchard,
Secretary.

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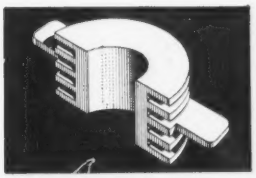

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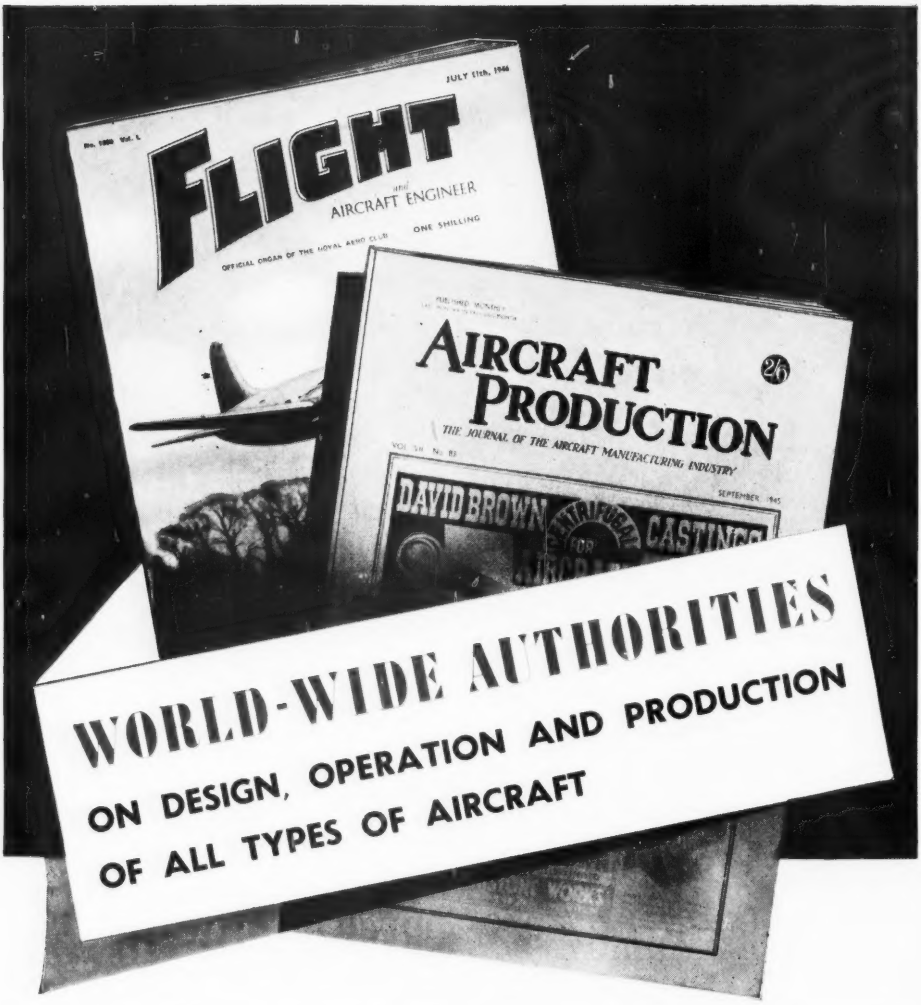

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Published in conjunction with these journals, FLIGHT HANDBOOK (212 pages, 7 6 net) is essentially a manual for the student, whilst GAS TURBINES AND JET PROPULSION (272 pages, 12 6 net) by G. Geoffrey Smith, has been widely adopted as the standard text on the subject by Universities, Technical Institutions and Training Centres everywhere.

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Extract from an address made by Mr. T. A. Macauley, Chairman of A. C. Cossor Ltd., on 14th August, 1946, at the company's 8th General Meeting.

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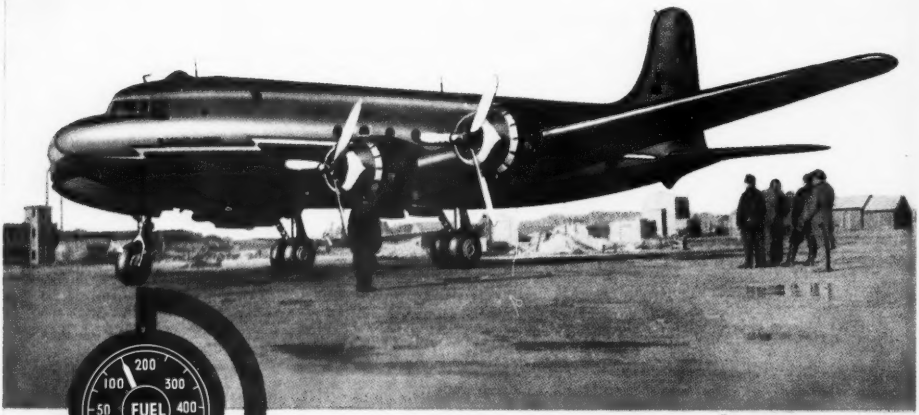

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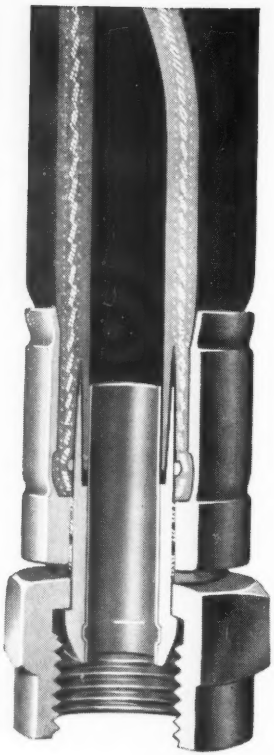

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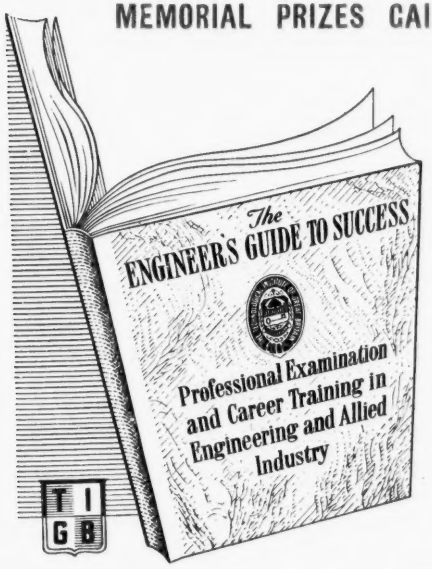

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A FULL DAY discussion on Engineering Problems of Future Aircraft, was held by the Royal Aeronautical Society in the Great Hall of the Institution of Civil Engineers, Great George Street, Westminster, London, S.W.1.. on Thursday, 14th November, 1946. Four papers were presented.

Sir Frederick Handley Page, C.B.E., President of the Society, occupied the Chair during the greater part of the proceedings.

MORNING SESSION

THE PRESIDENT, introducing the author, said he obtained his degree at Manchester University in 1922, after which he joined the Bristol Aeroplane Company, where he was engaged on engine design. In 1924 he joined the stress office, and in 1935 became Assistant Chief Designer; in 1944 became Chief Engineer on Turbines.

POWER INSTALLATIONS

by

F. M. OWNER, M.Sc., F.R.Ae.S., M.S.A.E.

Mr. Owner obtained his degree at Manchester University in 1922 *and then joined The Bristol Aeroplane Company Ltd., where he was engaged on engine design. In* 1924 *he had joined the Stress office and in* 1935 *became Assistant Chief Designer. In* 1944 *he was appointed Chief Engineer on Turbines at Bristol.*

1. INTRODUCTION.

1.1. OBJECT OF PAPER.

Never, at any time since the Wright Brothers made their first flight have the means of propulsion exercised a more dominant influence over the shape of aircraft or the shape of things to come, nor can it have been so difficult to forecast the form resulting from that influence. It has been a platitude for some time past to say that aircraft and engine design are mutually interdependent: now it is more than a platitude, it is a fact, and one to which we have not yet grown accustomed.

That power installations can no longer be procured "off the peg" is to-day an article of faith which is generally professed: what is rarer is an appreciation of the implications of this state of affairs. Pursued to its logical conclusion, the implication is that a new aircraft and its means of propulsion should originate in the same brain, and be designed, developed and manufactured under the guidance of that brain.

Unfortunately, this demands a degree of knowledge which is practically impossible to acquire in one lifetime, and hence we must content ourselves with seeing that the liaison between the aircraft and engine designers constitutes a marriage of ideas rather than an irregular and occasional interchange.

The advent of the gas turbine in practical form has led to some rather distorted ideas concerning its effect on aircraft performance

and this paper will attempt to review the various types of power plant now available, and to discuss their applicability to the various classes of aircraft, both current and projected. It is obviously impossible to cover such a vast field in detail, but even a general review may, it is hoped, give a clearer idea of the engineering problems confronting us.

1.2. Outline of paper.

A difficulty which arises in the preparation of this paper is that it has perforce to deal more with possibilities and probabilities than with matters of established fact; not for years have there been so many exciting possibilities of advance, both in aerodynamics, thermodynamics, and on the mechanical side, but practical experience is, as yet, available only to a very limited extent indeed. Under these conditions, the chief use of a paper must be to provoke discussion, and statements of opinion on controversial issues are certainly a good means of doing this. So long as it is clearly stated at the outset that only expressions of opinion are possible at this stage (since experience is as yet lacking in many of the fields discussed), it is hoped to avoid the charge of dogmatism.

The main headings of discussion are as follows:—

2. Classification of Aircraft Requirements.
3. Classification of Power Plants.
4. Possible Power Installations.
5. Installation Requirements.
6. General Trend of Development.
7. Conclusions on Engineering Problems.

1.3. Period under review.

The period chosen for review is from five to ten years beginning now; the nearer limit is chosen since it is felt that it will take at least five years to perfect and bring into general use the conceptions which are now so freely discussed; and the further limit of ten years, because he who would seek to peer farther into the future needs a very large and powerful crystal into which to gaze.

2. CLASSIFICATION OF AIRCRAFT REQUIREMENTS.

2.1. Military aircraft.

The advent of such revolutionary weapons as the atomic bomb and rocket-propelled missiles renders the task of foretelling the trend of military aircraft extremely difficult. While we are all united in our heartfelt detestation of war and all the horrors it entails, we cannot disregard the need for insurance against possible disasters of this type. It may be that the development of rocket-propelled, pilotless missiles together with atomic power, will entirely change the character of air power as we know it to-day, but such development necessarily takes time and in the interim the development of military aircraft will not stand still. The probable trend of military requirements must therefore be reviewed.

Whereas in times of peace, human life is probably the most precious possession, under the stress of war this usually becomes a secondary consideration; it is therefore obvious that military aircraft must be designed not so much from the point of view of safety and economy, but for the overriding condition of high performance, since the two characteristics of maximum performance and adequate safety cannot usually be maintained together.

Military aircraft must be designed primarily to fulfil one of the following functions:—

To act as long-range artillery.
To observe and control military objectives and operations.
To transport personnel and equipment.
To intercept and destroy any of the above types of aircraft.

2.1.1. *Fighters.*

A remarkable feature of present-day fighter aircraft is their similarity with speed record aircraft. In 1929, the speed record exceeded 350 m.p.h., but it was ten years before such speeds were reached during every

day performances of fighter aircraft. Nowadays the fighter follows closely behind the record-breaker, and in the last successful attempt on the world's speed record, it was an aircraft originally designed as a fighter which actually earned the distinction of being the fastest aeroplane to fly.

The main reason for this appears to be that both types of aircraft are approaching the barrier of sonic speed. Once this barrier has been crossed, there is no reason to suppose that record-breaking aircraft will not again out-distance the everyday military types. While it is inappropriate for an engine designer to attempt to foretell the time when this sonic barrier will be crossed, it may very well be several years before this is achieved experimentally, and considerably longer before it is a practical possibility; it is therefore reasonable to assume that the design of military fighters will tend to stabilise itself for a time. Meanwhile, the engine designer can supply his aircraft colleague with more power than the latter can possibly use. For instance, it would be easy to install four or more engines in a fighter in the manner shown in Fig. 1, but an aircraft of such fairly conventional design would probably become unmanageable before full power was applied. Furthermore the fuel consumption would be immense and the range commensurately small.

The question may arise, however, whether the limit of fighter development has not almost been reached because of the limit of the endurance of the human body. Increasing the speed of flight increases the acceleration imposed on the pilot by a given manœuvre; even nowadays it is quite common for the fittest of men to black out when pulling out from a steep dive, and while it is true that the number of *g*'s that a pilot can endure may be increased by varying the position in which he is placed (*i.e.*, prone position as opposed to a sitting one) the limit is only slightly postponed.

Another point which should be borne in mind is that with the advent of extremely high-speed aircraft, the time of engagement of a target in the case of a fighter is reduced considerably and may become so short as to be outside the range of human perception and action. From the preceding arguments, therefore, it may be concluded that pilot control of an interceptor missile is approaching the limit of its utility, and may well have to be superseded by robot control from a distance.

The interceptor is not the only possible type of fighter, although it has received the lion's share of attention up to date. There are other classes which may well assume even greater importance in the near future. It seems, in fact, likely that an escort fighter will be needed to accompany the future bomber on long-range missions, if not, indeed to supplant it, by taking over the remote control of long-range guided missiles. For either duty, emphasis will need to be given to the reduction of drag and specific consumption in the power installations rather than to the attainment of the maximum possible specific output.

Another special case is that of the naval fighter. Here again range is likely to assume a greater importance than it has hitherto, and the addition of this feature to an already difficult specification will tax the designers' ingenuity to the full.

2.1.2. *Bombers.*

A bomber aircraft has to be designed primarily for the function of dropping the largest effective weight of bombs with the greatest accuracy; another consideration, and an important one, is that it must avoid or successfully beat off any interception.

In the beginning of the last war, it was generally considered that high altitude flying was the best means of achieving invulnerability, but during the last stages the tendency was to fly at high speed and very low altitude. For the machine which could not achieve this, a very heavy defensive armament was provided at the cost of the bomb load.

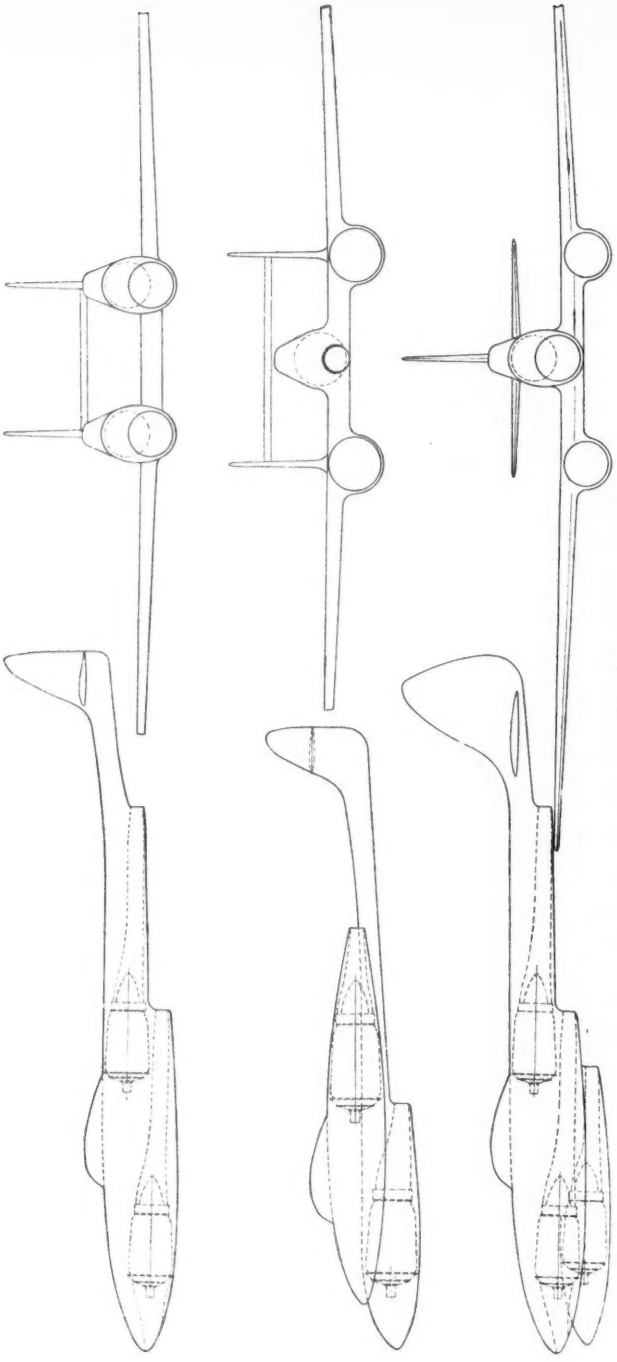


Fig. 1.
Suggested power plant dispositions fighter installation.

Again referring to the atomic bomb, it seems that the future bomber aircraft may have very much greater striking power for a much smaller weight of bombs, and the trend of development may well be towards the design of an extremely fast, but relatively small, aircraft carrying this new type of weapon. Boundary layer control will be one means by which such high speeds can be obtained more economically.

It is therefore likely that bombers as well as fighters may come up against the compressibility barrier, in which case interception by conventional types of fighters would be very difficult.

Clearly, the power plant required for the new vintage of bombers will need the best possible fuel consumption consistent with reasonable bulk (and therefore drag) since power plant weight is likely to be a small fraction of the fuel weight. A change of emphasis in the power installation design features is again indicated, together with the incorporation of measures designed to reduce the overall drag of the aircraft.

2.1.3. *Transport aircraft.*

This last war has shown the immense possibilities of transporting men and materials by air, and future policy in warfare may well be dominated by the supply of such aircraft. The extra mobility afforded by air transport is so valuable, and in any case, the time gained is so large, that emphasis on extreme speed seems unlikely. It will probably be more useful to transport the largest possible number of troops and weight of equipment, and to be able to land them safely on indifferent temporary aerodromes. The aircraft used will, therefore, be long range transports of either low or medium speeds, according to the variety of duty they have to perform. It seems certain that in this field propellers will be the chief means of propulsion, and that they will be driven by piston engines in a fair number of cases, and by propeller turbines in cases where the speed requirement warrants their use.

2.1.4. *Observation aircraft.*

As already mentioned under the heading of fighters, a type of observation aircraft which may play an increasingly important part in air warfare is the specialised machine required for directing a fleet of robot bombers or missiles, thereby combining a far more economical use of man power with greater accuracy. Obviously an aircraft of this kind would have to comply with the same kind of specification as the escort fighter, namely, very long range and the highest possible speed consistent with it.

A slow type of reconnaissance aircraft may still be needed for use at sea where duration of flight may be the criterion rather than range or speed. For this application, the piston engine, either in its present form or compounded, may be expected to survive.

2.1.5. *Trainer aircraft.*

As regards trainer aircraft, a sequence of types will obviously be required to provide step by step training to the most advanced stages. The elementary trainer must of necessity remain as simple as possible, and no important changes in present-day types seem warranted. As training progresses and speeds increase, however, a fairly early change to turbine propulsion would seem desirable, and a propeller turbine may well serve as an intermediary between the elementary trainer and the very high-speed jet engine machine.

2.2. CIVIL AIRCRAFT.

This is a field to which special attention must be given, and detailed consideration will therefore be devoted to the power plants required by all types of civil aircraft. Before doing this it may be as well to review the particular qualities needed for such aircraft.

Undoubtedly, the most vital requirement must be safety in operation. The wide-spread adverse repercussions of recent air crashes should bring home to all of us how supremely important this consideration is, and it is imperative that every effort should be made

in this direction, even at the expense of other qualities.

The next important qualification which a civil aircraft should have is that of economy of operation with low initial cost, since civil aviation has many other expenses to bear—such as airport and servicing facilities, all of which tend to increase the overheads—and uneconomic operation cannot be condoned indefinitely, by subsidising.

It has already been postulated that for economic high-speed long-range services, high altitude flight becomes essential, but we have had more highflown talk than highflown aircraft so far. Some time is likely to elapse before operational altitudes for civil airlines will attain the 35,000 to 40,000 ft. level, and in many instances the use of such altitudes will not be justifiable. The magnitude of the aircraft problems involved is perhaps not generally appreciated, while the air pressurising and conditioning systems alone have yet to be satisfactorily achieved and operated at such altitudes. It is as well, perhaps, to point out that very little flying in large aircraft has so far been done at altitudes in excess of 30,000 ft. and this only with specialised military equipment that would not be acceptable from the civil airline and passenger comfort point of view.

When these points are related to the safety maxim already discussed, it is evident that for some years to come extreme altitude will not be utilised in ordinary commercial aviation.

2.2.1. *Long-range aircraft.*

Since the still air range requirement for trans-Atlantic operation is of the order of 5,000 miles, the weight of fuel required is very high indeed, something like one-third of the all-up weight. This contrasts with a payload percentage in the region of 10 per cent., so it is abundantly clear that the prime requirement for the power plant is low fuel consumption, which consideration is so important as to outweigh almost every other. For this reason alone a limit is placed on the cruising speed attainable, but there are a number of other factors which are likely to restrict trans-Atlantic aircraft to medium speeds (*i.e.*, 300 to 400 m.p.h.) for some time. Among these is the passengers' disinclination to arrive in the middle of the night; a more technical, but equally cogent factor lies in the very great effect which gust load has on these very large aircraft, and the rapid increase of its severity with speed.

These considerations dictate the use of propellers, and the high powers required are a powerful inducement in the direction of gas turbines which are more compact and therefore more easily buried within the wing. Submergence of the entire power plant within the wing is most desirable in this class of aircraft, both because it is so important to reduce drag to the absolute minimum, and also to provide a measure of access to the engine in flight. Complete enclosure of the power plants puts a limit on the size and power of each individual unit, and it will, therefore, probably be necessary on this score to use more engines, a tendency which would be desirable in any case in a type of aircraft where the highest degree of reliability is so necessary. Although in this realm of large aircraft the gas turbine holds a special advantage over the piston engine in that it should be possible to design units of 10,000 h.p. if necessary, without serious technical complications, merely by "scaling up," the foregoing considerations will limit this tendency; *e.g.* the turbine version of the Bristol Type 167 will retain the coupled arrangement used for the piston engine version, although naturally differences will occur in the disposition and method of coupling the power units.

The advent of jet propulsion in this type of aircraft must await a considerable reduction in specific fuel consumption and aircraft drag, at the necessarily high cruising speeds involved. If the aircraft remain large, a solution to the special structural problems imposed by high speed and high altitude will also have to be found.

2.2.2. *Medium-range aircraft.*

While great strides are being made with the development of large and long-range aircraft, it seems reasonable to assume that the volume of traffic on these long routes will only be a fraction of the total number travelling by air on very much shorter journeys. Even the existing network of trans-continental airlines justifies this assumption, and aircraft catering for this type of travel will be more numerous and carry a larger volume of traffic than the large ocean going air liners. The medium-range aircraft, therefore, can be justifiably regarded as the mainstay of civil aviation. Ranges will vary considerably, and although perhaps not the case at the moment, it may well be that special classes of aircraft will be developed to cater for certain routes.

The major class of medium-range aircraft will undoubtedly be required for journeys such as the London to Lisbon run where ranges of the order of 800 to 1,000 miles will be encountered. Here the comfort of the passengers becomes a primary consideration and pressurisation may well be considered worth while. This, however, requires very careful thought, and is intimately bound up with the economics of flying at high altitudes. As regards speed, there is still no necessity for exceeding 400 m.p.h., in that incidental travelling and waiting time at the termini forms an appreciable proportion of the total. The most suitable power installation therefore appears to be the propeller turbine with improved fuel consumption.

Another class of aeroplane which may be required, although probably not to such a large extent as the previous type, is the air liner covering ranges up to 2,000 miles. From the economical standpoint, it will probably pay to fly at altitudes of the order of 40,000 ft., and pressurised cabins will be essential.

The most suitable power plant for this type of aircraft will be either the propeller turbine or the jet engine, but the final choice will depend upon many factors which may vary from route to route. These factors may be enumerated as:—

(*a*) Time wasted on formalities at airports.
(*b*) Incidental travelling time to and from the airports.
(*c*) Overall fuel consumption.
(*d*) Servicing.
(*e*) Prevailing winds at various altitudes.
(*f*) Utilisation of aircraft.

Of these factors, the first two tend to reduce the value of high-speed performance, and the last two enhance it; the ultimate choice of power installation can only be determined by a balance sheet which in turn can probably only be drawn up from actual operational experience.

An excellent forecast, however, is to be found in Mr. Clarkson's* paper at the Gas Turbine Discussions in March 1946. This demonstrates conclusively the economic advantage of the propeller turbine, and the only further comment possible within the limits of this paper is that the turbine consumption figures assumed by Mr. Clarkson, have yet to be attained in service.

2.2.3. *Short-range transport aircraft.*

It has already been mentioned in the preceding section that the likelihood of special aircraft types to cater for individual routes may become necessary. An outstanding example of such specialisation is on journeys less than 500 miles such as the London — Paris run where a high degree of passenger density will make such specialisation an economic proposition.

On such journeys, high utilisation of the aircraft is desirable but this should be referred mainly to the turn-about time rather than to the actual flying time. Over such journeys high speed is not necessarily the prime requirement and this is further influenced by the fact that for short ranges no flying can be economically undertaken at altitudes at which pressurisation becomes essential, since the climb to such altitudes

* "The application of Gas Turbine in its Forms to the Field of Commercial Aviation," by R. M. Clarkson, B.Sc., F.R.Ae.S. *Journal* R.Ae.S., Vol. 50, No. 425.

would take up the major proportion of the flying time.

Such an aircraft type would therefore be required to satisfy the requirements of reliability and economy rather than those of high performance, and propeller power plants are therefore likely to be the choice in this field. The low weight of the simpler types of propeller turbine may more than counterbalance their increased fuel consumption for the short range, thus giving an increase in available payload, in addition to improving the aircraft utilisation.

2.2.4. *Freight aircraft.*

A new conception has arisen in aeronautical development which will greatly influence the overall economics of air transport. This is the freighter aircraft specially developed as the result of war experience for the economic transport of bulky cargo.

Vast possibilities are presented, too many to enumerate here, by such an aircraft type. The opening up of new markets for perishable goods, made possible by the cutting down of transport time, is but one of these possibilities, as is shown by the operation of a freighter aircraft on a regular service from the South of France. This service has resulted in an appreciable decrease in the price of grapes in England.

Again, this type of aircraft can be subdivided into about two main classes: —

(*a*) The trans-oceanic large freighter travelling at moderately high speeds and altitudes, unpressurised except for the crew compartment, and landing at major airports.

(b) The trans-continental medium-sized freighter travelling relatively slowly and at low altitudes, capable of landing and taking-off from small airfields.

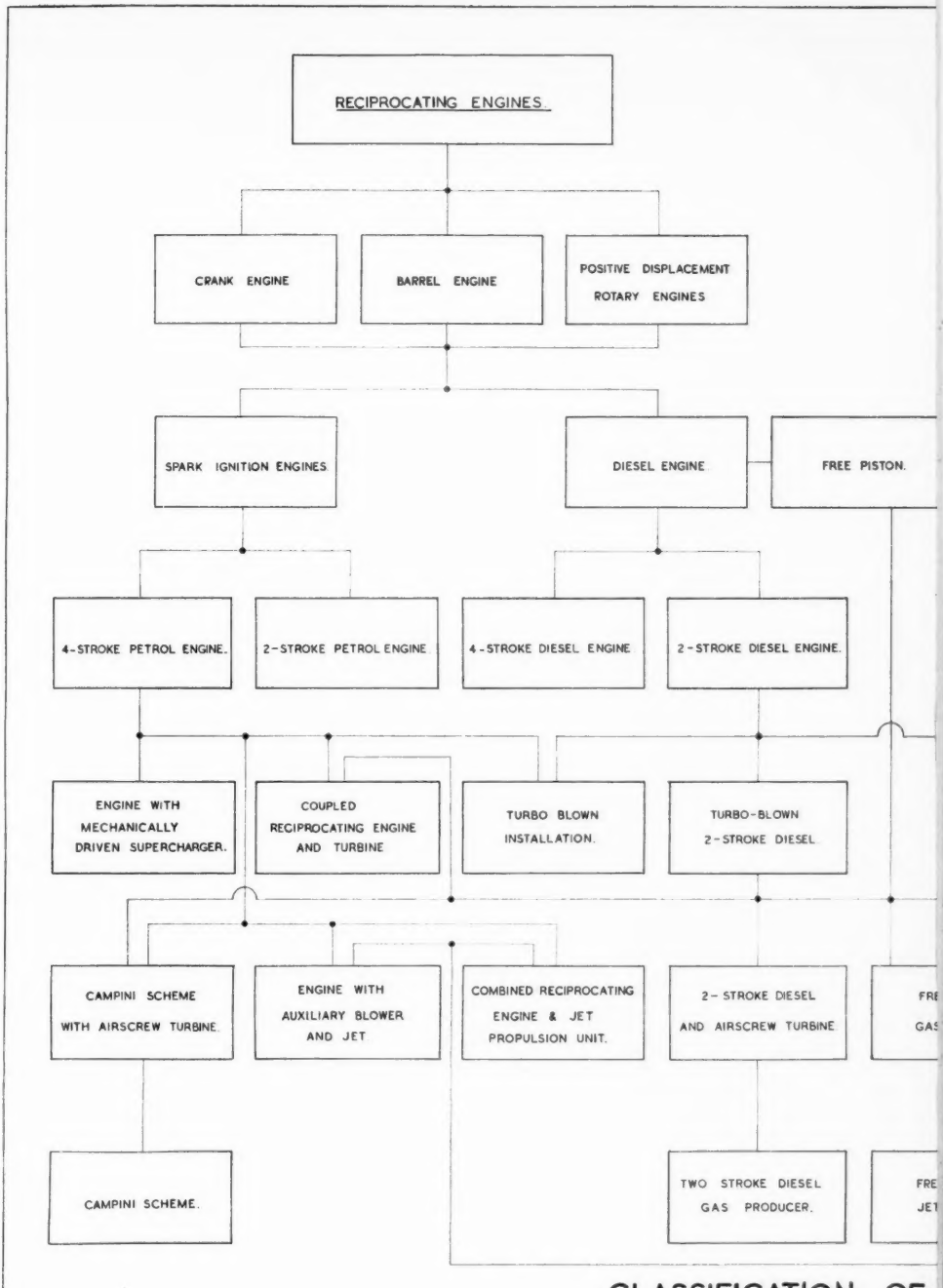
The first class may possibly resemble projected large air liners, perhaps equipped with gas turbine engines, although a version travelling much slower could be used for carrying bulky freight. The second class is primarily a slow machine of robust design capable of carrying all kinds of freight. The essential features of all such aircraft, however, must be low capital and running costs, robustness of design and capability to sustain continuous operation without undue maintenance. The propeller engine combination again appears to offer the best form of propulsion for the vast majority of such aircraft during the period under consideration, with piston engines for the slower classes, and propeller turbines for medium speeds.

2.3. FUNDAMENTAL AERODYNAMIC CONSIDERATIONS.

Leaving aside individual views as to the need for very high-speed transport, and the desirability of overtaking sound, it is indisputable that a very great demand exists for higher speeds. It is noteworthy in this respect that no important aerodynamic improvement has been introduced in order to reach these speeds, apart from the development of laminar flow wings, and increased performance is being achieved by the obvious methods of cleaning up designs and of providing more thrust.

We are now definitely approaching the region of diminishing returns by this method of progress, and if disproportionate increases in drag are to be avoided, some radically new method of approach is imperative from economic aspects. These effects make themselves seriously felt even at speeds of 400 to 500 m.p.h., and it is most desirable to travel at high altitudes if an exorbitant cost in power is to be avoided. This in itself is a great handicap since it means that much power must be expended in reaching high altitudes, and it is only for relatively long-range flights that the climb is justified.

The advent of the swept-back wing, while it offers an attractive means of postponing to higher Mach numbers the occurrence of prohibitive compressibility losses, introduces a new set of control problems, and does noth-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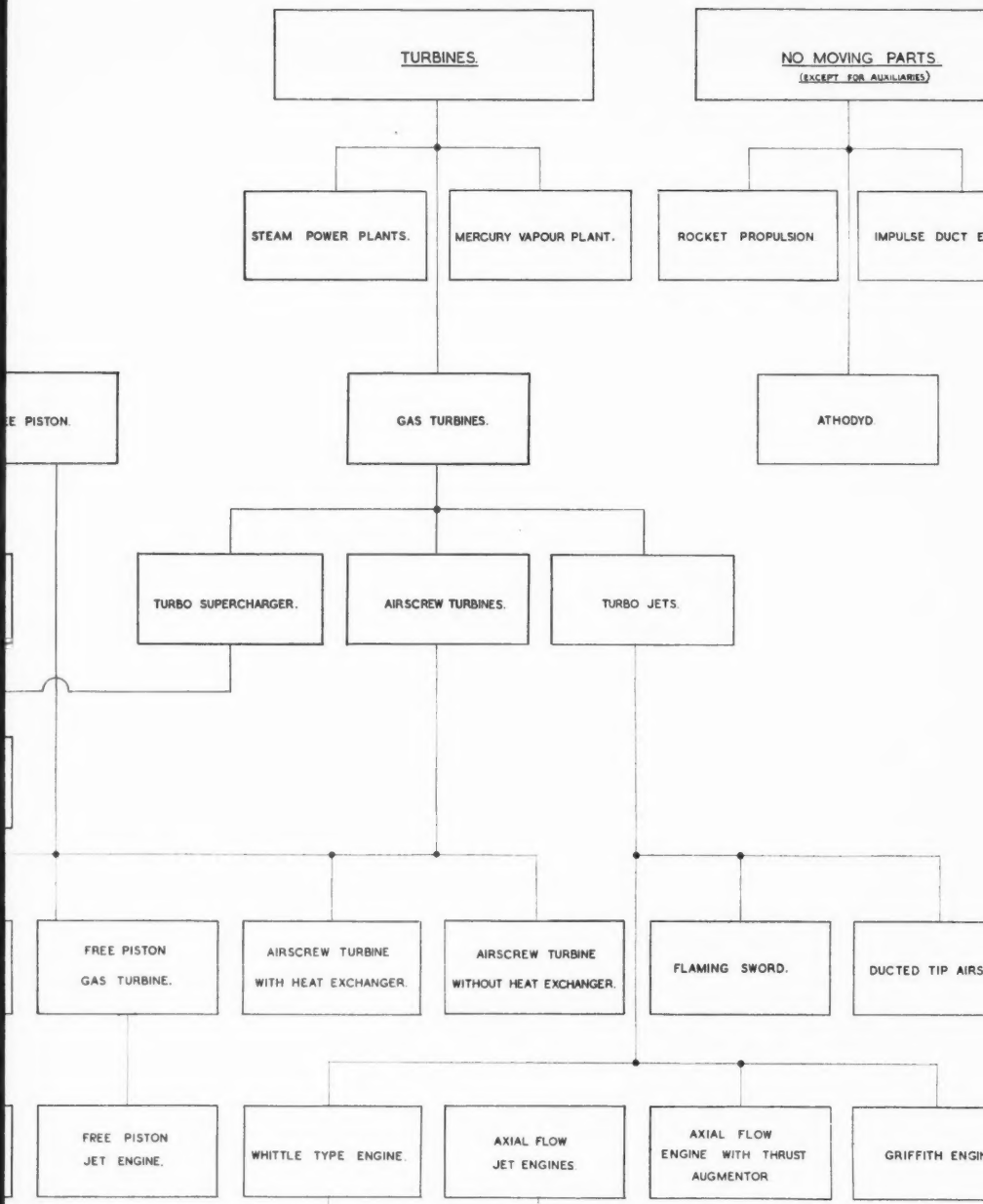
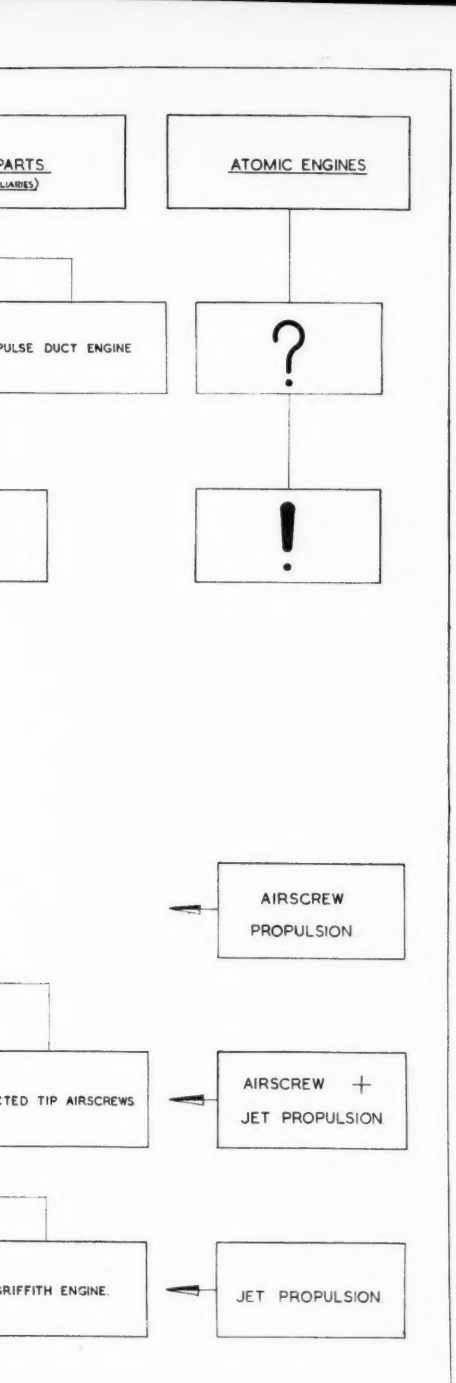
TWO STROKE DIESEL GAS PRODUCER.
CLASSIFICATION OF



N OF AIRCRAFT PROPULSION SCHEMES.

Fig. 2.


PARTS
(LIARIES)
ATOMIC ENGINES
PULSE DUCT ENGINE
?
!
AIRSCREW
PROPULSION
CTED TIP AIRSCREWS
AIRSCREW +
JET PROPULSION
GRIFFITH ENGINE.
JET PROPULSION

ing to reduce the normal penalty of increased speed irrespective of compressibility effects. What is required is a reduction of the total drag, so that higher speeds can be achieved without a disproportionate increase in power provided.

Laminar flow wings offer an attractive means of reducing drag but their applications would seem to be very much limited by their great susceptibility to the most minute surface irregularities. If a remarkable performance is dependent to a major degree on the absence of the slightest surface blemish or scratch, the practical value of an aircraft with such wings is very questionable.

More stable methods are therefore required, and the old idea of boundary layer control seems to offer the most promising approach. It is possible either to draw away a turbulent boundary layer or to accelerate it. Hence the suction and the energising methods each have their possibilities.

The suction method would seem to be the more straightforward, and the easier to realise, but it becomes only a truly practical proposition when combustion turbines are used because the large amount of air which has to be sucked in can more readily be utilised. It could, for instance, be used as combustion air or the exhaust jet could be arranged to operate as an ejector, sucking in large volumes of air through slots in the wings. This could provide not only boundary layer control, but would also reduce the jet speed and increase the mass of the discharge, and might in effect operate as a thrust augmentor. Alternatively, the pumping could be directly produced by a ducted fan, when again a thrust augmentor effect would result.

These and other similar devices offer great possibilities for important reductions in drag with the minimum of complication, but much research and development work will have to be done before the joint aircraft and power plant features are brought to a successful and practical realisation.

3. CLASSIFICATION OF POWER PLANTS

3.1. General review of fundamental characteristics.

Overlooking for the time being the case of the rocket, the principle of all heat engines used in aviation is the same. A gas which may be air, combustion gases, or even, perhaps, steam, is compressed, heated and expanded. The heating may be by direct combustion of fuel or by heat exchanger and in the case of direct combustion, the fuel must be injected and ignited. Subsequently the power produced must be converted to thrust.

The more important methods by which these functions may be accomplished are:—

(a) **Compression.**
- Ram.
- Piston compressor.
- Centrifugal compressor.
- Axial compressor.
- Rotary displacement compressor.

(b) **Injection of fuel.**
- Carburation.
- Pressure injection.

(c) **Combustion and heating.**
- Continuous.
- Spark ignition.
- Compression ignition.
- Heat exchange.

(d) **Expansion.**
- Piston engine.
- Turbine.
- Jet.

(e) **Propulsion.**
- Airscrew.
- Ducted fan.
- Jet.

By combining these methods in various ways, a very large number of forms of heat engines can be conceived and the choice becomes overwhelming if the engine is built up by selecting several items from each group. Thus the expansion may be performed partly in a piston engine, partly in a turbine and partly in a jet. A very large number of

cycles have also been studied at one time or another, but many of them are unsuitable for aeronautical applications because of their complications.

The suitable types can be classified in a number of ways, but difficulties are usually encountered in the tabular layout because the branches tend to tie themselves up in loops. The tree given in Fig. 2 mentions most of the important types of engines, but it is not complete and no doubt it will grow new branches. On the other hand, it carries much dead wood which is included for its historical associations.

Clearly it is impossible to analyse and contrast all these forms, and, therefore, excluding the types which are unlikely to come to fruition within the chosen period, we may classify those with which we are primarily concerned under the headings:—

Piston engine.
Propeller turbine.
Jet turbine.

3.2. Piston engine.

Practically every aspect of the application of piston engines to aircraft has been discussed already, and it is only necessary here to attempt an assessment of the part it is likely to play during the period under review. The main assets of the piston engine are its long background of development, flexibility of operation over a wide range of conditions, and high thermal efficiency; by the use of the constant volume cycle, very high pressures and temperatures can be utilised. The use of propellers has permitted high thrust efficiencies, at least up to speeds where compressibility effects on the aeroplane begin to be apparent. On the other hand, the very length of time which has been devoted to piston engine development means that a stage has been reached where progress has become increasingly difficult, and is in many cases incommensurate with the effort expended. This is particularly apparent in the expansion of the piston engine to very high powers and very high altitudes, in both of which cases the additional complications involved are indeed alarming. The same tendency is apparent in the increasing difficulties surrounding the design of the cooling systems, which are as obnoxious as ever in the losses they incur in throwing good heat away; and the other evils of vibration and noise have by no means been banished.

Nevertheless, the ability of the piston engine to retain its efficiency at low speeds and altitudes where the turbine cannot yet compete, is indicative of the roles it will continue to fulfil for a very considerable time to come. It is clear, however, that we have reached the end of the process of supercharging the piston engine to higher altitudes, and that any accumulation of exhaust energy must be diverted to other purposes. Moreover, since the utilisation of larger jet effect for propulsion is inefficient at the lower speeds, it would seem advisable to investigate other uses for the exhaust energy. For instance it can be used to assist in cooling, providing this can be done with a practicable installation from a noise and safety point of view.

In the heavy duty applications envisaged as the proper future field for the piston engine, it may be advisable to call a halt to the continuation of the process of improving the specific output, if this has a deleterious effect on reliability.

As an example of a modern version of piston engine installations, two applications of the 2,800 h.p. Centaurus engines may be cited. Fig. 3 shows the main features of a single power plant, while Figs. 4, 5 and 6 show how two Centaurus engines are coupled together to drive a pair of contra-rotating propellers, in the Bristol Type 167 power plants.

3.3. Propeller turbine.

The gas turbine as a prime mover has hitherto been discussed mainly in connection with jet propulsion, but although several references have been made to turbine driven

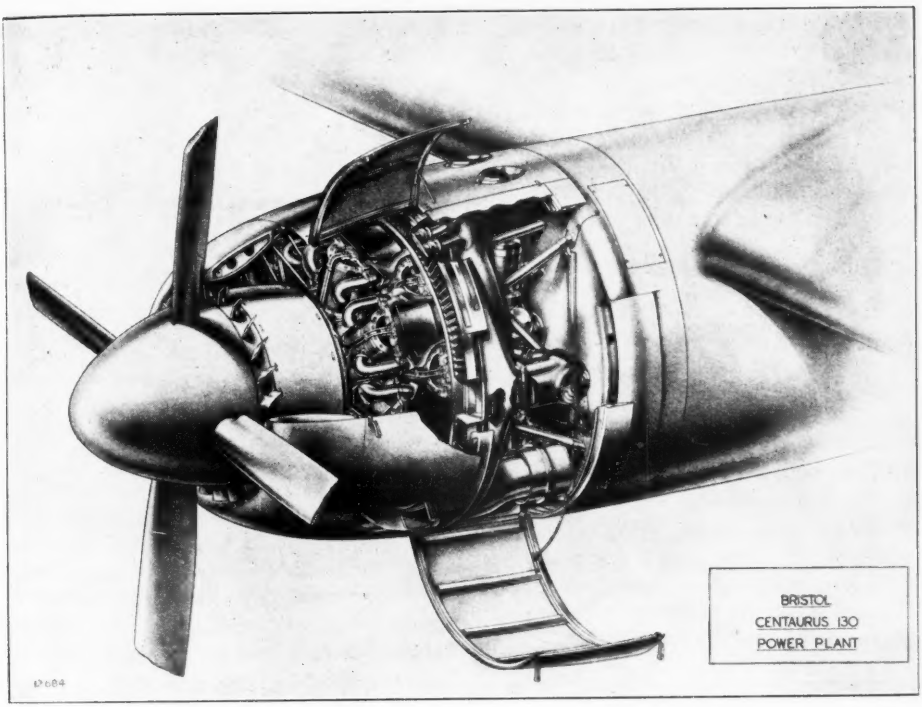


Fig. 3.
Installation of Centaurus 130.

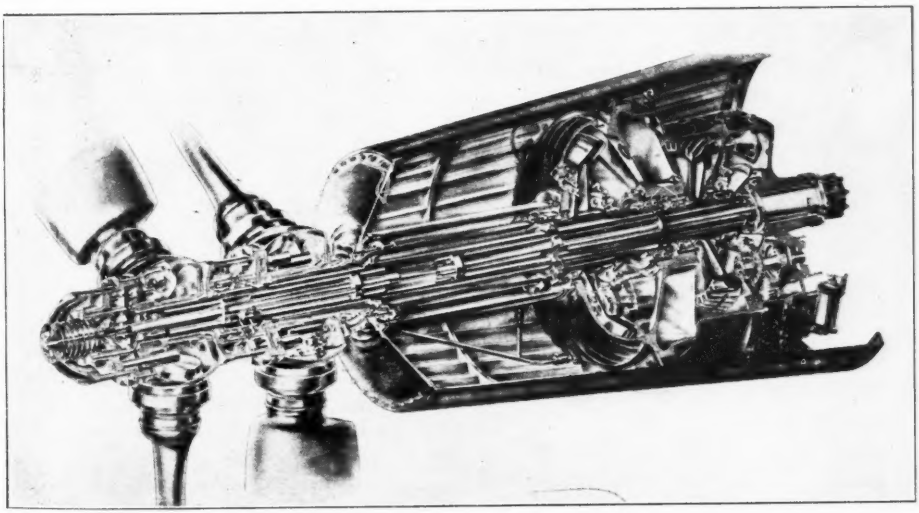


Fig. 4.
Gearbox for Brabazon I

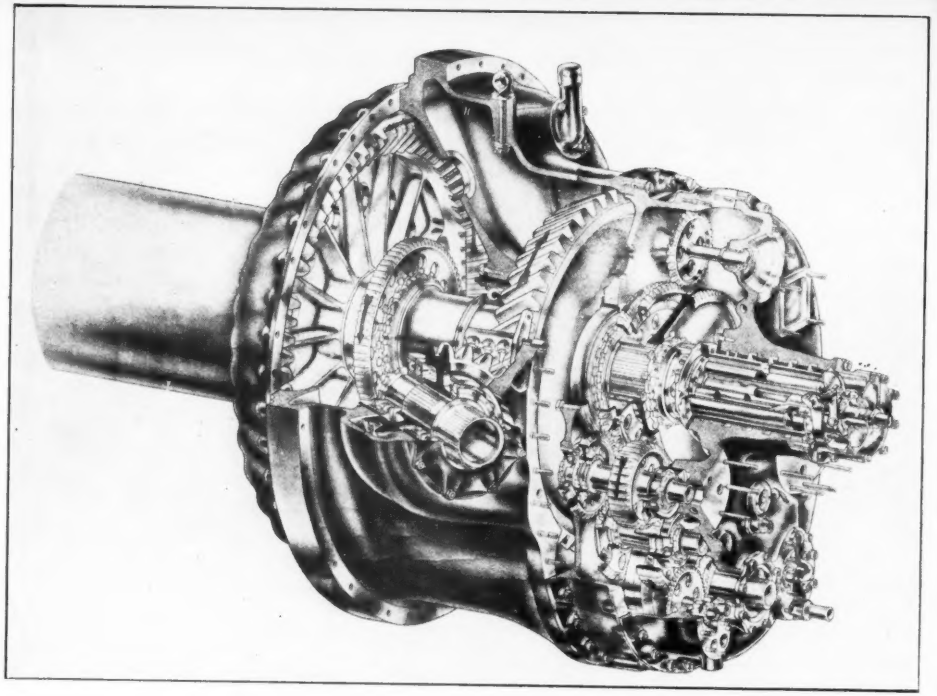


Fig. 5.
Gearbox for Brabazon I.

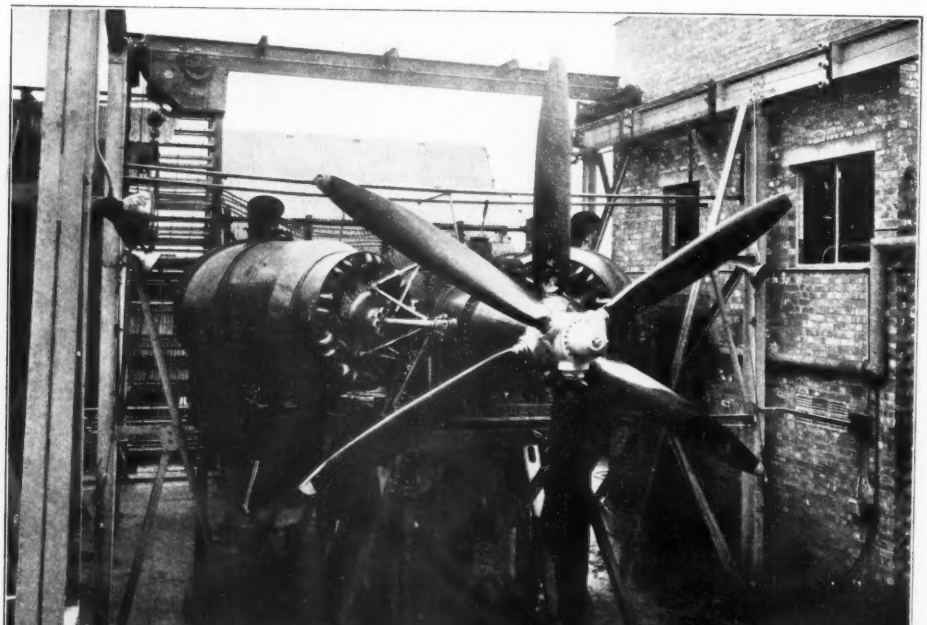


Fig. 6.
Brabazon I. gearbox test rig.

propellers, no detailed review of their design characteristics has yet been published.

While this is the case and although the need for clarification of this subject may be necessary, it is impossible within the scope of this paper to deal with the matter as fully as it deserves. It is proposed therefore to restrict comment on the assumption that the general principles are already recognised.

In the case of the propeller turbine there appear to be two main avenues of improving the fuel consumption.

(*a*) By the use of heat exchangers.

(*b*) By the use of high expansion ratios with correspondingly high compression ratios.

The Bristol Aeroplane Company is following both these lines of development since it is of the opinion that fuel economy is one of the most important considerations.

The Theseus propeller turbine (Fig. 7) is the only aeronautical example as yet in which a heat exchanger is incorporated in the design. The development of high compression turbine engines is also proceeding in parallel.

The absence of vibration which is a characteristic of the turbine engine has a large influence on the design of propellers, and this has been amply illustrated in a recent lecture by Mr. L. G. Fairhurst.* These contentions have already been proved at Bristol where practical strain gauge tests on propellers have been carried out by the de Havilland Company on a Theseus engine and the results obtained show that an immediate and appreciable reduction in propeller weight is possible. Recent information from the U.S.A. also indicates that intensive research is taking place with regard to propellers suitable for high-speed flight and it appears not unreasonable to expect efficiencies well over 80 per cent. at speeds of the order of 500 m.p.h.

It is easy to see that the gas turbine is a strong competitor to the piston engine even in its present form, and when consideration is given to the limitations of the piston engine with regard to higher powers, it is obvious that the propeller turbine has a very strong case.

Since the gas turbine already incorporates a compressor absorbing the maximum power available, it is difficult to see how such an engine can be supercharged, and so at first sight it appears to suffer from the disadvantage that sea level power cannot be maintained at altitude. This, however, can be overcome by designing the turbine for altitude operation thereby providing an excess of power at the lower heights, power which can be usefully employed both during take-off and climb. The fact remains, however, that the gas turbine is essentially a fairly high altitude power plant and cannot be so economically employed at lower altitudes. At the same time, since the propulsive efficiency of a propeller turbine at medium altitudes and speeds is very much higher than that of a jet, the propeller turbine is a more efficient power plant than the pure jet unit under these conditions, the latter engine only challenging it at near sonic speeds and very high altitudes.

As in the case of the piston engine, propeller turbines can be coupled together to provide a suitable power plant for such large aircraft as the Bristol Type 167, and in subsequent marks of this aircraft coupled turbines will be installed driving common pairs of contra-rotating propellers.

As also mentioned in the next section, the jet of the propeller turbine can be used to reduce aircraft drag by assisting in boundary layer suction.

3.4. Jet turbine.

Here again is a field which has been widely covered in past discussions, and it is only necessary to outline the main points by which a suitable assessment of the application of jet units can be made. In addition to this, several features have already been dealt with under the previous heading.

* "The Future Scope of Propellers," by L. G. Fairhurst, A.M.I.Mech.E. *Journal* R.Ae.S., Vol. 49, No. 420.

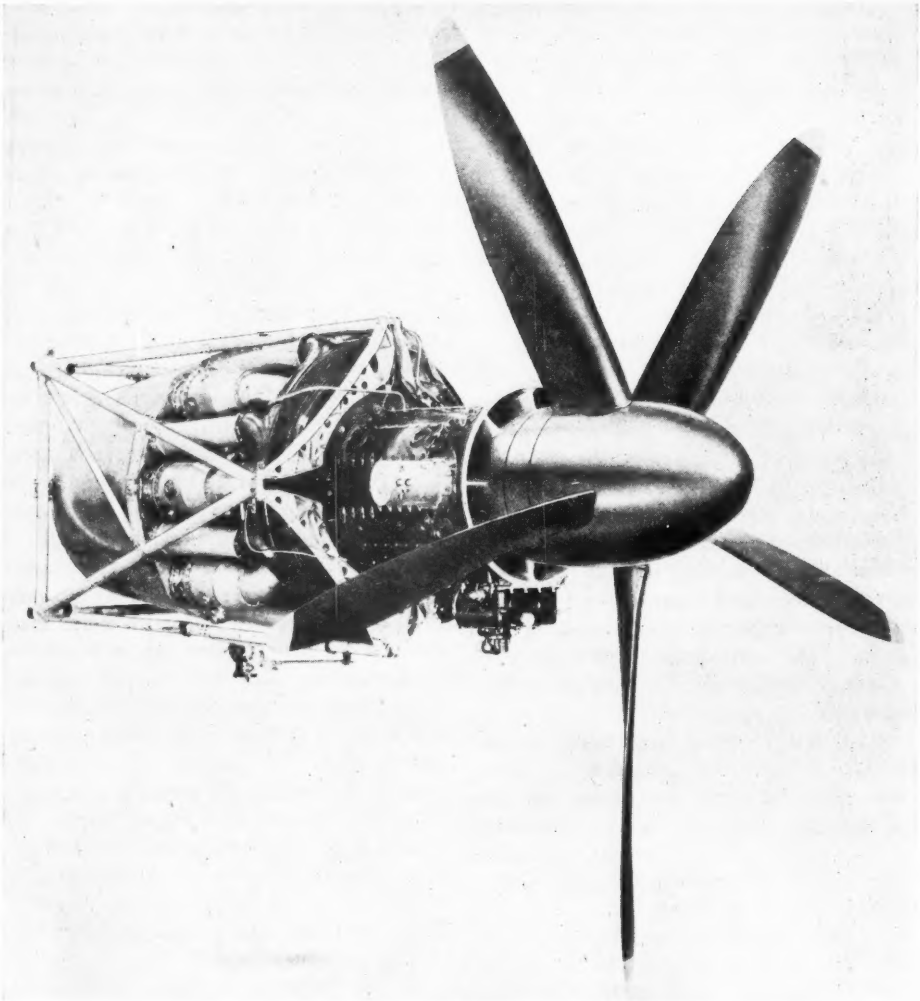


Fig. 7.
The Thesus I Propeller Turbine.

In its own particular field, the jet engine has not as yet been given a square deal in so much as the aircraft designer has not been able to produce a machine capable of utilising the advantages of this form of power plant to the full, and its application has so far been restricted to military high-speed fighters for which considerations of economy have of necessity given way to those of high performance.

Efforts towards improving the thermal efficiency of jet engines must, however, be made, and considerations towards this end are given in a later section.

As mentioned in connection with propeller turbines, the possibilities of reducing aircraft

drag by using the jet engine to suck off the boundary layer are even more attractive and should be given much greater thought than has hitherto been the case.

3.5. Compounded engines.

The idea of a compounded engine has been put forward in many forms, mainly on the score of its very low fuel consumption. While this is an undoubted attraction it is feared that an engine combining both reciprocating and turbine prime movers, may very well suffer from the disadvantages of both without having many of their advantages. A singular drawback of such an engine is its large bulk and weight and this may restrict its scope to a very narrow field. Furthermore it is considered that greater possibilities are offered in the successful development of the high efficiency propeller turbine rather than facing up to entirely new problems, such as would be encountered in a compound engine.

4. POSSIBLE POWER INSTALLATIONS.

Before going on to discuss such important items as installation requirements and the general trend of development, it is as well to review in the light of the preceding arguments what combinations of power plants and aircraft are likely within the period under review, although this has to some extent already been indicated in the preceding sections.

4.1. Military aircraft.

In general terms it is possible to forecast a gradual eclipse of the piston engine within the period under consideration. The future use of the piston engine in this field will probably be restricted to some forms of transport aircraft and initial trainers, since, to satisfy the requirements of extremely high-performance, the advantages of the gas turbine as a prime mover, permitting as it does very much higher powers, are immeasurably greater than those of the piston engine. The solitary exception may be the naval reconnaissance machine where the piston engine will survive for a long time to come because of its ability to cruise at very small fuel consumption per hour, at low speeds and low altitudes. It is possible that in this field the extremely low consumption which the compounded engine offers may warrant the extra weight and complication inherent in this scheme.

4.2. Civil aircraft.

4.2.1. *Long range.*

Here the propeller turbine should prove supreme and its size will only be limited by propeller considerations or room available for stowage within the wing envelope — the latter consideration will probably encourage the development of coupled units. The attainment of the best possible fuel consumption is a vital necessity as is also the utilisation of safer fuel (which the turbine can use). A possible additional type is the very high-speed passenger and/or mail-carrying jet aircraft flying at high altitudes. For this machine to be practical, the jet unit's fuel consumption must be drastically reduced and the aerodynamic efficiency of the aircraft greatly improved. Boundary layer control is likely to enable advance to be made on both counts, but the amount of research work involved is such that the result may not fructify until the close of the period.

4.2.2. *Medium range.*

In this class also the propeller turbine is likely to power the majority of aircraft, but in applications where speed is more important than economy, jet aircraft flying at altitudes of 30,000 ft. to 40,000 ft. may come into service; again, reduction of the jet unit's fuel consumption and aerodynamic refinement of the aircraft will determine what economic penalty, if any, is incurred by the attainment of high speed.

4.2.3. *Short range.*

For the reasons already outlined, the piston engine is likely to retain this market, unless

the lower weight of the propeller turbine can be made to outweigh its increased fuel consumption under the adverse conditions of low altitude.

4.2.4. *Freighters.*

As mentioned in previous sections, the conditions of operation are more likely to be favourable to the retention of piston engines, though with special classes of cargo combining urgency and long distance, the propeller turbine may find an application.

5. INSTALLATION REQUIREMENTS.

5.1. Relating to power plant.

The problems relating to installation requirements have also been discussed on many occasions, particularly in regard to those affecting the engine. A great step forward was achieved in the year or two immediately preceding the war by the industry's concerted efforts to standardise installations, an effort which, while it failed in the major aim, did succeed in establishing the quickly detachable power plant, and, more than this, succeeded in making the whole industry installation conscious. The advent of the gas turbine engine has given relief on some problems, has accentuated others, and has introduced a few new ones; in order to assess these, it is first of all necessary, briefly, to touch upon the piston engine power plant, but only briefly as it has been so adequately covered on many previous occasions.

The prime essential of a successful power plant installation varies somewhat in the different aircraft classifications in that, while the essentials themselves remain constant, the emphasis on any particular requirement is stressed or relaxed as dictated by the aircraft type. The following conditions must be met:—

(*a*) Interchangeability as a unit.
(*b*) Good component accessibility.
(*c*) Low maintenance requirements.
(*d*) Design for economic performance.
(*e*) Good relationship to wings and fuselage.

Out of each of these requirements many sub-sections arise: for instance, the latter two items need a study of the problems relating to aircraft C.G. position, effect on wing aerodynamics, engine cooling, noise level, vibration level, power plant drag, oil cooler location, and consequently undercarriage length, propeller diameter, and so on.

In a military high performance aircraft, such as a multi-engined fighter, emphasis is obviously placed on low body and cooling drag at some expense in the way of accessibility and interchangeability, while in a medium-speed commercial machine the problems of accessibility and maintenance would take precedence over those of drag and performance although it should not be deduced from this that there can be any great relaxation in modern aircraft of the highest possible standards of achievement in any of the qualities referred to previously. This shifting of emphasis to suit the different aircraft types was, of course, the primary reason for the break-down of the attempt at complete standardisation of power plants, but there is no reason why we should ignore the advantages to be gained from standardisation of fundamental components such as cowlings, exhaust systems and engine mountings for medium-speed commercial and military transport aircraft and the like. In this regard the possibilities of boundary layer removal from the wing surfaces which have been opened up by the use of gas turbine engines means that standardisation of power plant types in such highly specialised conditions is unthinkable as the engine is serving not only the purpose of propelling the aircraft but of providing aerodynamic control of the wing surfaces. This subject is touched upon elsewhere in the paper.

Although the piston engine has perhaps reached its zenith in regard to its development as a power plant—it would be invidious to select examples (the Hornet, the Halifax and the new Fury must, however, be mentioned) since we all know the high standard we have now reached—there are still certain

problems which have not been satisfactorily solved and which have been inherited by the gas turbine engineer. These are the problems of fire risk, anti-icing and air filtration and it is felt that an extra attribute of the good power plant should have been inserted at the beginning of this section, namely, safety. Far more attention must be paid in the early stages of new designs to these factors than has been done in the past and it has been observed with great gratification that the responsible authorities are at last making moves in organisation which will ensure attention, founded on adequate operational information, being given to these problems.

It is not proposed to discuss in this paper familiar problems such as engine cooling, vibration, oil dilution and the like as they are familiar to every aeronautical engineer—one suspects at times too familiar to receive due attention—but to concentrate rather on those which will require development to meet future trends in aircraft and engine design.

To begin with that which is perhaps the most difficult; it is regrettable that rather more is not known about the practical aspects of anti-icing now than was known at the beginning of the war despite all the thousands of flying hours which have been done under adverse weather conditions to meet operational needs and despite the great increase in theoretical knowledge. No effective and foolproof system has yet been devised to prevent carburettor (or injector) icing which one would have thought would be the easiest condition to meet when the whole field of aircraft and engine de-icing is surveyed. How then are we to achieve ice prevention of air intake entries and ducts, axial compressor blading in gas turbine engines, of propellers, aircraft wings, empennage and so on?

5.1.1. *Anti-icing and de-icing.*

Several varieties of wing de-icing are at present in use, the three chief methods being:—

(*a*) The ice being broken up after formation by the use of a pulsating rubber bag along the leading edge.

(*b*) Prevention of ice formation by means of fluids such as glycol being oozed on to the surface to be de-iced, through porous rubber or metal strips.

(*c*) The application of heat to the surface.

The latter is the most certain and desirable means of dealing with the icing problem and much work has been done both in the use of heat taken directly from the piston engine exhaust and by the use of a heat exchanger applied to the engine exhaust. Neither method has been successful because of practical difficulties such as high back pressure in the exhaust system, insufficient heat, and mechanical failure of piping and ducting.

The first really practicable proposition has been the development of combustion heaters which although they have not yet been thoroughly proved in operational service are sufficiently reliable as units to be scheduled for use on the prototype Bristol Type 167, where the heat required for the whole aircraft amounts to 3,000,000 B.Th.U.s per hour, 3/7ths of which is for main wing de-icing.

The generally accepted figure for effective de-icing is 1,500 B.Th.U.s per square foot of surface. The gas turbine engine offers a new source of heat for this purpose, perhaps the term " new practical source " should be used, in that the use of a muff or intensifier system on the jet pipe would give all the heat required.

It is intended moreover to use this heat for anti-icing purposes on and in the engine itself and various experiments are being made in an endeavour to introduce the heat uniformly into the compressor system. One method which is about to be attempted by the Bristol Company is to adapt the circular mesh stone guard which covers the entrance to the Theseus compressor by making hollow strip sections through which hot gas can be passed and from which a controlled mixing with the inspired air can be obtained.

The term anti-icing is used deliberately in preference to de-icing because it is felt that every effort should be made towards the prevention of ice formation rather than the removal of ice after it has once formed. To this end further research and practical development is required in the science of determining the conditions conducive to ice formation and of warning the pilot that such conditions are at hand so that he can immediately put into operation the ice prevention equipment. As the same set of icing conditions do not affect wings, carburettors, air intakes, etc., simultaneously, the problem of detection is going to be difficult and will take a long time to develop. Meanwhile, we must rely on the pilot's "feel of controls" and visual observation *after ice has formed* so that he can put into effect the various means of removing ice. For piston engines there appears to be no alternative to the use of existing methods, mainly heating the intake air or using alcohol spray on carburettor and blower elbows. The injector "carburettor" is, to a high degree, resistant to ice formation, but under extreme conditions still requires assistance from hot air or alcohol.

The establishment of the Winterisation Committee is a very long step in the right direction, but more support is required for the testing in Canada of non-operational types (*i.e.*, civil types) and for testing individual engines on the ground. It is considered that support from PICAO for the establishment of an aircraft station operating several representative types of civil aircraft in Canada with sufficient equipment, workshops and technical staff to investigate the problems on the spot, would constitute a sound practical step towards the solution of a problem which is almost universal. This will cost a lot, but it should not be impossible to secure funds for the purpose of dealing internationally with this and similar problems.

It is desired particularly to mention a restricted report by J. K. Hardy, of the R.A.E., which contains an admirable exposition of the whole problem.

5.1.2. *Fire risk.*

Fire prevention and control is now much more fully understood and, recently, issues of aircraft design memoranda from official quarters show that energetic attention is being given to this most unpleasant problem. It can not be too strongly emphasised, particularly in the commercial field, that where other installation problems interfere with the fire control arrangements, the former must suffer and precedence must be given to the latter.

The use of kerosene as a fuel introduces a new set of fire prevention problems despite the obvious advantages of kerosene over petrol. Its tendency to creep from what appear to be tight joints and the problem of explosive vapours forming in fuel tanks will need their own safety measures. There are, however, developments already in progress to introduce inert gases over the surface of the liquid in fuel tanks and this system will obviously be useful with kerosene, as this fuel under certain conditions of temperature and pressure is more liable to form an explosive mixture than petrol, since the latter produces an over-rich atmosphere. Nevertheless the use of kerosene is a great advance in safety as the rate of flame propagation is much less than that of petrol and in the case of accident more time for rescue work would, it is believed, be available before the spread of fire interfered with the rescue operations.

5.1.3. *Air filtration.*

The war in the desert directed urgent attention to the problem of efficient air filtration on piston engines, but, here again, although no criticism of the various filter manufacturers is intended, no solution better than the merely adequate has been found. The problem was undoubtedly beginning to

receive proper attention when the cessation of hostilities relieved the necessity for further intensive work to meet military requirements. The industry would, undoubtedly, welcome a classification of the results obtained from the many official reports and investigations which were made during the war on various locations of air intake, various types of air filter material, various methods of filtration and the like, and this might form the subject of a separate paper. It is certain that the civil airline operators should not be asked to provide all this information over again by their own bitter experience in view of the mass of data which should be available.

If the problem of air filtration in the piston engine is large, that in the gas turbine field is greater. The air mass flow is so great that no filter material within any reasonable area would cope with it without considerable pressure drop, even though some sand grain size larger than that now being filtered were to be adopted to obtain a coarser mesh material. It is, therefore, evident that the only effective way of dealing with this is to make a turbine engine "eat sand and like it" and in this regard it is understood that a "centrifugal" jet engine has performed quite happily under desert conditions with little apparent deterioration due to sand erosion.

The axial compressor type of turbine engine will need special attention as there are two problems to be dealt with, namely, that of blade erosion and that of loss in performance due to blade fouling by dirt adhering to the blade surface and spoiling its aerodynamic form. The first of these problems may be met by using hard blades, say steel or bronze, instead of dural and the second problem may be dealt with satisfactorily by preventing oil leakage into the compressor, thus providing no sticky surface to which the sand could adhere.

Propeller turbines having an annular air entry around the spinner, such as the Bristol Theseus, are likely to be good in this respect due to the fact that the entry is not usually in a region of high density sand concentrations; but a note of warning must be sounded as it is known that there is an inward flow of sand from the tip of the blade towards the hub which leaves the blade in highly concentrated fashion somewhere about two foot radius and it is necessary to design the outer perimeter of the air entry annulus to fall within this region. It is also important to ensure that oil and de-icing fluid do not leak from the propeller into the intake. The normal type of entry ducted from the wing leading edge should be good as this portion of the propeller disc is normally free from sand and, in any case, the centrifuge effect of the bends in the duct should, if properly used, throw out most of the larger grains of sand which give the worst erosive trouble.

Turbo jet engines having axial compressors will suffer more severely than the propeller turbine type as the low undercarriage and lack of propeller cleaning will emphasise the difficulties.

The problem of oil and fuel filtration needs attacking far more on turbine engines than has been the practice hitherto since, with regard to oil, the high-speed bearings simply will not stand up to a dirty lubricant and neither will the high-speed pinion bushes of the reduction gear of a propeller turbine. The fuel system must also be meticulously clean as the effects of dirt in the burner atomisers can be disastrous.

5.1.4. *Starting.*

The piston engine employs, chiefly, the electric motor and the cartridge starter systems and will, it is believed, continue to do so although two-stroke motor starters and the electric inertia starter—to mention but two varieties—have been designed and tried without making much headway. The turbine engine starting system is still far from being satisfactorily developed as an entirely different set of conditions are

presented. Essential requirements are the ability to accelerate the compressor system smoothly on starter motor alone until the fuel injection stage is reached when the starter will assist the engine's own gas system to continue this acceleration to the point where the gas system can take over and the starter motor then cuts out.

In the case of propeller turbines, with the exception of the Bristol Theseus, it is necessary also to accelerate the propeller as it is coupled directly to the compressor system. Consequently, there is a very high drain on batteries if an electric motor system is used and a very high, but smooth, "punch" is needed from a cartridge starter or similar system. The Bristol Theseus engine (Fig. 8) shows a decided advantage in that the free power turbine system employed relieves the starter of the necessity of turning the propeller, the compressor system only being accelerated, and the propeller picking up smoothly as soon as surplus power is generated by the turbine system. This advantage will be more keenly appreciated under low temperature climatic conditions owing to the fact that high viscosity oil drag in the reduction gear may have a considerable retarding effect on direct coupled propellers, but this will not be observable on the free power turbine engine.

The electric system is, nevertheless, unsuitable for military aircraft, particularly fighters, owing to the extremely rapid time cycle required for starting—namely five seconds, and the fact that the aircraft must rely to a great extent on its own batteries owing to aerodrome dispersal. Commercial users will not find the same disadvantages in the electric motor system of starting as ground electric supply should be available and the duration of the cycle is unimportant, within reasonable limits. In large aircraft such as the Bristol Type 167 the miniature power station carried for general services makes electric starting almost a *sine qua non*.

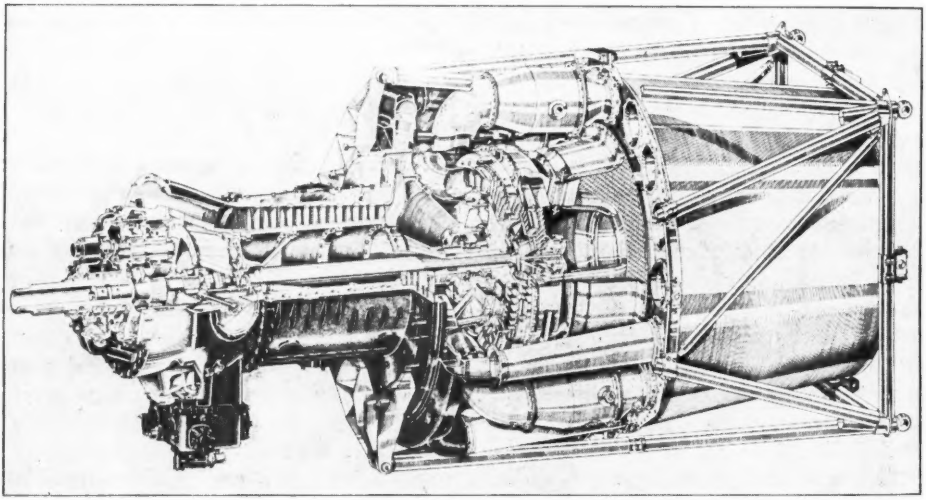


Fig. 8.
Perspective cut-away drawing of the Bristol Theseus I Propeller Turbine.

The development of a satisfactory starting system is not going to be an easy job, since if it were, this would have already been shown. It is, however, hoped that a slow burning cartridge system employing a compact high-speed turbine as the power transmitter will

be eventually adopted in preference to any form of reciprocating starter motor employing petrol as fuel. It will indeed be a retrograde step after having adopted a safe fuel for main engines to introduce petrol for the purpose of starting those engines, apart altogether from the nuisance of having to provide additional fuel tanks and pipe systems—small though they may be in proportion to the main systems.

Although the slow burning cartridge system is advocated here, it only receives this support because the electric system is inadequate for military needs and should, therefore, be used on commercial aircraft to avoid duplication of effort; it is not considered to be, in principle, better than the electric starting system.

5.1.5. *Engine cowling.*

The engine cowling system will present no new problems as far as can be seen, although the absence of vibration and internal air pressure should permit a lighter structure to be used than is at present necessary for piston engines, both liquid and air cooled—although there must still be sufficient strength and rigidity to prevent damage due to careless handling. There must, of course, be some flow of air through the cowl to provide continuous ventilation of all compartments, but this nowhere approaches the pressure required for piston engine cooling. It will be necessary to provide a small panel in the upper and lower sections of the cowl which automatically open up when the aircraft comes to rest so as to avoid unduly prolonged heat soaking within the engine compartment after shutting the engine down.

5.1.6. *General.*

In considering the installation of propeller turbine engines there are many factors affecting piston engines which are happily absent, such as cooling gills and radiator installations, cylinder baffles, liquid cooling systems, together with the need for high tension ignition maintenance. Many other factors are of course present, the engine location and undercarriage length being, as with the piston engine, fixed probably by considerations of propeller diameter rather than by aerodynamic factors. The effect of the propeller slip-stream on wing lift still has to be taken into account. Oil coolers will remain for a long time yet, but these will be much smaller than those required for a piston engine, power for power, due to the lower heat dissipation required in propeller turbines and still lower in turbo-jets consequent upon the elimination of the reduction gear. This may be disappointing to those who have thought that the gas turbine engine needed no oil cooling, but it is not yet established that air cooling for high-speed bearings is as satisfactory as oil cooling and until this can be established beyond shadow of doubt, the oil cooler must remain. Engine vibration is absent entirely and care must be taken to see that vibrations due to propeller buffeting against adjacent aircraft structures are not set up by bad layout.

The noise level of propeller turbine engines seems to be less than that of an equivalent piston engine, but this has to be proved in flight. It is nevertheless a fact that less fatigue is apparent in the test staff when testing propeller turbine engines than when testing piston engines due to the elimination of vibration and the reduction of noise. It is therefore reasonable to assume that there will be a corresponding effect on aircraft passengers.

The problem of disposal of exhaust efflux is a considerable one, both in piston engines and in propeller turbine engines. During the war the Bristol Company, in order to meet demands for glow suppression, improved engine cooling and a cleaned-up power plant exterior, introduced an exhaust system whereby individual pipes were taken from the cylinder and ejected their gases in the cooling gill annulus. This system has proved

far too noisy for passenger carrying aircraft under peacetime conditions and a reversion has been made to the manifold exhaust collector which successfully eliminates the staccato bark of direct individual exhaust pipes. The propeller gas turbine exhaust efflux is inherently a smooth roar, but the problem of taking away the exhaust jet pipe rearward through the wing is by no means an easy one to solve unless the aircraft nacelles are dropped so as to allow the pipe to pass below the main spar; this solution is also considered to have aerodynamic advantages in that it interferes less with the wing lift than does a high nacelle.

The fact that such a small proportion of the total propulsive effort is derived from the jet pipe in a propeller turbine engine means that it is possible to take greater liberties with the shape of the pipe than with a turbo-jet engine. For instance, in an inboard nacelle where the jet pipe must clear the undercarriage system, it is possible in the former type of engine to bifurcate the pipe quite sharply and to permit the gas to emerge from a rectangular aperture in each side of the nacelle at a relatively large angle to the direction of flight. As the exit velocity is subsonic, it is desirable to control the area of the jet pipe orifice so as to maintain a constant pressure ratio condition throughout the engine at various altitudes and this should be a fully automatic system. The simplest means of achieving this would appear to be to use a pair of bellows capsules to register the pressure difference across the turbine and to operate, by means of hydraulic or solenoid switches, an actuator motor which would open or close the orifice as required.

In passing it may be said that this problem has been met in the coupled turbine version of the Bristol Type 167 aircraft by matching the engine mounting and air duct system with the design of the wing spar proper, as a result of the extremely close collaboration which is possible by having both engine and aircraft designed in the same firm.

5.2. Relating to aircraft — auxiliary services.

It has been Bristol policy since 1935 to segregate the ancillary equipment not concerned with the direct functioning of the engine and locate it remote from the power plant, on an auxiliary gear box mounted separately and, where possible, behind the bulkhead in order that power plant changes may be accomplished with the minimum of disturbance of the aircraft services. The continuance of this policy will permit a wide variation of the services necessitated by the differences in aircraft types, without needing corresponding alterations of power plant.

The following auxiliary services are required:—

(a) Hydraulic—for operation of undercarriage flaps.

(b) Pneumatics—instruments, flaps, and other possible applications.

(c) Electrical services—lighting, radio, undercarriage, and other possible applications.

(d) Additional systems—air conditioning, pressurisation.

Items (a) and (b) are the conventional systems employed, and need not be discussed in this paper.

5.2.1. *Electrics.*

Electrics in the small and medium aircraft may absorb only a small fraction of the auxiliary power load, and except perhaps in the case of all-electric aircraft, may be adequately dealt with by the existing gear box system. However, in the case of large aircraft, particularly those employing electric power actuation of controls, electric power loading may assume very much larger proportions, giving rise to additional problems of speed control and the like. Further reference to this case will be made later.

5.2.2. *Air Conditioning.*

Refrigeration can be obtained by the intermediate cooling and expanding down to cabin pressure of air obtained from the main compressor system and further compressed in a separate stage. The power obtained from the expansion of the air is utilised in a turbine to drive the separate blower necessary to complete the compression. Such equipment should be easily removable from the aircraft as a complete sub-assembly if not required. Humidification of the air may be effected by injecting a quantity of water into the second stage of the main compressor. A system for metering such small quantities of water is already in use in reciprocating engine power plants, and its control may be arranged to be either manual or automatic.

For tropical use, dehumidification is achieved by utilising the refrigerating system to cool the air below the dew point required in the cabin, when the requisite amount of water vapour is deposited, the air being re-heated to a suitable temperature.

5.2.3. *Pressurisation.*

When it is realised that to reproduce G.L. conditions in an aircraft at 20,000 ft. a compression ratio of 2:1 is required, at 30,000 ft., 3:1, at 40,000 ft., $5\frac{1}{2}$:1, and at 50,000 ft., $8\frac{1}{2}$:1, it will be seen that the magnitude of the problem of pressurising aircraft to these altitudes is indeed large, and perhaps not fully appreciated. It is clear that the step from the present experimental civil altitudes of the order of 25,000 ft. or so, to operational altitudes of 40,000 ft., represents a large advance in pressurising equipment and technique, not to mention the multifarious aircraft problems involved.

For compression ratios much in excess of 2:1, the conventional displacement compressor is unsatisfactory for aircraft installations, and since a wide range at high efficiency is essential with the larger powers involved with these high altitudes it would seem that the centrifugal blower becomes the only suitable type. This is more evident when it is realised that a centrifugal blower operating at normal output up to 40,000 ft. has to accommodate intake volume flows varying by 400 per cent.

To meet the wide variations of conditions of pressure and mass flow imposed by these altitude requirements, a variable gear system is required. Ideally an infinitely variable speed mechanism is indicated, which may, it has been suggested, be achieved by the use of a fluid coupling. However, this system can be shown in its practical form to be wasteful of power, since it necessitates additional means for dissipating heat generated by the slipping coupling. Such a system would require considerable development.

A more direct approach is perhaps the use of a multi-gear change speed system utilising the existing technique of supercharger two-speed clutches in suitable combination. It has been shown that a four-speed system should satisfy the demands of a 40,000 ft. cruise altitude, where a compression ratio of at least $4\frac{1}{2}$:1 is required, and such a ratio can best be achieved for this purpose by means of a two-stage centrifugal blower. This type of compressor has a wide range of operation and it is envisaged that three main categories of compressors can be arranged to cover the range of airflows required for most of the future aircraft. Thus, for large air liners of the 100 passenger type, duplicated 80 lb. per minute compressors are suitable, while for the medium size, 40/50 passengers, a similar 30 lb. per minute system can be used, and the smaller aircraft can be catered for by a 15 lb. per minute compressor. Intermediate sizes can be satisfied by these three types without fundamental changes.

5.2.4. *Alternative auxiliary drives.*

A difficulty arising with the larger aircraft, in which the tendency is to use A.C. electric generators, is that of reconciling the varying engine and propeller speed demands with those of constant speed operation required by

the alternators. To this end it has been suggested that an A.A.P.P. be provided for all the ancillary equipment, and for the above reason, and others, it is especially attractive. However, in the case of large aircraft, the difficulty of providing powers of the order of 250 h.p. at 40,000 ft., which implies a potential G.L. power of about 1,000 h.p., and providing it economically, has yet to be solved. Moreover, the need for a variable speed drive to the centrifugal compressor will remain.

In passing, an alternative method of cabin pressurising by means of air bled from the main compressors has been proposed, and while in small aircraft this may be possible due to the very small percentage of air required, in the large installations amounts considerably greater are involved with consequent marked effect on individual turbine performance. Additionally, it is extremely difficult to avoid contamination of such air, and variations of supply due to fluctuation in engine demands must accrue. This system is considered, therefore, to be unsuitable.

In the case of turbine engined aircraft, it has also been suggested that the difficulty may be overcome by tapping off hot gas from one engine, to drive a separate turbine for these auxiliaries. This, however, introduces problems of main compressor and turbine matching, which are aggravated by the fluctuating demands, together with the added complication of high temperature ducting and throttle valves.

It would seem that the auxiliary services will for some time to come be driven from the conventional shaft-driven gear box and the difficulties of constant speeding A.C. alternators be dealt with separately, perhaps by the inclusion of a separate constant speed device. In turbine installations having separate propeller drives it might be advantageous to take this shaft drive from the compressor system, in which case the variation in speed of the alternators is reduced, and consequently it may be possible to delete the separate constant speed device for the electrics. However, other difficulties arise with this scheme, and it is felt that the best solution is to utilise the propeller system as the power source.

Since those concerned with gas turbine design are accustomed to thinking in terms of high altitude, and are continually trying to improve engine performance, the tendency is to assume that suitable aircraft with pressurised cabins will be available when needed. Little imagination is required, however, to envisage the vast engineering problems which beset the aircraft constructor in furnishing a commercial aeroplane with certificate of airworthiness, covering operation at the altitudes required to suit the characteristics of the jet engine. The effect on payload may be serious when it is considered what additional structure weight will be involved in withstanding the pressurising conditions, and the weight of the auxiliary equipment to maintain those conditions, and the development time to achieve the proposed height, say, of 40,000 ft., or over, may well extend over the ten-year period under review.

What the incidental problems may be are not known, but some can be anticipated, such as noise of the plant, leak drags, temperature and pressure effects on structures, controls, and accessories.

The object in posing these problems is not to detract from long-range planning, but to plead for sane progressive effort in surmounting the technical obstacles, stage by stage.

6. GENERAL TREND OF DEVELOPMENT OF POWER PLANTS.

6.1. Piston engines.

It is not proposed to discuss at length the general trend of development of the piston engine since there is little to add to previous forecasts. Since, however, it is most important that development on this form of prime mover be pursued without any relaxation of

effort on the part of the designer, it is considered desirable that the main items be enumerated and briefly discussed.

6.1.1. *Fuel economy.*

The fields of application to which the piston engine will be confined during the period under consideration will cause particular emphasis to be laid upon fuel economy. This will naturally lead to the use of fuels of high octane value in conjunction with high compression ratios. Subsidiary developments are to be expected in improved combustion chamber design, mixture distribution, cooling and supercharger design.

For high altitude cruising the application of turbo-blowers has received much consideration in the past. In view, however, of the serious complications which arise in the installation of these units and the absence of any serious development effort to date in this country, it is not considered likely that a suitable turbo-blower installation for high altitude work in civil aircraft will be produced in time to compete with projected turbine engines.

Arising from the turbo-blower, of course, there is the ultimate development of the compound engine in which the reciprocating section becomes the gas generator. This engine and its particular applications are discussed elsewhere.

6.1.2. *Reduction in drag.*

The main sources from which we may expect a reduction in piston engine power plant drag appear to lie in cooling refinements and the utilisation of exhaust thrust. Particularly in the case of air cooled engines it is considered that there is still very considerable scope for development towards the efficient utilisation of cooling air. Coupled with this is the possibility of using exhaust ejectors quite apart from the utilisation of exhaust thrust in the normal sense.

A third factor in drag reduction is of course the geometric form of the engine itself. It is not, however, considered that we are likely to see any revolutionary designs from this point of view, since the mechanical developments involved are likely to be such as to render them unwarrantable at this stage in the history of the piston engine.

6.1.3. *Economy of operation.*

Development in this direction is perhaps almost as important as fuel economy, and if piston engines are to continue to compete with the turbine, it is essential that efforts be made towards the simplification of the engine as a whole and of its installation. This affects not only the first cost by easing production problems, but affects maintenance also. Particular attention should be paid in the installation to the necessity for rapid engine changes. Additionally, overhaul periods must be increased.

6.1.4. *Specific output.*

The developments under this heading are closely related to those concerned with fuel economy. There are, however, the additional factors of methanol-water injection and increased rotational speeds. When considering the latter expedients, however, it must be borne in mind that take-off power must have a direct relation to cruising power under all normal conditions in the future. An exception may occur where runway lengths are restricted.

6.1.5. *Safety and reliability.*

The most important development to be followed with regard to safety is the use of safety fuels and direct injection into the cylinders. The intermediate step of direct injection with normal fuels is already well under development. Improved reliability will follow from design refinements, better methods of inspection and manufacture and refinement in materials.

6.2. PROPELLER TURBINE ENGINES.

In dealing with the propeller turbine engine it must be remembered that, unlike

the piston engine, it operates at maximum efficiency under full load conditions. Therefore for maximum economy of operation its cruising speed will be 100 per cent. of the maximum r.p.m. except in cases where altitude is limited by other factors, such as pressure cabins.

This feature gives rise to two special problems, namely, reliable operation at high rotational speeds and the selection of suitable gear ratios for the propeller in order to avoid excessive noise at altitude while providing adequate take-off thrust at ground level. This latter difficulty is largely overcome in the case of the Bristol Theseus engine which employs an independent power turbine, thus allowing a relatively wide variation in propeller speed while keeping the compressor speed constant. The alternative to this, should future experience prove propeller noise to be a major problem, is the use of two-speed reduction gears.

The more specific items of development are discussed under the same headings as have been used for piston engines.

6.2.1. *Fuel economy.*

To obtain maximum fuel economy from propeller turbines the operator will turn towards a cruising condition of full power at operational ceiling since forward speed and low air temperature are essential factors.

Apart from increasing the maximum turbine entry temperature, there are two basic means of improving the fuel economy of these engines, namely, increasing the compression and expansion ratio and/or heat recuperation from the exhaust gases. Both these methods are under development, and in very general terms the conclusions to date are that the use of heat exchangers or regenerators results in an engine which is less heavily stressed but is considerably more bulky and of rather higher specific weight than the high compression engine.

Increasing compression ratio while retaining a heat exchanger is a development which can only be pursued to a limited extent since the temperature difference between charge air and exhaust diminishes as compression ratio is raised. Where heat exchangers are concerned it is considered probable that development will tend towards high thermal ratios in conjunction with low compression ratios.

The importance of development to improve component efficiencies on the propeller turbine cannot be over-emphasised, particularly in the case of high compression engines. As an example of the importance of this it may be mentioned that an improvement of 1 per cent. on the compressor or compressor turbine efficiency of the Theseus engine would result in an improvement of 3 per cent. on the specific fuel consumption. This effect will be even more pronounced when higher compression ratios are used.

With the improvement of the general aerodynamic efficiency of the engine in mind it will be necessary to devote much development work to the airflow in intake ducts, diffusers, combustion chambers, etc. So far as the combustion efficiency of the latter is concerned extremely good results have been achieved already and only small improvements here can be expected.

The relative insensitivity of the gas turbine to fuels opens a new avenue of development, and research will be directed towards the production of fuels having a high calorific value per pound for use on long-range aircraft. Alternatively, there may be applications, in cases where tank space is restricted, for fuels having a high calorific value on a volume basis.

6.2.2. *Reduction in drag.*

The introduction of the gas turbine brings about an immediate reduction in power plant drag due to the smaller overall dimensions of these engines, and to the small amount of cooling that is necessary. Development in this direction, therefore, will tend towards further reduction in frontal dimensions to enable the

power unit to be completely submerged within the wing, improved intake design and refinements to the internal cooling of bearings, discs and blades.

The design of cooling air ducts for jet pipes will also require development, and it is anticipated that this cooling may ultimately be done at no cost in drag. Alternatively this cooling air may be ducted along the leading edge of the wing or admitted to the air intakes for de-icing purposes.

6.2.3. *Economy of operation.*

While it would not be true to say that turbine engines at the moment are any cheaper than reciprocating engines, it is nevertheless a fact that they are basically simpler and will ultimately lend themselves to cheap production. The most serious obstacles in our way at the moment are the high cost of the special materials which have to be used in the turbine, together with the high cost of forging and machining these same materials.

This applies in particular to turbine blades and extensive development is required both in the forging and machining of these components using materials which by their very nature must be difficult from this point of view. It is perhaps ironic that this development must be pursued very vigorously in order to produce turbines during the next five to ten years, while we know that ultimately it will be necessary to use materials which will operate at appreciably higher temperatures than are in use at the present, and accordingly may become impossible to forge. This will lead to the use of cast blades either in metallic or ceramic materials.

It will be necessary to apply the lessons we have already learned on the installation of reciprocating engines to the turbines and to take full advantage of the relative simplicity of the latter to give improved accessibility and to reduce the time for servicing or changing an engine to a minimum.

So far as overhaul periods are concerned, it is not yet possible to give any figures based on experience. The Theseus, however, was designed with an ultimate target in mind of 1,000 hours before major overhaul. The principal factors affecting this period will be blade and disc creep, bearing life and combustion chamber endurance.

In considering these factors the specific performance of the engine must be borne in mind since the demands of performance will necessarily lead towards higher temperatures which in turn will increase the creep problems, and the fuel saving resulting from higher temperatures may economically counterbalance the shorter overhaul period.

The turbine engine should show an appreciable economy over the piston engine so far as fuel costs are concerned. Fuels which have already been shown satisfactory for the turbine, such as kerosene, low grade petrols and Diesel oil, are essentially cheap to produce, and in view of the reduced fire hazard should be cheaper to transport.

6.2.4. *Specific output.*

There are two lines of development to be pursued in the case of propeller turbines, namely, increasing the maximum temperature at the turbine entry and increasing the mass flow through the engine by raising the rotational speed. For emergency operation at take-off, or for evasive action in the case of military aircraft, ammonia injection may also be used.

6.2.5. *Safety and reliability.*

It is considered that as development proceeds the turbine engine should prove safer and more reliable than the reciprocating engine. The advent of the turbine presents an opportunity of using fuels which will appreciably reduce the fire risk, and it is desired to emphasise that every effort should be put to the development of such fuels.

So far as the reliable operation of turbines is concerned, it is considered that the problems are considerably eased by the smooth torque of these engines, and the almost com-

plete absence of alternating stresses. The very high rotational speeds, however, demand extreme precautions where inspection of such items as turbine discs are concerned in order to eliminate the possibility of a catastrophic failure due to bursting. Further development of high temperature materials and heat insulants will be necessary to ensure complete reliability of combustion chambers over long periods of operation.

6.3. JET TURBINE ENGINES.

The operating conditions and trend of development of the pure jet turbine are somewhat different from those of the propeller turbine. If the specific output of the jet engine is increased by working at higher temperatures the energy available in the jet increases, but the jet velocity (and hence thrust) only increases as the square root of the available energy. Since the amount of fuel increases in direct proportion to the energy increase in the jet, the ratio of fuel used to thrust obtained (*i.e.*, specific consumption) is increased.

The fact that the jet discharge velocity is far in excess of any aircraft forward velocity which can be foreseen within the next ten years, also has a great influence on the operating conditions of the jet turbine. Any increase of thrust obtained by increasing the working temperature means a higher jet velocity and a lower Froude efficiency increasing the consumption still further.

Froude efficiency also has an important effect in widening the range of cruising speed, and for this reason, with present day aircraft, jet units can be cruised at a much lower percentage of their maximum speed than propeller turbines.

The two factors considered above are really complementary as they both concern the efficiency of the conversion of gas energy into propulsive thrust.

In view of the interdependence of the factors affecting fuel economy and specific output of jet engines, these two subjects have been grouped together.

6.3.1. *Fuel economy and specific output.*

To improve the economy of a jet propulsion unit two main factors may be varied, *i.e.*, peak temperature and compression ratio. An increase in peak temperature improves the efficiency of any practical thermodynamic cycle, but in the case of a jet propulsion engine the overall propulsive efficiency must be considered in conjunction with the internal efficiency of the engine.

Thus, as shown above, for compression ratios and aircraft velocities at present in use, an increase of peak temperature does in fact lower the overall efficiency of the engine due to the reduction of propulsive efficiency. It is interesting to note that the optimum temperatures are even now appreciably below the maximum permissible from the metallurgical point of view. However, it is necessary to use a reasonably high gas temperature to keep up the specific output of the engine, and, therefore, a compromise must always be made between these factors.

Turning now to the effect of compression ratio, it can be shown that for a given aircraft condition of speed and height, an optimum specific fuel consumption occurs which also varies with the gas temperature of the cycle. In a specific case of an aircraft travelling at 500 miles per hour at an altitude of 40,000 ft. the best fuel consumption is obtained at a compression ratio of 10:1 (ground level), and a gas temperature at entry to the turbine of about 750°C. These values will increase slightly with speed and altitude. In considering the use of high compression ratios, however, it must be borne in mind that the specific output tends to decrease and the specific weight will increase before the optimum condition for fuel consumption is reached.

From the foregoing it will be seen that it will be necessary to develop jet engines in relation to the performance of the aircraft classes for which they are intended. In the case of long-range aircraft of speeds below a Mach number of 1, it would appear that

the jet engine should be designed with as high a compression ratio as can be reconciled with a reasonable power/weight ratio. In the case of very high-speed short-range aircraft of the interceptor fighter type, lower compression ratios and higher peak temperatures will be used. The ultimate trend in this field will be towards the propulsive duct.

Other developments to improve fuel economy and specific output will follow the lines already suggested for propeller turbine engines with the exception of course that heat exchangers cannot be usefully employed on jet engines.

Development towards reduction of drag, economy of operation and safety, will also follow that indicated for propeller turbine engines.

6.4. General development items for turbine engines.

The two foregoing sections have dealt with the more fundamental lines of development of turbine engines, and it is proposed in this section to deal with some of the detail problems which will arise as a result.

6.4.1. *Turbine materials.*

The demand for higher temperatures and operating speeds will call for improved material from the metallurgist, and research work will be necessary for the production of suitable ceramic materials for blades. As an intermediate step hollow turbine nozzle blades and rotor blades with air or liquid cooling will be developed.

The use of castings in high temperature materials for turbine casings necessitates the development of a new foundry technique to obtain the necessary accuracy and thin sections.

6.4.2. *Compressors.*

The design of axial compressors will be directed towards the use of higher Mach numbers without loss of efficiency with a view to reducing the overall dimensions and increasing the pressure rise per stage of this component.

Experience is not yet available to justify any assessment as to the relative merits of alternative compressor blade materials, but it is evident that it will be necessary to ensure that the blades will operate satisfactorily under desert conditions. This factor combined with the higher temperatures likely to be encountered in the later stages of the compressor may lead to the abandonment of light alloys for compressor blades, at any rate in some of the stages.

An alternative to changing material to avoid sand erosion will be the development of a suitable surface treatment.

6.4.3. *Combustion and fuel injection.*

The combustion and fuel injection systems of the gas turbine engine as we know them at present will require much development to increase their reliability, reduce pressure drop, improve combustion at very low engine speeds, and ensure satisfactory lighting characteristics under conditions of altitude and high forward speeds. Furthermore, it is necessary that the control system be developed towards the ideal of a single lever. Speed control, particularly in the case of propeller engines, must be developed to give a very high degree of accuracy.

6.4.4. *Anti-icing.*

Anti-icing problems have been briefly discussed in a previous section of this paper dealing with installations, but the necessity for considering means for preventing the icing of air intakes and axial compressor blades when laying down the basic design of an engine, cannot be too strongly emphasised, and this is not a problem that can be left to the installation engineer to deal with as an "afterthought." Here again no experience is yet available on propeller turbines, but considerable thought has been given to this subject and various proposals have been made such as the use of alternative air

intakes to allow hot air to be taken from around the jet pipe, and the actual tapping off of some of the exhaust gas and by-passing it to the compressor entry.

6.4.5. *Reduction gears.*

The design and development of reduction gears presents certain new problems. The first is to produce a light and compact gear with the high reduction ratio required. This usually leads to the adoption of an epicyclic gear with its associated problems of planet cage design and high centrifugal loadings (unless the form of epicyclic gear is carefully chosen).

In view of the absence of large torque fluctuations it should be possible to make the reduction gear generally lighter than that for an equivalent piston engine. There are, however, indications that troubles may be experienced due to high frequency vibrations, although these have not been common to all types of propeller turbine.

6.4.6. *Bearings.*

The development problems associated with turbine bearings are considerable by the very nature of the conditions under which they are called upon to operate. These conditions are very high rotational speeds, high end thrusts and high temperatures.

The direction in which development will take place will be towards improved heat insulation and cooling of the bearings and mountings, improved lubricants, alternative materials for the bearings having higher tempering temperatures and better standards of manufacture and inspection. The latter items are of particular importance, not only in increasing bearing life, but in ensuring consistency of performance.

6.4.7. *Seals.*

The trend towards higher compression ratios will place increasing importance upon the design of air and oil sealing. Development of these items becomes an essential factor to reduce oil consumption and bearing end loadings by pressure balancing. It is also vital to provide adequate oil sealing at the entry to the compressor to avoid fouling of the blades.

6.4.8. *Lubricants.*

The gas turbine generally, and the jet type in particular, has given rise to a demand for lubricants having a very low pour point. While this problem may not be so difficult for the jet engine, the propeller turbine presents serious difficulties and involves the use of additives to provide a lubricant which has a sufficiently low pour point while being satisfactory for reduction gear lubrication. However, it is felt that the need for low pour point is not so pronounced in the case of the propeller turbine, and it may be desirable to have separate specifications for the two types of engines.

It has long been the designer's (perhaps pious) hope that a lubricant will be found which will retain its viscosity at high temperatures, and will enable him to use what are in effect grease-packed bearings for his turbine wheels. Silicones have been suggested for this purpose, but to date have been disappointing.

6.4.9. *Propeller control.*

Propeller controls may have to be divided into two classes (a) those for engines having independent power turbines; (b) for engines having the propeller driven directly from the compressor shaft.

In the first case the control is relatively simple, and may, if desired, be done by a normal type of constant speed unit. Whether this or a speed ratio type of control between the compressor and propeller turbine is used, provision will be made to vary the propeller speed in relation to the compressor over the widest range permissible on grounds of turbine efficiency. This will enable the propeller to operate at improved efficiencies for take-off and cruising and will reduce noise pro-

blems as already mentioned. Experience to date on the Theseus engine indicates that the rate of acceleration with the independent power turbine is so great that increased rates of pitch change will become necessary.

Where the propeller is driven directly from the compressor shaft the problem of control becomes much more difficult since the propeller speed cannot be varied in relation to the compressor and propeller and fuel controls cannot be divorced from one another.

Fig. 9.
Compressor and Turbine Test House.

A further difficulty arises under starting conditions, where in order to decrease the starting torque it may be necessary to put the propeller into zero pitch.

7. CONCLUDING OBSERVATIONS ON ENGINEERING PROBLEMS.

7.1. RESEARCH.

Pride of place among research requirements must be occupied by the study of aerodynamics which lies at the root of progress in both turbine and aircraft. High-speed wind tunnels and air flow test rigs are needed, and these can conveniently be combined with large-scale apparatus which is an imperative necessity for carrying out research work on compressors and turbines. The magnitude of the equipment needed and its very large power requirement must not deter us from pushing on with its provision and the Bristol Company have undertaken this work on a large scale, as shown in Figs. 9 and 10. A first stage provides motors with a maximum power of 4,400 h.p. for driving compressors and return-feed generators capable of absorbing an equal power output from a gas turbine.

Two more similar stages of expansion are planned so that the final equipment will provide a total of over 13,000 h.p. for driving compressors, with a corresponding power absorption capacity for turbines. The amount of compressed air which can be supplied from such a plant should enable very useful air flow rigs to be constructed, not to mention high-speed wind tunnels, when we have determined how these may be designed and how their results can usefully be interpreted.

Research work on combustion will also proceed in parallel, together with its associated subject of fuel investigation.

The importance of material research to turbine development needs no emphasis, but notwithstanding the enormous amount of

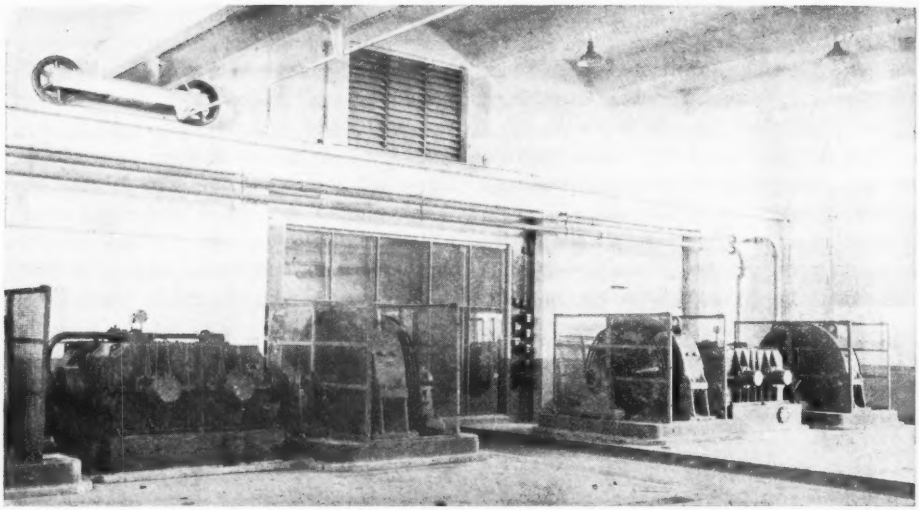


Fig. 10.
Motor Room—Compressor and Turbine Test House.

vital work already carried out, notably at the National Physical Laboratory, a good deal remains to be done in extending this work to cover the actual types of stress combinations to which a turbine blade is subjected at its operating temperature. Fatigue and creep research equipment at Bristol for investigation of the physical properties of turbine blading materials is shown in Figs. 11 and 12.

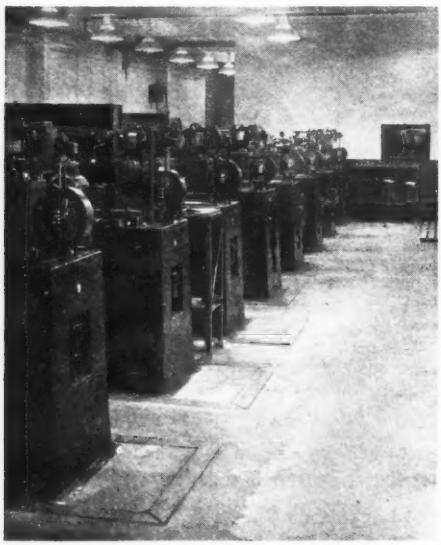


Fig. 11.
Fatigue Test Machines for high temperature material research.

7.2. Design.

Little can be added at this stage to the review given in Section 6, but it should be commented that a great deal of new design will have to be initiated. The provision of adequate air and oil sealing devices to cater for speeds of the order of 10,000 r.p.m., and pressures of the order of 100 lb./sq. in. or more at temperatures which may be in excess of 200°C. is a case in point. The much greater extent to which gas turbine design is amenable to scientific calculation should not be allowed to blind the designer to the fact that the possession of a great deal of ordinary engineering cunning still remains one of the most important of his essential attributes. The designer, in fact, is still, in certain respects, an artist as well as a scientist, and is likely to remain so for a long time to come. He will, however, need both art and science in his liaison work with the aircraft designer

in tackling the general problems which the future holds in store, and as mentioned at the outset of this paper, it is impossible to exaggerate the necessity for the closest co-operation between them.

7.3. DEVELOPMENT.

In development, an enormous amount of work has to be done on the gas turbine engine before its background can be in any way comparable with the piston aero-engine on which 40 years' unremitting effort has been lavished. The jet engine is piling up a considerable background of very satisfactory operation, but none of this is, as yet, in commercial service, and the propeller turbine has very little development background indeed.

It cannot be too forcibly underlined that a new type of power installation has to begin where its predecessor leaves off in matters such as reliability, easy maintenance, service life, and so on, if it is not to receive adverse criticism from its users.

The age of the gas turbine in service is nearer 40 months than 40 years, and its development youth is a defect which time will cure in due course, provided that intensive and intelligent work is carried out on the numerous types of the engine and its installation, some of which have been mentioned in this paper. In this connection, a fact which might well receive earnest attention is the need for improved instrumentation. An excellent example of this is the progress

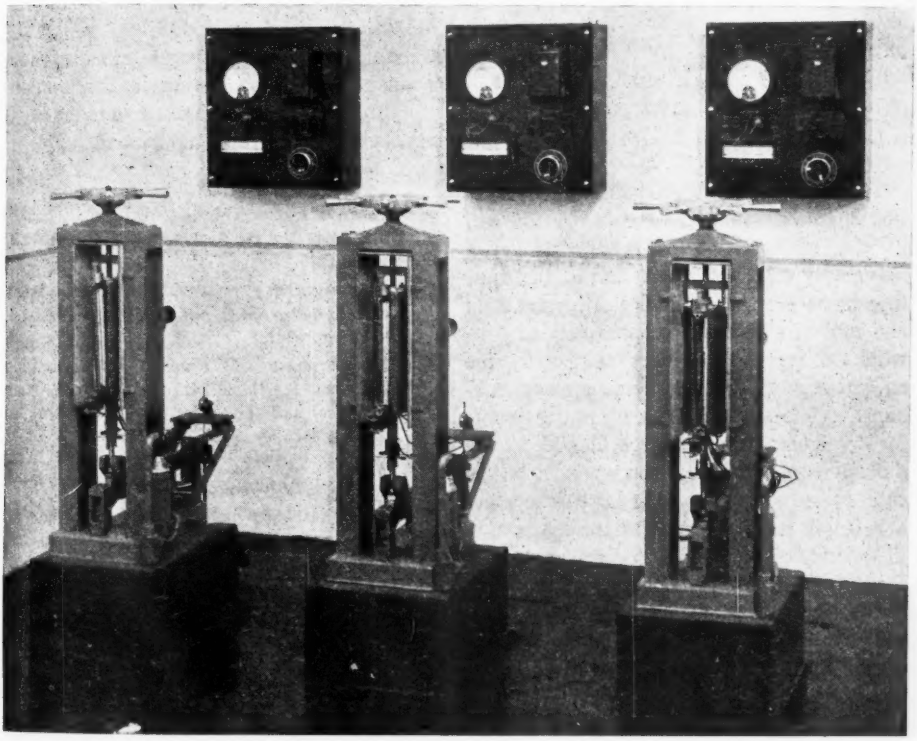


Fig. 12.
High temperature creep testing machines.

which the art of propeller design made as soon as strain gauging was developed; a similar technique is urgently required now for both compressor and turbine blading, and the manifest experimental difficulties should not be allowed to form a deterrent.

A final word must be said on the importance of continuous liaison with aircraft development in order to ensure that new types of power installations are provided by the time they are needed in a new type of airframe. Again, boundary layer control forms the perfect example in that it can only be achieved in practice by the maximum co-ordination of aircraft and power plant, research, design and development.

7.4. Production.

The chief problem confronting the production engineer during the coming period will be the maintenance of production on a reasonably economic basis, and at the same time preserving the flexibility of organisation and tooling which will be necessary to cope with the repeated changes caused by the enormous strides made in power plant technique. The specialisation of power plants for particular applications will add to the difficulties of the production engineer in this case, by reducing the size of his batches, and a halt will have to be called somewhere in the process of " tailoring " the power plant to the individual aircraft, if very high costs are not to result.

The development of new techniques, as for example precision casting, has proved of the utmost value in turbine design, and with the possible advent of such items as ceramic blades will have to be extended into even more unusual fields. This factor, together with the introduction of high temperature materials as regards casting and machining will offset, to some extent, the attractive simplicity of turbine power plants from the production engineer's aspect.

Turbine blades are, however, likely to remain the most outstanding and expensive items to manufacture, and these are required in quantities which almost reach the scale of mass production, so that special techniques are justifiable.

Broadly speaking, the gas turbine should in due course, represent as acceptable an advance to the production engineer as it does to his design and development colleagues.

The author's grateful thanks are due to Messrs. G. J. C. Davies and E. I. B. Marples for their invaluable assistance in the preparation of this paper, and to his colleagues, Messrs. S. W. Mansell, J. Cutts and A. C. Clinton, for their general collaboration. Thanks are also due to the author's Company for permission to include data concerning their products and equipment.

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DISCUSSION

Group Captain G. M. Buxton (Associate Fellow): He questioned the statement that the performance of the future fighter would be beyond the limits of the human body. As far as time limits were concerned, the human brain could react in the time of flight of a cricket ball, and the time available for action, although shorter than before, might well be greater than this; and the accelerations to be endured might also be moderate when at high altitudes.

Mr. Owner: He was not sure that the limits of endurance of the human body were known with any certainty. He had pointed out that the adoption of the prone position and of special medical devices might delay the point at which the human body became inadequate to deal with the problems involved; but he felt that the rate of progress in aircraft would outstrip the capacity of medical devices to overcome the fundamental limitations.

There had been a lot of discussion before the last war on the relative importance of a single shot well planted and of spraying the space in front with lead. The answer was fairly conclusive that it was not possible to plant a single shot well, and the development of fighter design was concentrated on making the space ahead of the aircraft as lethal as possible over a certain area. Presumably that was because the time available for action was getting so short; and he could only imagine that in the course of future development the time available for interception would become very much shorter still. He was no expert on those matters, but he imagined it would be of an order comparable with the human being's reaction time, and at least was pretty well constant and could not be improved very much.

He could not answer the question factually, but it was his impression that they were fast approaching the fixed limitations of the human body, about which they would be able to do very little.

Dr. H. Roxbee Cox (Vice-President, Fellow): When discussing the types of power plants for long-range transport aircraft, the author had correctly made the point that they must, at least for the time being, use the propeller turbine, and had also suggested that, in their efforts to instal the engines inside the wings, they should use a greater number of engines than formerly. What, in the author's view, was the limit to the number of engines that ought to be installed in a large aeroplane? He believed that with piston engines, it was felt to be inconvenient, from the point of view of the pilot controlling the aircraft, to have more than four engines. Did the author consider that the turbine engine was sufficiently simple as a type to enable them to instal as many as they liked? Obviously, they could not increase the number *ad lib*; the limitation, for example, in the case of the airscrew turbine, was the number of airscrew discs that they could get across the span of the wings. But personally he hoped that later they would move away from the propeller turbine. It seemed very difficult to instal it satisfactorily except with a tractor screw, and for general aerodynamic reasons they would prefer to use a pusher screw, but that appeared to be a difficult thing to do. While they were developing the jet engine to achieve the fuel consumption they expected at present from the ordinary propeller engine, could they not use the ducted fan engine, because that should be capable of installation in a manner which would give less drag than the airscrew turbine. While its overall efficiency might be proved to be less than that of the propeller version, the difference might perhaps be made up by the better aerodynamic form of the aeroplane in which it was installed.

Although the author had pointed out that combustion efficiency was already at a very high level indeed, one imagined he did not wish to imply that nothing much remained to be done in combustion research, for there was still a great deal to be done. They would like to complete combustion in shorter distances to get away from the plurality of combustion chambers and to use single annular chambers, which were more convenient and compact. That type of chamber had not yet been developed to the same degree as the smaller chambers, of which there was a cluster around nearly every engine. Further, there were the combustion problems of high altitude flying. So that there was much work to be done on the combustion component as well as the other components—the turbine and the compressor.

On the question of materials, the author had referred with perhaps rather more optimism than most of them would exhibit to ceramic materials. Had the author anything up his sleeve which would suggest that they could produce ceramic materials having tensile properties which would render their use practicable for the purposes they had in mind?

Mr. Owner: There were several factors which determined the practical limit to the number of engines that could be installed on an aircraft. The difficulty about the number of airscrew discs which could be accommodated across the span, and the uncomfortably large proportion of the span which would come into the slipstream, could be obviated, as on the Bristol Type 167, where only four airscrews were used for eight engines. That was the type of installation he had had in mind when he had referred to the advantage of using smaller engines which could more easily be buried in the wings. An advantage of such an installation was that the reliability of eight engines could be obtained while confining the disadvantages to the slipstream of four airscrews.

One of the most powerful factors limiting the number of engines would be that of cost. Broadly speaking, the turbine engine incorporated the same number of parts, whether one made it half or double the normal size; therefore, the cost would vary only slightly with size. Thus, if they used a very large number of engines, it might be

aerodynamically convenient to stow them within the wing, but it was likely to be more expensive.

He did not think the control problem would prove the limitation, because they were developing turbines which should have substantially single-lever controls. They had eliminated things such as mixture controls, and every other type of control should be as automatic as possible. Automatic control of the propeller speed was being arranged. In the Bristol Type 167 there were only four controls for the pilot to handle, each pair of engines being ganged. The installation of eight engines seemed perfectly practicable.

His personal impression was that there might be a case for the use of the ducted fan as an intermediate stage between the normal propeller engine and the jet engine only at speeds between, say, 500 and 550 m.p.h. The installation problems in connection with the ducted fan were considerable, in ducting the air streams, while the fundamental fact remained that the efficiency of a propeller was likely to be higher.

Interference with the flow over the aeroplane depended on how well the ducts could be arranged within the aeroplane surfaces. If they were exterior, the drag of the ducts might be a very serious drawback to the use of ducted fans. There was the further difficulty that the small higher-speed slipstream obtained with the ducted fan would not give as good a take-off as would the normal propeller.

On combustion efficiency he did not intend to convey the impression that nothing remained to be done; indeed, he agreed fully with Dr. Roxbee Cox in that respect. An enormous amount remained to be done to improve mechanical reliability, design, and the convenient disposition of combustion chambers. What he had meant to convey was that, thanks largely to the heroic pioneer work of Air Commodore Whittle and the extent to which the Lucas Company had carried on that work, they had very little more to come in the way of combustion efficiency *per se*; it was already 99 per cent. or so, but improvements in the design of combustion chambers were desirable. The attainment of maximum reliability and the prevention of mechanical failure ought to come first, and for this reason he expected individual combustion chambers to remain for some time, because a jet from a spray nozzle was apt to be circular, and it was easy to fit a cylinder around it.

His remarks on ceramics reflected, not so much optimism, but rather a pious hope. They had some things up their sleeves, but there was also considerable difficulty in shaking them down! Probably the advent of ceramics would follow some years of patient detail application on metallurgical testing apparatus such as he had illustrated. He would only drop the gentle hint that, although perhaps they could not make the whole blade of ceramic, there might be opportunities in some sort of ceramic coating.

Mr. L. W. Rosenthal (Saunders-Roe Ltd., Associate Fellow): What was the weight penalty resulting from a design in which the engines were buried in the wings, and how far had the design of the structure to be modified to enable that to be done? Also, would the author enlarge on the question of the disposal of the ducts?

Mr. Owner: Perhaps the best answer he could give to the question concerning the burying of engines was to quote the remark of a former colleague, that aeroplanes should not have "boils" on them. He believed it was W. S. Gilbert who had said that he admired a manly man, and adored a womanly woman, but he detested a boily boy! It was for the designers to ensure that aeroplanes in the future should not have boils on them, for they could not afford them.

He was not armed with precise figures with regard to the weight penalty; but in any case, whether they used piston engines or propeller turbines, they must have a reduction gear and they must also lubricate it and put

it somewhere. The propellers had to be carried at a minimum of 12-15 per cent. forward of the leading edge; and it was his opinion that the combination of two reduction gears into one, and the improvement made possible by placing the engine within the wing, where it could be carried more easily than if it were on a long stalk in front, had resulted in very little weight penalty in the whole installation.

Considerations of secrecy precluded him enlarging on the question of the disposition of ducts, particularly ducts for turbines; but if the ducts were to be disposed reasonably well aerodynamically they must be dealt with in a way which was not affected by the burying of the engine. In the investigation of the turbine edition of the Bristol Type 167, no material difficulty had been experienced in disposing of the ducts; indeed, the buried installation should lend itself or lead to very considerable improvement in respect of air intakes, especially when one included problems such as de-icing, air filtration, and so on. Further, when they came to do some serious work on boundary layer control, as he hoped they would do in the near future, the disposition of the necessary ducts and passages was likely to be greatly facilitated if the engines were buried.

THE ROYAL AERONAUTICAL SOCIETY

TAILLESS AIRCRAFT DESIGN PROBLEMS

by

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Mr. Lee studied physics and aeronautics at Imperial College from 1931 *to* 1934, *after which he spent three years with Saunders-Roe, Ltd. He then joined Handley Page Ltd., and since* 1940 *has been in charge of research and is closely associated with the work done on tailless aircraft.*

INTRODUCTION

A FEW of the outstanding problems connected with the design of tailless aircraft are discussed. The paper does not set out to be in any way a comprehensive dissertation of the whole field of tailless design, as it was considered that it would be more useful to concentrate on some of the recent problems which are exercising the minds of designers at the present moment.

Since the problems discussed are those which are currently causing some degree of speculation, it follows that in the majority of cases only possible means of solution may be suggested and only tentative conclusions drawn. In attempting to deal with the matters that are causing concern to designers to-day, it is inevitable that this paper must run the risk of drawing inferences which later data may prove to have been incorrect, and of making false predictions, but it was thought that a paper dealing with controversial issues would be of more value and would provoke a more useful discussion than one which dealt with problems that are now more or less cut and dried and so hold far less general interest than the untidy questions discussed below.

The paper is divided into the following sections:—

(1) Low speed stalling and high lift devices.
(2) Compressibility effects.
(3) Aeroelastic problems.

Although the special needs of the tailless aeroplane are kept in mind, the majority of the discussion applies generally to sweptback wings, whether they have a tail behind them or not.

1. LOW SPEED STALLING AND HIGH-LIFT DEVICES

1.1. THE STALL IN GENERAL.

The main problem in the stall of wings with more than about 25° of sweepback is the avoidance of a premature tip-stall with consequent loss of maximum lift and, more serious, pitching instability at low speeds.

The tip-stalling propensity of sweptback wings may be attributed to:—

(*a*) The increase, due to sweepback, in local C_L near the tips.

(*b*) The negative induced camber near the tips.

(*c*) The outwards drift of the tired boundary layer on the top surface of the wing.

(*d*) In the case of part span flaps, the flap outer end may produce a particularly bad interference; effects (*a*) and (*b*) occur and there may be a serious outwards flow of air on the lower surface (pushed off the front of the flaps, snow-plough fashion) which causes a premature stall just outboard of the flap outer ends.

1.1.1. *Explanation of phenomena promoting the tip-stall.*

(*a*) *The increase in local C_L near the tips due to sweepback.*

The reason for this may be seen in a crude way by considering Fig. 1. Suppose

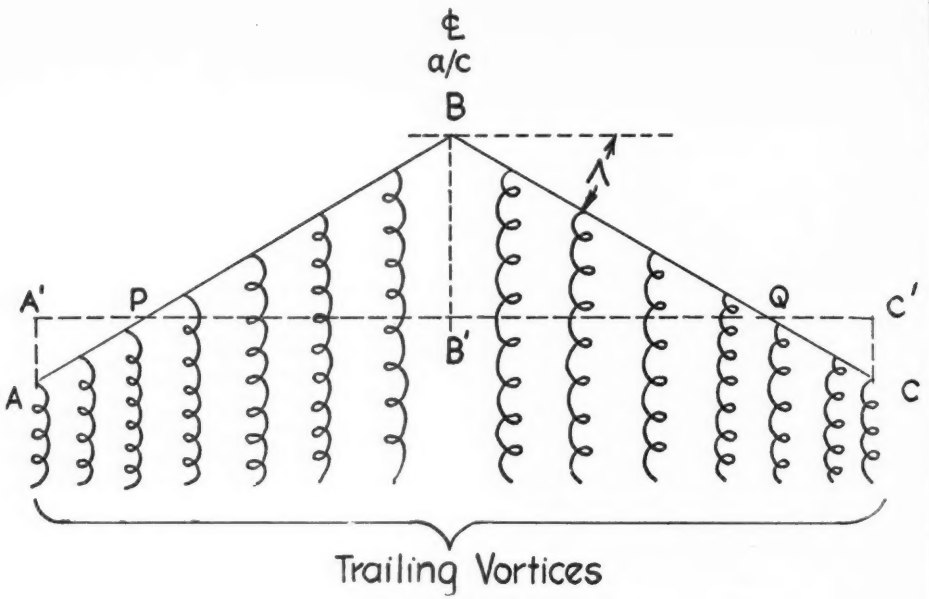


Fig. 1.

A B C represents a sweptback bound vortex (lifting line) and suppose the trailing vortex system shown corresponds to the load grading that the wing *would* have if it were straight (*i.e.*, *sheared* forward so that Λ = zero). Then for the straight wing the upwash induce at P would be that due to this trailing vortex system originating along the line $A'PB'QC'$, whereas in the sweptback case the trailing vortices originate along $APBQC$, so that comparing swept and unswept cases it is seen that the vortices which occupy (or would occupy) the triangles $AA'P$, PBB' and QCC' tend to increase the upwash at P, while those in $B'BQ$ tend to reduce it. Clearly when P is near the tip the net result is to increase the upwash when the wing is swept back and hence to increase the local C_L.

(*b*) *The negative induced camber at the tips.*

This was predicted by Falkner (1). The idea of negative camber is simply a way of saying that if the chordwise pressure distributions of an aerofoil profile in two-dimensional flow and of a section of the same profile near the tip of a sweptback wing are compared at the same (local) lift coefficient it will be found that the tip section distribution corresponds to a profile having less (positive) camber than it actually has, *i.e.*, roughly the suction peak at the nose is higher than would be obtained with the same section at the same lift coefficient in straight flow. Since reduction in camber reduces $C_{l,max}$, it may be expected that the induced negative camber will reduce the available local $C_{l,max}$ and so will tend to give an earlier stall.

(*c*) *The outwards drift of the tired boundary layer on the top surface of the wing.*

This outwards drift is due to the fact that the isobars (lines of equal pressure) on the top surface of a sweptback wing are oblique to the direction of the main airflow. Fig. 2

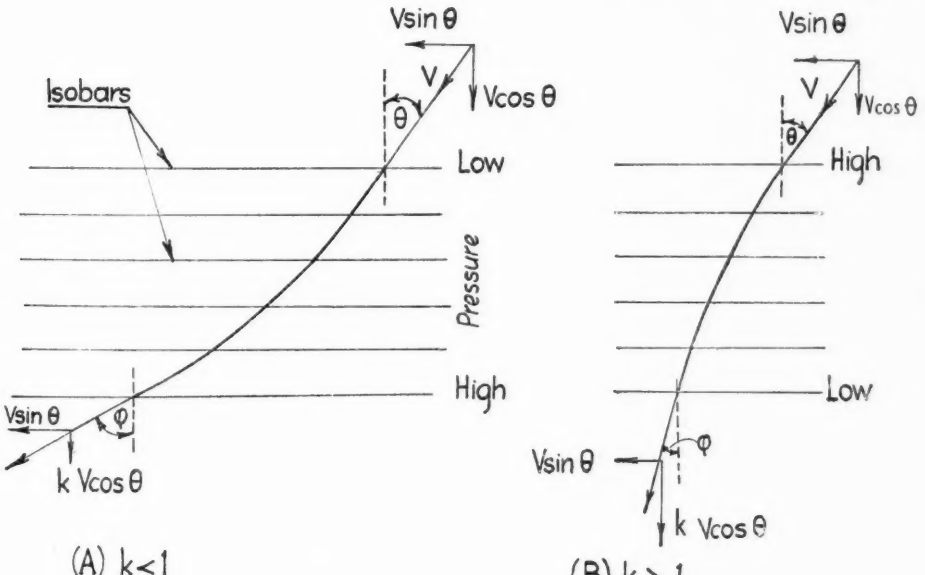
shows how an oblique pressure gradient will deflect a streamline, sketch (A) depicting the effect of an adverse gradient and sketch (B) the effect of a favourable one.

Suppose that the incident streamline with velocity V is inclined at an angle θ to the normal to the isobars, then the component velocity parallel to the isobars is $V \sin\theta$ and this component will be unaffected by the pressure gradient; the normal component, $V\cos\theta$, however, will be changed by the pressure gradient to, say, $kV\cos\theta$, so that on emergence the inclination, ϕ, of the streamline to the normal to the isobars will be given by

$$\tan\phi = (1/k)\tan\theta$$

With an adverse gradient $k<1$ so that $\phi>\theta$ and the streamline is more parallel to the isobars, becoming in fact exactly parallel to them when $k=0$. For a favourable gradient, on the other hand, $k>1$, so that $\phi<\theta$ and the streamline emerges more at right angles to the isobars. It will also be seen that if passing through a series of adverse and favourable gradients, ϕ is dependent only on the final value of k. It should also be noted that $\tan\phi$ is proportional to $\tan\theta$, so that the smaller the initial obliquity the smaller the deflection, until when $\theta=0$, $\phi=0$ also.

On the top surface of a conventional section at high lift there is a short very steep favourable gradient from the stagnation point to the suction peak followed by a long adverse gradient, usually steep at first and thereafter more gentle, see Fig. 3. It is clear that in the absence of viscous friction k would be close to unity and the resultant deflection of the streamlines small. However, in the boundary layer viscosity is of primary importance so that k will be decidedly less than one when the flow reaches the trailing edge and hence the



$$\tan\varphi = \frac{1}{k}\tan\theta$$

Fig. 2.
Deflection of streamline by isobars.

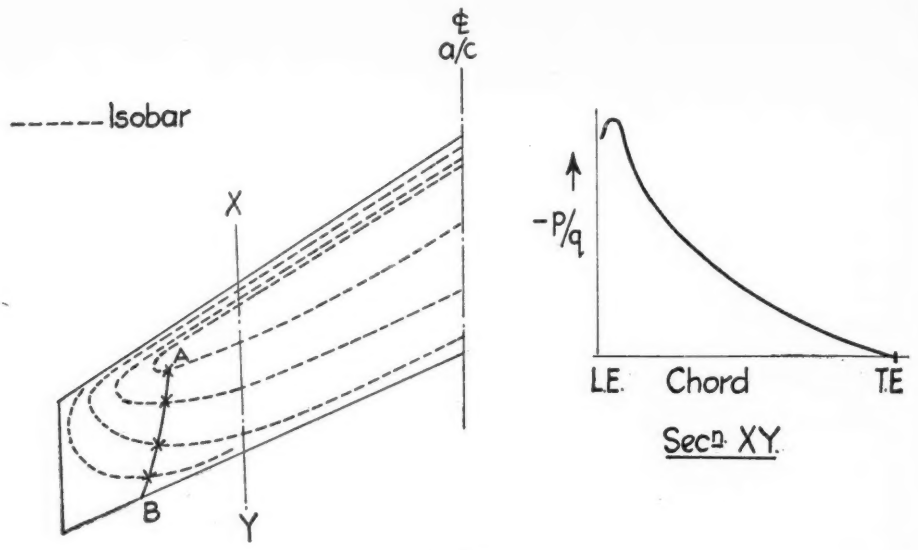


Fig. 3.
Pressure distribution on sweptback wing.

boundary layer flow near the trailing edge will tend to set itself parallel to the local isobars and so will, over most of the wing, tend to drift outwards towards the tips. As Griffiths points out (2), the boundary layer will tend to drift from both directions towards the line AB (Fig. 3) the locus of points where the isobars are normal to the flow direction.

Thus there will be an accumulation of "tired" boundary layer air near the tips, and this, very crudely, tends to "lever off" the air flow in that region, while at the same time there is, in effect, a favourable "boundary layer suction" effect on the centre section and this will tend to delay the stall over this central portion of the wing.

(*d*) *Part Span Flap Interference.*

Silk-tuft observations during wind tunnel tests of part span flaps on wings with large sweepback indicated that there could be a very violent outflow from the ends of flaps; this is dealt with more fully in para. (1.3).

1.1.2. *General note.*

It should be noted here that wind tunnel evidence indicates clearly that in order to get really good tip-stalling behaviour the outer parts of the wings must be more "favourably" designed than the central parts. For example, a wing with outer part span nose flaps has a much better stall than one with full span nose flaps; or if L.E. slots are used they should be confined either to the outer part of the span, or, better still, tapered so that the slat chord is a bigger percentage of the wing chord at the tip than at the centre.

1.2. THE FLAPS-UP STALL.

1.2.1. *Relative significance of the various causes of the tip stall.*

Effect (*a*), the increase in local tip C_L, is undoubtedly present, but does not seem to be of primary importance as devices which operate so as to reduce tip C_L do not seem to be very effective as cures for the tip-stall. Thus Ref. 3 shows that whereas an untwisted wing of 40° sweepback and 1.47 to 1 taper was slightly unstable at the stall, it needed 12° of washout to render it slightly stable at

maximum lift. Also, Ref. 4 reporting tests on a tapered wing of 36½° sweepback with the outer 30 per cent. of the span capable of rotation as a whole about a spanwise axis showed that even 15° of washout on this outer 30 per cent. did not really cure the tip-stall.

There is some evidence that effect (*b*), the negative induced tip camber, is present, as pressure plots (5) show an increased "peakiness" of the chordwise pressure distribution near the tips; however these results have not been analysed so that nothing definite can be stated.

One is thus forced to the conclusion that the real "nigger in the wood pile" is the outwards drift of the boundary layer air on the top surface. This contention is supported by the fact that devices, such as the Brennecke contra flap (6) which render this outwards drift innocuous, are successful in preventing tip-stalling and that all flow pictures of a sweptback wing at the stall do, in fact, show a very pronounced outwards flow.

The bad effect of taper ratio on tip-stall may also be explained in terms of outwards drift. There is plenty of evidence (7) to show that for wings of about 40° or 45° sweepback a parallel planform (taper ratio 1/1) is stable in pitch at the stall, while with any taper greater than about 1.5/1 the pitching moment is unstable at the stall. At first sight this might be credited to the increase of local C_L at the tips with increasing taper, but the washout tests quoted above (3 and 4) are not consistent with this explanation, so that one is forced to conclude that the real explanation lies in the fact that with a high taper the volume of boundary layer drifting down towards the tips is proportionately greater (since the tip chord is small) than is the case on a wing of slight taper, so that its disruptive effect on the flow is proportionally more serious.

Further evidence in support of this theory of the importance of outwards drift is

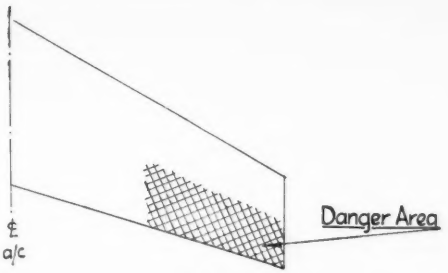


Fig. 4.

furnished by pressure plots of a sweptback wing at the stall, which show that, with flaps up, the $C_{L\max}$ attained at the centre of the wing is about twice that reached at the tips. This is in accordance with the argument given in the last paragraph of item (*c*) of para. 1.1.1.

1.2.2. *Boundary layer behaviour.*

In view of the great importance of outwards drift of the boundary layer, it is important to investigate its behaviour in more detail so as to see how one may expect to control it.

Test information indicates that the outer rear quarter of the wing upper surface is the "danger area," see Fig. 4. Thus the flow pictures of Ref. 8 show that near the stall the outwards drift is most pronounced in this region, while the wake survey of Ref. 9 indicates that the accumulated boundary layer streams away more or less uniformly over the outer halves of the semi-span. Tests also show that most of the outwards drift is confined to the boundary layer and does not extend to the flow above it.

Attempts to influence the stalling behaviour of the wing by manipulation of the boundary layer within this danger area do not seem to be effective. Thus boundary layer suction, or air ejection, applied to a slot near the trailing edge has very little effect on a sweptback wing, whereas it is quite efficacious on a straight wing (10). The second report quoted in Ref. 10 also deals with ejection from a forwardly located slot and this,

although still far from satisfactory, is more potent than ejection from a rear slot. Again, ordinary slotted flaps applied to sweptback wings do not give results substantially different from those of split flaps, which is consistent with the idea that a slot at 75 or 80 per cent. of the chord from the leading edge is too far aft to be of any value.

The fundamental cause of the outwards drift is that the air has to traverse an oblique gradient over the rear part of the top surface of the wing. There is also strong evidence to suggest that, although the outwards drift takes place over the rear part of the wing and is directly due to the oblique adverse gradient there, a factor of prime importance is the strong adverse gradient (at the stall) just behind the suction peak near the leading edge. The evidence for this is to be found in the success of devices, such as the Krueger nose flap and the Handley Page L.E. slot (11) which reduce this adverse gradient and in the successful outcome of the practice of the Horten brothers in using at the tips a profile with a large leading edge radius (which reduces the suction peak, and hence the steep adverse gradient, at high lift coefficient) (12).

It must be noted that the slot and nose flap raise the local stalling angle as well, but, as the comparative ineffectiveness of washout showed, merely to increase the gap between local C_L and available $C_{L\max}$ is not sufficient alone, hence one deduces that the reduction of the suction peak is a vital part of the whole business.

It is clear that if there are two profiles having the same adverse gradient over, say, the rear 2/3 of the chord, then, from the argument given under item (*c*) of para. 1.1.1, in oblique flow, the greater outwards drift will occur on that profile for which the boundary layer is most retarded over the first 1/3 of the chord. Since high suction peaks imply high local velocities and so large viscous forces and high loss of boundary layer momentum, it follows that one would expect the results quoted just above. To put it colloquially, one may say that a strong adverse gradient must seriously thicken and retard the boundary layer and apparently so enervates it and weakens its will that it succumbs to the slightest temptation to drift outwards.

1.2.3. *Designing to avoid outwards drift.*

It is interesting to speculate on the ways in which the outward drift might be avoided, or, at any rate, reduced.

Although L.E. slots and nose flaps are very efficacious they have certain obvious disadvantages, so that it is natural to ponder on the possibility of finding some more elegant solution in terms of a suitable design of aerofoil section profile or of wing planform.

From 1.2.1 and 1.2.2 it follows that there are three ways of reducing outwards drift:—

(i) By reducing the adverse gradient over the rear part of the profile.
(ii) By reducing the steep adverse gradient on the forward part.
(iii) By reducing the obliquity of the isobars.

The "stepped" suction profiles of the Griffiths type, which have a velocity distribution at ideal C_L and are shown by Fig. 5a, may be expected to have a velocity distribution, as shown by Fig. 5b, at high lift; the velocity distribution for a conventional section is shown in 5b for comparison. From this it may be seen that such a profile should give a smaller adverse gradient at the rear than the orthodox type and so would be expected to reduce outwards drift.

Even more significant are the new type of nose suction aerofoils developed initially by Lighthill (14). Fig. 6 shows the comparison between a Lighthill and a conventional section at high lift and it is at once apparent that the Lighthill profile offers the opportunity of eliminating the steep adverse gradient behind the suction peak and so, if the argument of 1.2.2 is correct, of much reducing outflow.

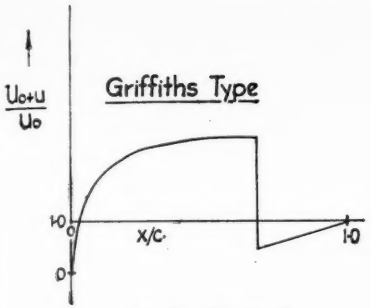


Fig. 5a. Design C_L.

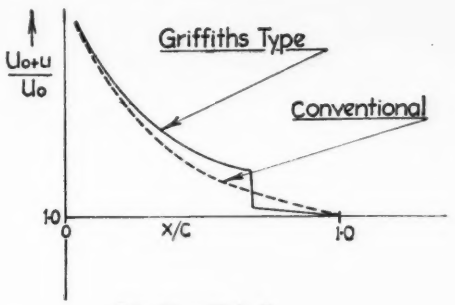
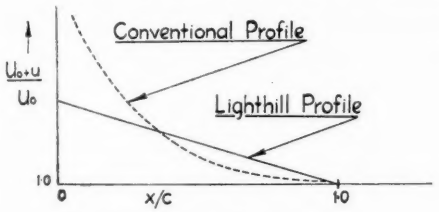


Fig. 5b. High C_L.

Uo = Free stream velocity.
u = Supervelocity in direction of Uo.
x = Distance from L.E.
c = Chord.

Fig. 5.
Velocity distributions.

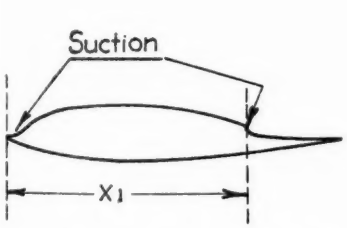
To carry the argument logically a stage farther, one could contemplate a double-suction profile as shown by Fig. 7a which, at high C_L, should have a velocity distribution of the type shown by Fig. 7b and so should have the benefit of no steep forward adverse gradient and also of a reduced rear gradient. Whether this degree of complication is worth while or justified is not at present known.



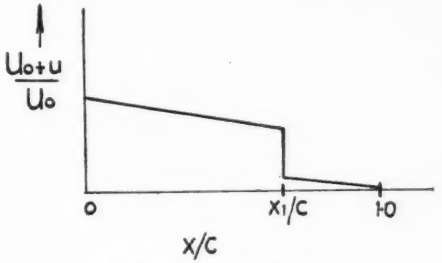
Uo = Free stream velocity.
u = Supervelocity in direction of Uo.
x = Distance from L.E.
c = Chord.

Fig. 6
Lighthill profile at high C_L.

It may also be noted that preliminary test evidence shows that the Lighthill profile possesses quite a good maximum lift coefficient even without suction, although it is undoubtedly improved by suction. This fact suggests that it would be possible to meet the requirement of 1.1.2, that the tips shall be more favourably designed than the centre



Possible appearance of section.
Fig. 7a.



Velocity distribution at high C_L.
Fig. 7b.

Uo = Free stream velocity.
u = Supervelocity in direction of Uo
x = Distance from L.E.
c = Chord.

Fig. 7.
" Developed " Lighthill profile.

section, by applying greater suction to the outer parts of the wing than to the inner parts.

Another approach is to try and reduce outflow at the tips by reducing the obliquity of the isobars for the outer part of the wing, and this clearly suggests a progressive reduction in sweepback from the centre line to tip. Such a wing would be awkward structurally, and at the tips the gain in critical Mach number from sweepback would be reduced, but for moderately high speeds a reasonable thickness/chord ratio could still be retained outboard while full advantage could still be taken of sweep at the structurally much more important centre section.

1.3. The flaps-down stall.

The main problem in obtaining a high $C_{L\max}$ with highly sweptback wings (greater than 35°, say) in addition to the low $C_{L\max}$ flaps up, is the fact that with flaps down the stall occurs much earlier than with flaps up. This undesirable phenomenon occurs with both part span and full span flaps. Although test results are very scattered, there is evidence to show that the loss, due to sweepback, in ΔC_L at small incidence is less than might be expected. Thus, the very simple (and nearly always incorrect) theory which considers merely a yawed infinite wing (16) implies that ΔC_L due to flaps should be proportional to the square of the cosine of the angle of sweepback whereas in fact a rough deduction from test results is that ΔC_L actually rises for small sweeps and at 20° has the same value as for a straight wing; at 40° of sweep, ΔC_L is about 15 per cent. less than for a straight wing (compare $\cos^2 40° = 0.59$).

Some typical results are given in Ref. 15; this report shows that the fall in ΔC_L with sweep is less for part span flaps than for the full span type and this is presumably due to the fact that with increasing sweep the ΔC_L induced outboard of part span flaps (by the strong vortices trailing from the outer ends of the flaps) gets progressively greater, as would be expected from the argument given in relation to span load grading in paragraph 1.1.1 item (*a*).

Hence, the $C_{L\max}$ troubles are principally due to the early flaps down stall.

The fact that the stalling angle is reduced for both part and full span flaps suggests that the trouble is due to a combination of two effects:

(i) The fact that lowering a flap increases the top surface adverse gradient at the rear of an aerofoil and so must accentuate the outwards drift tendency.

(ii) An effect or effects essentially linked with part span flaps.

Considering first the case of full span flaps, it is obvious that effect (ii) cannot be present and therefore the problem is essentially the same as that of the flaps-up stall considered in paragraph 1.2 above.

Regarding part span flaps, the phenomena included under item (ii) above seem to be as follows:

(*a*) The fact that lowering flaps over the central part of the wing increases the top surface suction there and so renders the "boundary layer suction" or "cleaning-up" effect of the tips less powerful, hence, at the stall the centre wing is relatively less favourably situated flaps down than flaps up and so may actually develop a smaller $C_{L\max}$ with flaps than without them.

(*b*) The fact that just outboard of the flaps on a sweptback wing, the vortices from the flap outer ends will induce a high local angle of attack and so would be expected to promote an early stall there.

(*c*) The fact that Falkner predicts considerable induced negative camber on a sweptback wing just outboard of the flaps (1).

(*d*) The fact that tunnel tests with silk tufts have shown that part span flaps, especially if arranged so as to run across the wing normal to the plane symmetry, push outwards on the lower surface of the wing a very considerable flow of air and this, in the crudest way possible, seems to inter-

fere with the flow outboard so much that it promotes an early tip stall.

As a corollary to the items (*a*) to (*d*) it should be noted that (*a*) should tend to give a centre-wing stall, whereas (*b*), (*c*) and (*d*) would promote a tip stall. As a result it is hard to predict whether the pitching stability at the stall should be better flaps down than flaps up; the evidence is very scattered but indicates that usually the wing is more stable (or less unstable) at the stall with part span flaps down. This suggests that effect (*a*) is very powerful.

1.3.1. *Flap design.*

There is a big field for investigation here. Clearly the schemes suggested above for improving the flaps-up stall by reducing outflow should apply in large measure here also.

There seems to be general agreement that ordinary split flaps are not satisfactory for wings with large sweepback and the German work (17) does not hold out much hope for ordinary slotted flaps. The Germans seemed to be inclined to favour flaps which extended the chord and latterly were applying Zap flaps in their tests. One would expect good results from Fowler-type part span flaps, as such flaps would produce a suction peak to the rear of the section and this should "clean up" both the centre section itself and also the tips. Preliminary tests have confirmed this prediction.

Devices to deflect the outflow from the front of part span flaps away from the tips have been tried and do produce some improvement, thus supporting theory of item (*d*) above.

1.4. CONCLUDING REMARKS ON STALLING.

The multitude of differing, and often conflicting, effects discussed above make it clear that there is still much work to be done in order to establish the quantitive relationship of these various phenomena. There is a mass of wind tunnel data (mainly German) which it is suggested might profitably be analysed against a background such as is furnished by the suggestions already given.

If it could be established beyond reasonable doubt which effects are important and which are not, a great step would have been taken towards finding a satisfactory solution.

2. COMPRESSIBILITY EFFECTS

This is a very large subject and only a very small part of it will be dealt with here. The following subjects will be discussed briefly:—

(1) The variation of the aerodynamic characteristics of a sweptback wing with Mach number.
(2) The raising of the critical Mach number by sweepback.
(3) The stalling of a sweptback wing at high Mach number.

Although the main attraction to sweepback at high speed lies in the fact that it raises the critical Mach number of a wing, it is somewhat more logical to deal first with the general behaviour of a sweptback wing at high subsonic Mach number.

2.1. GENERAL AERODYNAMIC CHARACTERISTICS AS AFFECTED BY COMPRESSIBILITY.

The phrase "general aerodynamic characteristics" is intended to cover such items as slope of lift curve, wing no-lift pitching moment, elevon effectiveness and so on. The effect of compressibility is considered at Mach numbers up to the critical.

The approach to the problem is by way of the "equivalent incompressible flow" analogy. It may be shown for subsonic flow (18) that where the velocities induced in the flow by a body are small compared with the flow velocity (*i.e.*, the body must be slender, or the aerofoil of small thickness) it is possible to deduce the flow round a given body in a compressible medium if the flow round a different body, obtained from the

actual body by the application of definite rules, in an incompressible medium is known. There are many versions of this analogy, but one of the most convenient is that known in Germany as " Prandtl's rule " and this form is used in the present paper. The rule is as follows:—

The flow round a given body in a compressible fluid is related to the flow of an incompressible fluid round a body obtained from the original body by reducing its dimensions normal to the flow direction by the factor β, with dimensions in the direction of flow unaltered, by the rule that near the body perturbation velocities normal to the flow direction are $1/\beta$ times, and perturbation velocities parallel to the flow $1/\beta^2$ times, the respective perturbation velocities at the corresponding point in the incompressible flow about the analagous body: where $\beta = \sqrt{1-M^2}$, M, being the Mach number.

Consider the rule applied to a sweptback wing. Let the real wing, whose characteristics in compressible flow are required, be specified by the following:—

Span $= b$
Wing Area $= S$
Aspect Ratio $= A$
Sweepback Angle $= \Lambda$
Typical Chord $= c$
„ Thickness $= t$
„ Incidence $= \alpha$
„ Camber $= h$
„ Angle of washout $= \epsilon$
„ Flap or control surface angle $= \eta$

To obtain the analagous wing in incompressible flow, the normal dimensions must be reduced by the factor β, that is span, thickness, incidence camber, washout and control surface angle must be so reduced, while the chord and the fore-and-aft extension of the wing are unaltered. From this it follows that if primes are used to denote the properties of the analagous wing (*e.g.*, b' = span of analagous wing) then the following relations hold:—

$$b' = \beta b$$
$$S' = \beta S$$
$$A' = \beta A$$
$$\Lambda' = \tan^{-1}(1/\beta)\tan\Lambda$$
$$c' = c$$
$$t' = \beta t$$
$$\alpha' = \beta\alpha$$
$$h' = \beta h$$
$$\epsilon' = \beta\epsilon$$
$$\eta' = \beta\eta$$

Since the aerodynamic forces on a wing are proportional to the surface supervelocity, u say, in the direction of flow (true within the limitation of the theory, that u must be small) it follows that if the variation of the supervelocity, with α, h, ϵ, t, η and angle of sweepback are known, then it is possible to calculate the forces on the real wing in compressible flow. For if the typical supervelocity, u, in an incompressible flow of velocity Uo may be written, by virtue of the linearity assumed,

$$u = Uo\left[f_1(\alpha,\Lambda) + f_2(h,\Lambda) + f_3(\epsilon,\Lambda) + f_4(t,\Lambda) + f_5(\eta,\Lambda)\right]$$

then clearly

$$u' = Uo\,[f_1'(\alpha',\Lambda') + f_2'(h',\Lambda') + f_3'(\epsilon',\Lambda') + f_4'(t',\Lambda') + f_5'(\eta',\Lambda')]$$
$$= Uo\left[f_1'\{\beta\alpha, \tan^{-1}(1/\beta)\tan\Lambda\} + f_2'\{\beta h, \tan^{-1}(1/\beta)\tan\Lambda\} + \ldots \text{etc.}\right]$$

where $f_n(p, q)$ signifies some function of p and q.

Hence from application of the rule that supervelocity, say u_M, in the flow direction in compressible flow is $1/\beta^2$ times that in the analagous incompressible flow it follows that:

$$u_M = u'/\beta^2 = (1/\beta^2)\,Uo\left[f_1\{\beta\alpha, \tan^{-1}(1/\beta)\tan\Lambda\} + f_2'\{\beta h, \tan^{-1}(1/\beta)\tan\Lambda\} + \ldots \text{etc.}\right]$$

Hence u_M can be found if the functions f_1, f_2, etc., are known. The following examples of the application of this method will help to make it clear.

2.1.1. *Variation of lift curve slope with Mach Number.*

In this case let u be the supervelocity at a certain point on the aerofoil due to the fact that the aerofoil is lifting, *i.e.*, write:

$$\frac{u}{Uo} = k\,C_L \text{ where } k \text{ is some constant}$$
$$= k\ a\ \alpha$$

where $a = dC_L/d\alpha$

In incompressible flow, for a straight wing,

$$a = \{\ \pi A/(\pi A + a_\infty)\ \}\ a_\infty \text{ (approx.)}$$

where a_∞ = 2-dimensional lift slope

A = Aspect ratio of wing.

The analysis of many wind tunnel tests shows that for a sweptback wing it is a fair approximation to write:

$$a_\Lambda = a_0 \cos^{\frac{1}{2}}\Lambda \tag{3}$$

where a_Λ = slope of lift curve of sweptback wing

a_0 = slope of lift curve of straight wing of same aspect ratio.

Λ = Angle of sweepback.

$$\therefore\quad a_\Lambda = \{\ \pi A/(\pi A + a_\infty)\ \}\ a_\infty \cos^{\frac{1}{2}}\Lambda \tag{4}$$

$$\therefore\quad \frac{u_\Lambda}{Uo} = k\,a_\Lambda \alpha = k\ \{\ \pi A/(\pi A + a_\infty)\ \}\ .\ a_\infty\ .\cos^{\frac{1}{2}}\Lambda\ .\ \alpha \tag{5}$$

Thus

$$f_1(\alpha,\Lambda) = k\ .\ \{\ \pi A/(\pi A + a_\infty)\ \}\ .\ a_\infty\ .\cos^{\frac{1}{2}}\Lambda\ .\ \alpha$$

Considering now the analagous wing in incompressible flow:

$$\frac{u'_\Lambda}{Uo} = k\ .\ \{\ \pi\beta A/(\pi\beta A + a_\infty)\ \}\ .\ \alpha_\infty\ .\cos^{\frac{1}{2}}\Lambda'\ .\ \beta\alpha$$

Since $\tan\Lambda' = (1/\beta)\ \tan\Lambda$, with $\beta = \sqrt{(1 - M^2)}$ it follows that $\cos\Lambda' = (\beta\cos\Lambda)/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}}$, so that

$$\frac{u'_\Lambda}{Uo} = k\ \{\ (\pi\beta A)/(\pi\beta A + a_\infty)\ \}\ .\ a_\infty\ \{\ (\beta\cos\Lambda)/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}}\ \}^{\frac{1}{2}}\ .\ \beta\alpha$$

$\therefore$ The typical supervelocity $u_{\Lambda m}$ of the real sweptback wing in compressible flow is given by:

$$\frac{u_{\Lambda M}}{U_0} = (1/\beta^2)\frac{u'_\Lambda}{U_0} = (1/\beta^2)\ .\ k\ .\ \{\ \pi\beta A/(\pi\beta A + a_\infty)\ \}\ .\ a_\infty\ \{\ (\beta\cos\Lambda)/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}}\ \}^{\frac{1}{2}}\ .\ \beta\alpha \tag{6}$$

$$\text{But } \frac{u_{\Lambda M}}{U_0} = k(C_L)_{\Lambda M}$$

$$\therefore\quad \left(\frac{dC_L}{d\alpha}\right)_{\Lambda M} = \frac{d}{d\alpha}\left(\frac{u_{\Lambda M}}{U_0 k}\right)$$
$$= (1/\beta^2)\ .\ \{\ \pi\beta A/(\pi\beta A + a_\infty)\ \}\ .\ a_\infty\ .\ \{\ (\beta\cos\Lambda)/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}}\ \}^{\frac{1}{2}}.\ \beta$$
$$= \{\ \pi A/(\pi\beta A + a_\infty)\ \}\ .\ \{\ (1 - M^2)/(1 - M^2\cos^2\Lambda)\ \}^{\frac{1}{2}}.\cos^{\frac{1}{2}}\Lambda\ .\ a_\infty \tag{7}$$

From this it follows that

$$\frac{\left(\frac{dC_L}{d\alpha}\right)_{\Lambda M}}{\left(\frac{dC_L}{d\alpha}\right)_{\Lambda}} = \frac{\{\ \pi A/(\pi\beta A + \alpha_\infty)\ \}\ \{\ (1 - M^2)/(1 - M^2\cos^2\Lambda)\ \}^{\frac{1}{2}}(\cos^{\frac{1}{2}}\Lambda)\alpha_\infty}{\{\ \pi A/(\pi A + a_\infty)\ \}\ (\cos^{\frac{1}{2}}\Lambda)a_\infty}$$
$$= \{\ (\pi A + a_\infty)/(\pi\beta A + \alpha_\infty)\ \}\ \{\ (1 - M^2)/(1 - M^2\cos^2\Lambda)\ \}^{\frac{1}{2}} \tag{8}$$

where $\left(\frac{dC_L}{d\alpha}\right)_\Lambda$ = Slope of lift curve for swept wing in incompressible flow.

With $\Lambda = 0$ this expression reduces to

$$\frac{\left(\frac{dC_L}{d\alpha}\right)_M}{\frac{dC_L}{d\alpha}} = \frac{\pi A + a_\infty}{\pi\beta A + a_\infty} \tag{9}$$

which is the usual formula for straight wing, which, in turn when $A = \infty$ becomes the well known Glauert Rule.

A comparison of equation (8) with such test data as was available (24) gave quite good agreement, being practically exact in a case with $\Lambda = 35°$ and throughout better than the usual expression of equation (9).

Qualitatively, equation (8) means that for a sweptback wing the increase in $dC_L/d\alpha$ with M is less than for a straight wing.

2.1.2. *Variation of* $\frac{dC_{mo}}{d\eta}$ *with Mach Number.*

This example is considered as a further illustration of the general method.

The quantity $dC_{mo}/d\eta$ is the change in no-lift pitching moment coefficient with flap angle for full span flaps, thus it may be regarded as the rate of change of section no-lift moment with flap angle.

For a straight wing the appropriate supervelocity in incompressible flow may be written:

$$\frac{u_\infty}{U_o} = k\eta \ (k = \text{constant}).$$

Reliable information on the effect of sweep has not been found; but Ref. 15 suggests that $dC_{mo}/d\eta$ is very roughly proportional to $\cos\Lambda$ so that:

$$\frac{u_\Lambda}{U_o} = k\eta\cos\Lambda \tag{10}$$

$$(i.e., f_\infty(\eta,\Lambda) = k\eta\cos\Lambda)$$

$$\therefore \quad \frac{u}{U_o} = k\beta\eta\left\{\beta/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}}\right\}\cos\Lambda$$

hence:

$$u_{\Lambda M}/U_o = u'_\Lambda/\beta^2 U_o = (k\eta\cos\Lambda)/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}} \tag{11}$$

$$\therefore \quad \frac{u_{\Lambda M}}{u_\Lambda} = \frac{(k\eta\cos\Lambda)/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}}}{k\eta\cos\Lambda}$$

$$= 1/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}}$$

Thus obviously

$$\frac{\{(dC_{mo})/(d\eta)_{\Lambda M}\}}{\{(dC_{mo})/(d\eta)_\Lambda\}} = 1/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}} \tag{12}$$

This therefore compares with the usual $1/(1 - M^2)^{\frac{1}{2}}$ for straight wings.

Comparison of (8) and (12) shows that:

$$\frac{\text{Mach No. correction for } dC_L/d\alpha \text{ for sweptwings}}{\text{" " " " " " straight wings}} = \{(1 - M^2)/(1 - M^2\cos^2\Lambda)\}^{\frac{1}{2}} \tag{13}$$

whereas:

$$\frac{\text{Mach No. correction for } dC_{mo}/d\eta \text{ for swept wings}}{\text{" " " " " " straight wings}} = \{(1 - M^2)/(1 - M^2\cos^2\Lambda)\}^{\frac{1}{2}} \tag{14}$$

and the reason for the difference between (13) and (14) lies in the fact that a different rate of variation with sweepback of the characteristic considered was taken for the two cases.

2.1.3. *Comment on method.*

The method described in paragraph 2.1 and used in paragraphs 2.1.1 and 2.1.2 on pp. 119 and 120 to allow for compressibility effects is very rough and ready, but it does possess the great advantage that it enables use to be made of calculations or of empirical data obtained from wind tunnel tests in incompressible flow. At the present time low-speed wind tunnel tests have to provide nearly all the basic information, so the method may be acceptable as a temporary expedient until such time as more precise methods become available; another advantage is that it is simple and easy to apply.

2.2. INCREASE OF CRITICAL MACH NUMBER BY SWEEPBACK.

The usual simple yawed infinite wing explanation given for this effect is unsatisfactory for a number of reasons, *e.g.* (*a*) it obviously breaks down at the centre line of the wing, (*b*) it involves having different t/c ratios (in the flight direction) for the straight and swept wings considered and since the t/c for the swept wing comes out to be less than that for the straight one it therefore makes the sweepback explanation less convincing; and finally (*c*) it cannot be applied simply to compare straight and swept wings carrying equal lifts, obviously the important case.

A better start on the problem is furnished by a calculation done by Ludwieg* of Göttingen who evaluated the supervelocities in compressible flow at the wing centre line (which he appropriately called "the most endangered centre section") for a sweptback wing with a symmetrical bi-convex section. His calculations can be generalised to show that at the centre line the supervelocity on a swept wing in incompressible flow may be written:—

$$\frac{u}{Uo} = k \,.\, t/c \,.\, \cos\Lambda. \qquad (15)$$

Where

k = constant.

t/c = thickness/chord ratio in the flight direction.

Λ = Angle of sweepback.

This is analagous to expression (10), so that by comparison with (11)

$$\frac{u_{\Lambda M}}{Uo} = k(t/c)(\cos\Lambda)/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}} \qquad (16)$$

Hence comparing compressible supervelocities for swept and straight wing:

$$\frac{\text{Compressible supervelocity for swept wing}}{\text{" " " straight "}} = \frac{u_{\Lambda M}}{u_M}$$

$$= (\cos\Lambda)\,\{\,(1 - M^2)^{\frac{1}{2}}/(1 - M^2\cos^2\Lambda)^{\frac{1}{2}}\,\} \qquad (17)$$

From this it is clearly possible to compare the critical Mach numbers for straight and swept wings. The result of this calculation is given in Fig. 8 in which the abscissa is the critical Mach number of a straight wing and the ordinate the corresponding critical Mach number of a swept wing having the same section in the flight direction and having the sweepback indicated. In this figure the full lines correspond to Ludwieg's theory, and for comparison the dotted lines corresponding to the simple $\sqrt{\ }$cosine rule are also given: while as a check some experimental points are plotted from Ref. 19. The $\sqrt{\ }$cosine rule means simply that, as a rough guide one may write:—

$$\text{Critical Mach Number of Swept Wing} = \frac{\text{Critical Mach number of straight wing}}{\text{square root of cosine of angle of sweep}}$$

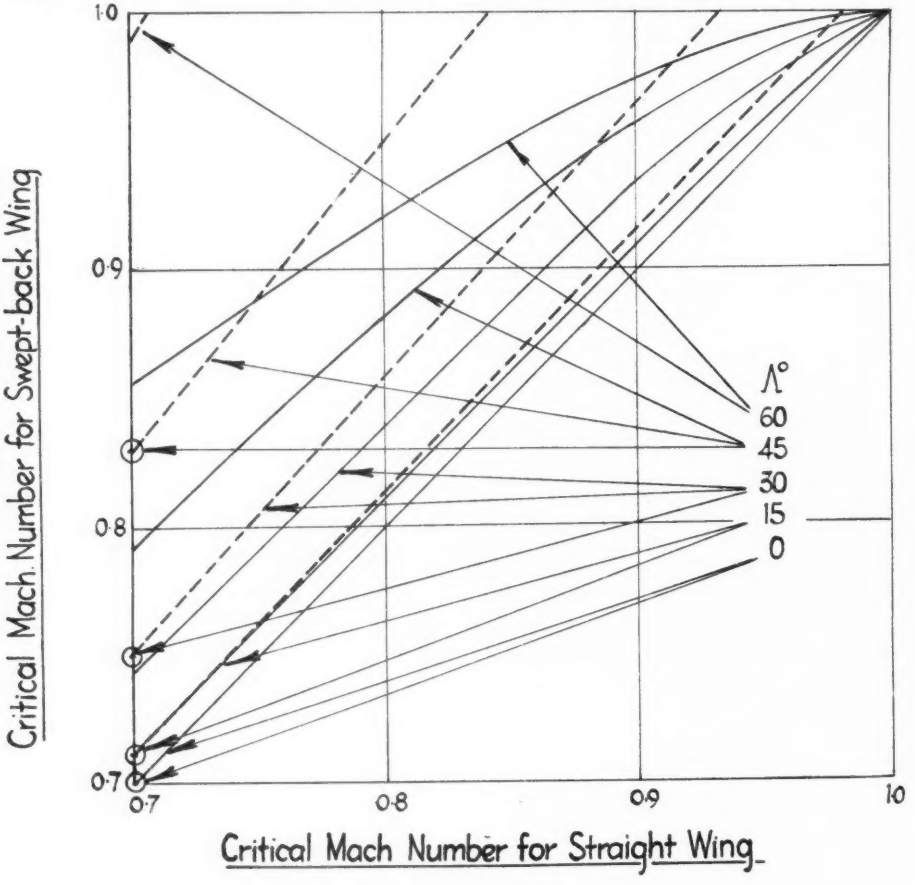
It will be seen that the $\sqrt{\ }$cosine rule seems to fit the very meagre experimental data better than Ludwieg's more elaborate theory, so that, until more data become available, one feels inclined to stick to the simplicity of the empirical $\sqrt{\ }$cosine method. The experimental points are the Mach numbers at which the drag began to rise.

It is clear that Ludwieg's calculation can form the basis of a method for calculating

* It should be noted that since writing this paper information has been received which shows Ludwieg's calculations to have been in error.

the critical Mach number of a sweptback wing carrying lift, since a combination of Ludwieg's calculation and any reasonable method of calculating the lift distribution of a sweptback wing will give the incompressible supervelocity on a wing which can then be turned into the compressible supervelocity by Prandtl's rule.

The process would be, essentially, to choose a Mach number, derive the analagous incompressible flow wing, work out the no-lift supervelocity (using equation (15)) and then find what C_L would be necessary to bring the total incompressible supervelocity up to such a value that the corresponding compressible supervelocity would make the chosen Mach number the critical one. This implies that the supervelocity in incompressible flow is the sum of the no-lift supervelocity plus a supervelocity proportional to the local C_L; for a given section the factor of proportionality would be known. Perhaps it is more logical to use the "ideal" C_L instead of zero lift to get the basic supervelocity. Having thus found the incompressible C_L required, it follows, from Prandtl's Rule, that the corresponding compressible C_L will be $1/(1-M^2)$ times this C_L.



——— Ludwieg's method. — — — — $\sqrt{\text{cosine}}$ rule. (·) Experimental points.

Fig. 8.

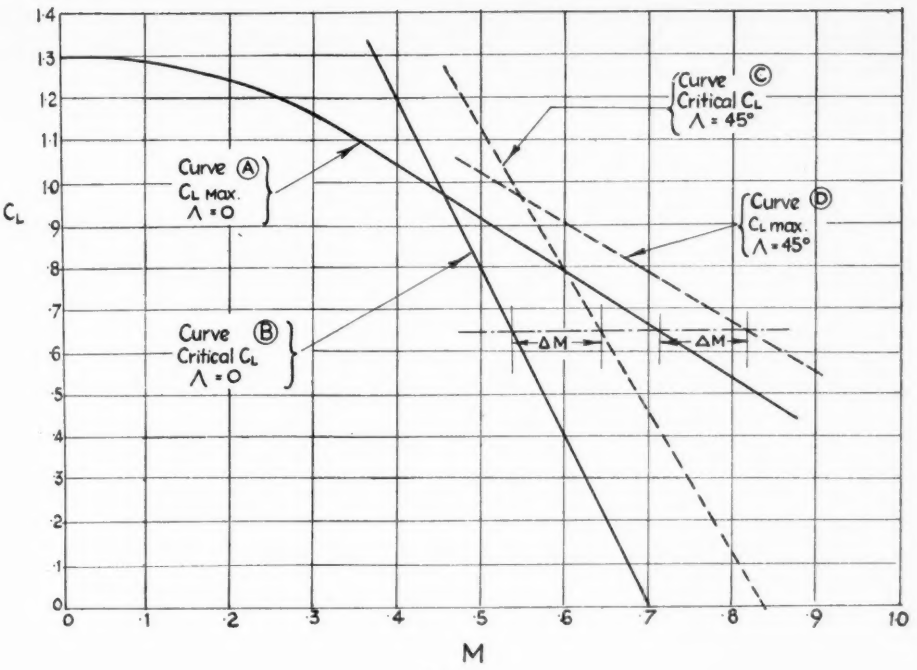


Fig 9.

However, since Ludwieg's calculations have not yet been substantiated by tests (rather the reverse in fact) it is doubtful if such a long process is worth while. If a really satisfactory method could be found of getting the complete supervelocity distribution on a sweptback wing in incompressible flow, then the problem could be considered as solved. The fact that Ludwieg does consider only "the most endangered centre section" is a drawback, since usually there is a body or nacelle there in any case, so in that case the critical area may be elsewhere on the wing; Goethert (20) suggests that a well designed body could raise the critical Mach number of a sweptback wing. Clearly in this field there is a wide scope for more tests and also more theory.

2.3. The stalling of sweptback wings at high Mach number.

In the design of aircraft to operate at high subsonic Mach number the maximum lift coefficient available at high speed is very important because of its influence on the manœuvrability of the aircraft, since it limits the normal acceleration that can be developed.

If $C_{L\max}$ for a straight wing is plotted against Mach number a curve such as curve A of Fig. 9 is obtained (this curve is empirical). Another curve, curve B, may also be plotted on Fig. 9; this is the curve of critical Mach number against C_L, *i.e.*, at any given Mach number, M_1, the C_L read from curve B, C_{L1}, is the C_L for which M_1 is the critical Mach number.

It is clear that at low M's the $C_{L\max}$ is determined by the ordinary stall, while it is also clear that at high M's the stall will be principally governed by shock waves. Curve B of Fig. 9 corresponds to the onset of shock waves and it is clear from this figure that there is a considerable interval between critical C_L and $C_{L\max}$ at high Mach number.

The details of the stalling process in the presence of shock waves do not seem to be clearly understood at the present time and this constitutes quite a serious gap in our knowledge.

Turning to sweptback wings, there are two opposing tendencies, firstly the fact that at low Mach numbers the $C_{L\max}$ of a swept wing is less than that of a straight one, and secondly the fact that the onset of shock waves is delayed by sweepback. From this it can be argued that at high Mach numbers the $C_{L\max}$ of a swept wing should be higher, since under these conditions of high Mach number the reduction in $C_{L\max}$ is brought about by shock waves. Reference to curves B and C of Fig. 9 will help to make this clear. Curve C is the critical $C_L - M$ curve for a wing with 45° of sweepback derived from curve B by the application of the simple $\sqrt{\text{cosine}}$ rule. Consider conditions $M=0.7$: here the straight wing has shock waves just starting at $C_L=0$, so that all the available C_L must develop in the presence of shock waves; on the other hand, the 45° wing does not begin to show shock waves until $C_L=0.45$ is reached. Therefore, surely it is reasonable to contend that the sweptback wing will reach a higher maximum C_L before stalling. Or to put it in another way, at a given C_L, sweepback delays the onset of shock waves to a certain higher Mach number; one is then inclined to suggest that a similar displacement to higher Mach number could be expected in the $C_{L\max}$ curve.

The displacement process described above has been applied in Fig. 9 to derive curve D from curve A. Test data are very scanty, but such evidence as there is suggests that curve D is not far out at high Mach numbers (24).

Another point of interest regarding the stall of sweptback wings is the fact that tests show that the tip stall may persist to high Mach numbers (21). At first sight this is unexpected, since the shock waves would be expected to occur first at " the most endangered centre section," so that it is here that the lift would be first expected to break down. Two modifying influences may be suggested: (i) the fact that at moderate C_L's the reduction in local C_L at the centre line may imply that shock waves do not in fact first occur here when the wing is near $C_{L\max}$; and (ii) the fact that a shock wave constitutes a very strong adverse gradient and so will greatly aggravate the outwards drift of boundary layer and so increase the tendency to tip stalling (see paragraph 1.2.1).

2.4. Concluding remarks on compressibility effects.

The outstanding impression obtained after surveying this aspect of tailless aircraft design is that a great deal more data is required and that much work needs to be done.

In order to deal satisfactorily with subcritical phenomena much more systematic information on the effect of sweepback on the important aerodynamic properties of a wing is needed in order to make " Prandtl's Rule " the very effective tool that it might be.

It is clear that a great deal more must be discovered regarding the trans-sonic region as this effects such matters as $C_{L\max}$ at high Mach number and also pitching behaviour in this critical condition. The trans-sonic problems are not, of course, confined to sweptback wings, but in this case, too, such wings are, as usual, a degree more complicated than wings without sweep.

3. AEROELASTIC PROBLEMS

3.1. General.

The fundamental difference between sweptback and straight wings from the aeroelastic point of view is that, for the sweptback wing, spar bending produces a change of incidence, whereas for the straight wing it does not do so.

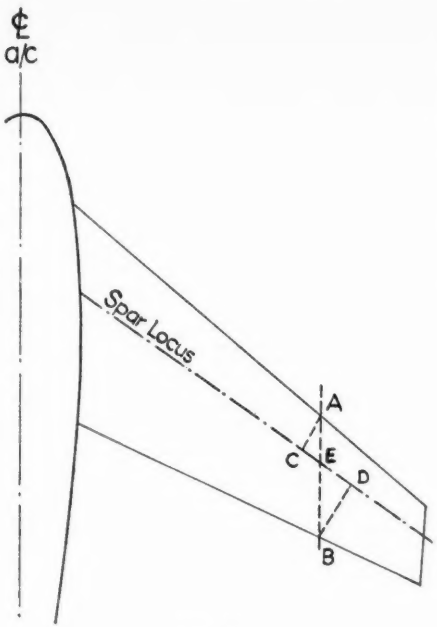


Fig. 10.

The reason for this may be seen from Fig. 10. The total change of incidence $\Delta\alpha$ of the chord AB may be written:

$$\Delta\alpha = (1/AB)(\theta_C \,.\, AC + \theta_D \,.\, BD + \delta_C - \delta_D) \quad (18)$$

when the angles concerned are small, and where:

θ_C = Twist of wing about spar locus at station C

θ_D = Twist of wing about spar locus at station D

δ_C = Upwards bending deflection of wing at station C

δ_D = Upwards bending deflection of wing at station D

The expression for $\Delta\alpha$ given above is obtained by dividing the vertical displacement of A relative to B by the length AB. Structurally it is convenient to consider torques about an axis parallel to the spar locus and bending moments about an axis normal to the spar locus, since in this way the wing can be treated in a perfectly ordinary manner.

It is thus seen that part of the elastic incidence change, $\Delta\alpha$, is due to the torque terms, θc and θ_D, and part to the bending deflections, δ_C and δ_D. Under an upload δ_D will be greater than δ_C, so that up-bending reduces incidence.

To quite a good approximation it is possible to write:—

$$\Delta\alpha_E = \theta_E \cos\Lambda - \phi_E \sin\Lambda \quad (19)$$

where:

$\Delta\alpha_E$ = change of incidence at station E, $+ve$ with $L.E.$ up.

θ_E = Twist at station E, $+ve$ with $L.E.$ up.

ϕ_E = Slope of spar at station E, $+ve$ when rising towards tip.

Λ = Sweepback of spar locus.

The importance of this dependence of incidence on flexure as well as torsion may be seen by considering two examples.

3.2. Manœuvre point.

Consider first an aircraft with sweptback wings pulling out from a dive. The pull-out will increase the up-load on the wings so that upwards deflection of the tips will occur and this will reduce the tip incidence (the root being unaffected), thus producing a washout along the wing and so giving rise to a positive change in aircraft C_{mo} which must result in an increase in the rate of pull-out. That is to say, wing bending shifts the manœuvre point forwards. If, as is usual, the flexural axis lies behind the locus of aerodynamic centres the torsional deflection in pull-out will increase the tip incidence and so will tend to reduce the rate of pull-out. Thus in this instance, torsion and flexure act in opposing senses. As in all such cases, the importance of elastic distortion increases with increasing speed.

3.3. Aileron reversal.

Secondly, consider the question of aileron (or elevon) reversal.

Let:

Λ = Angle of sweepback of the spar.

M_o = Root bending moment of spar about axis normal to spar locus.

T_o = Root torque about axis parallel to spar locus,

then:

Rolling moment from half wing
$$= M_o \cos \Lambda + T_o \sin \Lambda \qquad (20)$$

At reversal the rolling moment must be zero, hence:

$$M_o \cos \Lambda + T_o \sin \Lambda = 0 \qquad (21)$$

Since, in general, T_o is not zero when the aileron is deflected it follows that M_o will not be zero, so that bending deflections clearly enter into the problem. In the usual case lowering the aileron twists the leading edge downwards and produces an up-load on the outer part of the wing and so results in upwards bending of the tips. Thus under application of aileron both torsion and flexure combine to reduce rolling effectiveness so that a given reversal speed fixes lower limits to both torsional and flexural stiffnesses.

Figure 11 shows a typical plot of the necessary conjugate values of torsional and flexural stiffness to give a certain specified reversal speed. In this figure, m_θ is the usual stiffness measure (torque at a reference section to produce a twist of one radian) while l_φ is similarly a measure of the bending moment necessary to produce unit slope of the spar at the reference section. It can be seen from Fig. 11 that there must be a certain combination of m_θ and l_φ which will give the minimum weight of material necessary to meet the reversal stiffness requirements, for clearly infinite m_θ or infinite l_φ both imply infinite wing weight so that a minimum would be expected to lie somewhere on the curve near to the origin. There is little practical information available on this subject, but the indications are that there is quite a sharp minimum in the stiffness weight required, *i.e.*, stiffness weight is quite sensitive to the conjugate pair of m_θ and l_φ values taken.

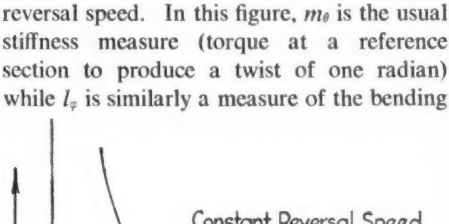


Fig. 11.

Another matter of great importance in connection with the reversal of sweptback wings is the values to be given to the aerodynamic derivatives

$$\left(\frac{dC_L}{d\gamma} = a_1, \frac{dC_L}{d\xi} = a_2, \text{ and } \frac{dc_{m_0}}{d\xi} = m \right)$$

especially at high speeds where the compressibility corrections are large.

The standard procedure for estimating reversal speed has been to apply simple strip theory, using two-dimensional values of a_1, a_2 and m in order to determine the aerodynamic loading on the distorted wing with aileron deflected; to allow for compressibility effects, the Glauert factor $1/(1-M^2)^{\frac{1}{2}}$ was applied. To begin with, the same procedure was applied to a typical high-speed sweptback wing with the result that very high value of m_θ was determined, resulting in a very high wing torque skin weight (about 6 or 7 per cent. of the all-up weight) being found necessary. Considered critically, it becomes clear that the standard procedure is very pessimistic, since:—

(*a*) It does not allow for the reduction in a_1, a_2, and m which must accompany sweepback.

(*b*) For the above reason, the Glauert correction for Mach number is very severe.

(*c*) The whole idea of using two-dimensional derivatives for strip theory is severe (and this applies equally to unswept wings) since it is clear that with a finite aspect ratio much lower values will apply. In fact Schrenk (22) brought forward evidence to show that with an aspect ratio of about 6, a reasonable approximation to the lift distribution could be obtained by taking the effective value of a_1 to be half its two-dimensional value.

In order to demonstrate the importance of the values assigned to the derivatives, an account is given below of calculations done with differing values of a_1, a_2, and m.

Calculations were done in which empirical allowance was made for the variation of the derivatives with sweepback, which same empirical rules were used to estimate compressibility effects by the method described in paragraph 2.1; allowance was also made for aspect ratio, by a method similar to that suggested by Neumark (23).

These calculations showed that a great reduction in the required stiffness followed from the more logical approach. In the first instance (call it case A) the derivative $m\,(=d_{c_{mo}}/d\xi)$ was assumed to be proportioned to cos Λ (Λ = angle of sweepback) as this was indicated by some German tests (15) which were, however, very scattered and unconvincing; the variation of the derivative a_1 was taken to be of the form given by equation (7). This calculation showed a big reduction in the required values of m_θ and l_τ and it was decided to investigate the effect of changing the assumptions regarding m. In this next instance (case B) the relationship between the m used and the two-dimensional value was assumed to be the same as for a_1, and led to the result that, for a given l_τ the m_θ for case B was about 50 per cent. greater than for case A.

Roughly, the results seem to be:—

Stiffness required with case A assumption = 30 or 40 per cent. of the value from two-dimensional strip theory.

Stiffness required with case B assumption = 50 per cent. more than case A.

At the time of writing, this reversal question is under general discussion so that it is difficult to draw conclusions. However, in view of the great weight penalty involved by following the established procedure with regard to derivatives it was thought to be worth while to refer to the preliminary *ad hoc* calculations mentioned above, as they do demonstrate clearly the vital importance of getting the correct derivatives as well as emphasising the need for really systematic knowledge of the way in which aerodynamic characteristics vary with sweepback. It is suggested that tests of representative elastic model sweptback wings in the wind tunnel might well be worth checking whatever theories are developed.

3.4. Concluding Note on Reversal and Manœuvre Point.

Considering only manœuvre point and aileron reversal (ignoring completely the important field of flutter) it is reasonable to suggest that the optimum values of m_θ and l_τ from the reversal point of view should be worked out and that then one should use a value of l_τ somewhat greater than that (with m_θ correspondingly smaller) in order to assist matters from the aspect of manœuvre point.

4. CONCLUDING COMMENTS.

Despite the considerable advances made in the technique of tailless aircraft design during the past twelve months or so, the conclusion, "yet much remains to conquer still," is inescapable, especially with regard to the modern conception of the high-speed, high-altitude aircraft with heavily sweptback wings.

No really satisfactory solution has yet been found for the tip-stall problem and it is

suggested that some fundamental re-design of aerofoil profiles may be necessary.

It appears that in order to deal with the calculation of sub-critical compressibility effects it is going to be necessary to extend considerably our systematic knowledge of the effect of sweepback in incompressible flow. An aircraft designed to fly at just less than critical speed will, at times, have to venture into trans-sonic region, so that knowledge of the phenomena to be encountered there is vital and, is unfortunately at the present time, very scanty.

In the domain of aeroelasticity, too, there is wide scope for both fundamental study of phenomena and for the development of a practical technique of calculation when sweptback wings are considered. Special emphasis is laid on the importance of determining accurately the aerodynamic derivatives.

Lateral stability has not been mentioned at all in this paper, and upon investigation may well prove to contain problems of considerable magnitude when high sweepback is considered. The large increase in $-l_v$ with C_L when the sweepback is heavy constitutes a possible source of difficulty.

Clearly, indeed, the difficulties of tailless design are considerable, but equally clearly, in many cases, we can see the way which will lead to their solution. Surely it is not too much to hope that the remaining obstacles likewise will surrender to our attack.

In conclusion, I should like to express my thanks to Handley Page Ltd. for permission to make use in this paper of work done on their behalf and to the Ministry of Supply (Air) for permission to refer to restricted reports.

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DISCUSSION

Mr. W. G. A. Perring (Director, Royal Aircraft Establishment, Fellow): Mr. Lee's hypothesis on the question of maximum lift at low speeds was extremely interesting and probably was not very far from the truth. He supported the proposals made in the paper for exploiting to the full the possibilities of the application of suction in the attempt to prevent wing tip stall.

Work on the problem of maximum lift by the use of the nose flap had provided that maximum lift at very much increased incidence, and, therefore, provided it in a form which was not very attractive to the aircraft designer. But the combination of nose suction and rear suction might well overcome both the stalling and the incidence problems.

On the problem of maximum lift at high speeds, the difficulties were still greater. At high speeds, so far as the results known to him were concerned, they had not been able to make use of the maximum lifts that were possible, because on tailless types it had been found impossible to provide sufficient trimming moment; therefore, although the paper was centred on and written around the tailless types, he had a feeling that, for flying at high speed and using swept-back wings, they might be forced to reintroduce a tail in order to provide the trim necessary to utilise the full lift which they knew to be possible. That was quite a serious problem, because when one considered the low maximum lift coefficients at high speeds and remembered the serious gust conditions to be catered for in any design it became almost impossible, within reasonable dimensions, to design an aircraft to meet the requirements.

In the high-speed field, too, there were serious experimental difficulties. The high-speed tunnel which had been used so far to explore high-speed work was beginning to break down. The high-speed tunnel at Farnborough was used with fair confidence up to Mach Numbers of .8 and .85; but when testing designs which introduced angles of sweepback of the order of 45° or more, one clearly put the critical Mach Number effects well above a Mach Number of unity, and the tunnel was powerless to handle the work. That was true, not only in respect of the lift and pitching moment work, but in respect of derivatives, about which, the author had pointed out, there was quite inadequate information available. The opportunity that was open to them, of using free flight models, although it might give them information on lift and trim, would give rise to prickly and

thorny problems if they had to tackle derivative work in that way.

Mr. Lee: He agreed with most of Mr. Perring's remarks. With regard to the omission of the problem of trim, he recalled his earlier statement that his paper was not intended to be a comprehensive summary; and—trim was one of the subjects he did not " comprehend " in the paper.

He had commented also on the argument as to whether or not a tail should be used because there was no doubt that the problem was one of the most knotty that they had to deal with at present. The question of tail or no tail would be decided by problems of pitching balance. At low speeds the need for high lift flaps might result in a tail being necessary to balance their pitching moment, although the considerable fore and aft extension of a heavily sweptback wing would probably result in a tail being unnecessary on this account. The high-speed pitching balance question was very obscure at the present time because of the lack of data on the behaviour of aircraft in the trans-sonic regime. A tailplane would certainly increase pitching control and damping, which was in its favour, but on the other hand would be influenced by downwash changes and so might possibly cause trouble; thus a loss of downwash due to the main wing shock stalling would result in an undesirable diving tendency.

The remarks by Mr. Perring on the limitations of the high-speed tunnel were of great interest. It would be very difficult to deal with derivatives at high-speed with free flight models; but they had to get down to those derivatives somehow, and it was for someone to make really bright suggestions for alternatives to the making and flying of many piloted machines, which was a long-winded and expensive process anyhow. They might have to be content with getting the low-speed derivatives for a good range of angles of sweepback and then rely on the Prandtl Rule to allow for compressibility (see 2.1.3.).

Mr. E. N. B. Bentley (Percival Aircraft, Associate Fellow): The word " tailless " in relation to aircraft apparently had not the same meaning for everybody. In most cases it seemed to imply the abolition of the horizontal tail. He and others would like to know Mr. Lee's opinions on the need, or otherwise, for vertical surface. The Northrop aircraft was perhaps one of the few which seemed to have flown successfully without a vertical surface either at the wing tips or sprouting out from the end of the fuselage.

Mr. Lee: At the moment he was in favour of tip fins. In the first place, unless one had a real all-wing aircraft it was necessary to have vertical surfaces somewhere. As soon as a body was stuck on to any kind of a sweptback wing, the whole thing became unstable directionally, unless there were vertical stabilising somewhere. A wing alone had a favourable directional stability; but if a body was put on it, it became unstable.

Discussing the reason why he favoured wing tip fins, he said that with wing tip fins one should be able to cash in on a reduction of induced drag. Again, if a lateral stability calculation for a tailless aeroplane was done —he was speaking of ordinary slow-speed— one of the things it was found to be short of was damping of angular velocity in yaw. If the arrangement was made that the fins carried an inwardly directed load, and if one worked through the calculations, it was found that such fins gave an increase in damping of angular velocity in yaw. That was another good job the fins could do.

Furthermore, a tailless aircraft with wing tip fins had a rear fuselage that was free from major aerodynamic loads, an advantage that was lost if a rear vertical fin and rudder were mounted on the fuselage.

A fin should not just be slapped on the end of a wing. If fins were designed with the care with which wings were designed, much of the disfavour of the wing tip fin would disappear. *(See also reply to Mr. A. R. Weyl).*

Mr. G. T. Dobson (R. Malcolm Ltd., Graduate): By moving the fin inwards along the span, could improvement be effected with regard to wing tip stall?

Mr. Lee: That would go some way towards clearing up tip-stall problems. But the main drawback was that the fin had then to be larger and heavier, and that gave rise to more drag. There was little data available with regard to it. Basically there was no doubt that, if the fin were large enough, it could improve matters in respect of tip-stall; but the fin must be big—a chord or so in diameter if it were circular.

Afternoon Session

Dr. H. Roxbee Cox (Vice-President) in the chair.

FLYING BOAT PROBLEMS RELATED TO PRODUCTION, CONTROLS AND PRESSURISATION

by

C. P. T. LIPSCOMB, F.R.Ae.S.

Mr. Lipscomb served his apprenticeship at the Royal Arsenal, Woolwich, and joined Short Brothers Ltd. in 1914. *From* 1916 *to* 1921 *he was Chief Draughtsman at the Airship Works at Bedford and then rejoined Short Brothers at Rochester. In* 1943 *he was appointed Chief Designer and in* 1945, *Director.*

THIS paper was requested to deal with (1) the future relations between designers and production; (2) the special problems to be faced by the production people and the assistance to be expected from the designers; and (3) how such design problems as pressurisation, silencing and the fitment of turbine engines were being tackled.

These terms of reference are very broad and probably can only be touched upon in a general way. I, personally, have been engaged entirely on the design side for the past thirty years and therefore it might have been more fitting if a member of the production organisation had been asked to give this paper.

The experience of the recent war has emphasised the importance of the production aspect of the problem and although design is still pre-eminent yet we can no longer afford to neglect every possible consideration which will assist us to produce aeroplanes quickly in times of emergency. The balance between design and production is a delicate one and designers as a whole are very loath to see their structure weight percentage being increased in order to simplify production as in the past light structure weight has usually been achieved at the expense of cost. Fortunately with new methods of processing material such as the rubber press, Ceco stamp, stretching press, modern forming tools, etc., increased cost does not necessarily result from a light structure.

From the general aspect the problems relating to the flying boat are not dissimilar to those associated with the landplane, the main difference probably being that the quantities required are usually very much less. The same line of thought is applicable to the general design but the range of tooling and breakdown need not be so extensive.

Experience has proved that time is of even more value than money in times of crisis and therefore a planning system is vital in every factory. With the modern complexity of design and processes of manufacture a properly organised methods and planning department with co-ordinated liaison with other sections of the factory is an essential requirement.

A detailed discussion on the wide ramifications of the production department involving the analysis of the design, programme planning, material control and all the interlocking sections that combine to produce an efficient aeroplane in a reasonable time is outside the scope of this paper.

Our concern is the part played by the designer and how he can assist in simplifying the work of the production department.

It is clear that the closest liaison must exist between the design and production departments and I strongly advocate that planning engineers should be resident in the drawing office all the time that a design is on the drawing board. The draughtsman's work is onerous enough as he must produce something from a blank sheet of paper and I think it is unreasonable to expect him to produce the most efficient job without constant help and assistance from the planning engineers.

Planning for production is by no means easy and it is always a debatable problem as to the best way that this should be tackled. It is common practice to produce a minimum number of prototypes by semi-tooled methods in order that flight experience can be obtained as early as possible. Opinion is hardening towards producing about thirty of these pre-production aeroplanes in order that the full service requirements, including essential modifications, can be ascertained before a fully comprehensive tooling programme is undertaken.

This implies an extremely active development department as it is most essential that these pre-production aeroplanes are made available as quickly as possible in order that final production tooling can safely proceed.

In the case of the large flying boat or landplane such a large pre-production order would probably be impracticable and the number of prototypes might be limited to about three. This does imply, however, that essential modifications are continually being incorporated into the design during most of the useful life of the aeroplane.

The late Director General of Technical Development (D.G.T.D.) arranged for a series of interchange visits early in 1946 between certain firms for the purpose of discussing the whole problem of building prototypes followed by pre-production contracts and finally to the full production programme. To those firms who were privileged to take part in these discussions the visits proved extremely valuable as a frank interchange of views was possible and I believe that the majority of the participants learned something from one another. As a comprehensive series of reports was issued on the visits I need not discuss these in detail but I do feel, and I think many will agree with me, that such interchange of visits between firms is stimulating and should be encouraged in the interests of British aviation as a whole.

The design staff, in particular, are usually very pre-occupied with their own difficult problems and perhaps are inclined, and rightly so, to give them preference over the other important branches of the work. The production aspect must, however, be given its rightful place in the general scheme as even the finest aeroplane from the design and operational point of view may be a failure if its design features render it difficult or impossible to produce in a reasonable time and at a reasonable cost.

The guiding principle of the production team is to translate theory into reality, a problem which is becoming increasingly difficult inasmuch as higher quality of finish is being demanded, more complex techniques are being introduced and, of course, the customer does not want to pay more for the improved article which results. The prime enemy of the production team is cost and the work of this team is largely centred around producing the design at a cost commensurate with the price which the customer is prepared to pay. It is therefore impossible to divorce design from production and the two departments should be linked together with the closest of ties.

In actual fact production does not follow very far behind design because, even in the early stage of a new conception, the engineering side of production has been in close collaboration with the design section and therefore the project should, from the beginning, fit the production requirements and existing shop techniques.

With this collaboration all sections of the

works should have a clear idea of what is to be produced and how it is to be produced.

It is frequently said in the Drawing Office that "nothing that is ever designed is considered suitable for production." This statement may seem a little hard on the production department as, although given adequate tooling any complicated design can be produced at a cost, yet in the overall interests of the job it is obvious that the simpler a component is designed the better will be the opportunity of obtaining rapid and efficient production.

The first step therefore in any organisation is to maintain a close and continuous liaison between the design and producing departments.

Perhaps the most serious aspect of aeroplane design, again from the production department's point of view, is the multiplicity of pieces which go to make the modern aeroplane. With current methods of planning involving as they do, an elaborate and somewhat complicated system of paper work it is essential that the number of parts be kept to the absolute minimum in order to reduce cost in every department of the Works. This is a point often overlooked by the Drawing Office and it is only when we follow the progress of a part from the time the drawing leaves the Drawing Office until it appears in the finished part stores that one realises how necessary it is to keep this always in mind. At the same time, however, the main principle underlying large-scale rapid production is to build the aircraft in components, each capable of being as complete in themselves as possible before being joined to larger components.

This enables the largest possible labour force to be employed simultaneously on the particular design, which is finally reflected in the speed of production.

The task of co-ordinating these often diverging requirements falls on the Chief Production Engineer whose first job is to decide methods of manufacture with due regard to the size of the contract, the general techniques to be adopted and the grade of labour to be employed, all of which have to be geared to cost and time. Whether the design is a commercial or military type makes no real difference, price is a governing factor and so is time.

The use of photographic and lithographic reproduction in the Drawing Office has considerably expedited tool making and there is no doubt that these processes have greatly eased the Production Engineer's problems, particularly insofar as accuracy of contours is concerned. In the past the difference of interpretation on contours between the design office and the Works has been a major problem and has caused high assembly costs and endless modifications involving much delay in flight testing.

Uniformity is also achieved, as many reproductions can be obtained from the original full-scale layout, each identical with the other, thus eliminating errors due to several sections of the Works producing their own layouts and templates. The S.B.A.C. have issued much information on the subject and, I think, all are agreed that the advantages to the production side are so great that the process will be continued and expanded in the future.

The degree of tooling to be embarked upon for any given design is probably the most difficult decision the Production Engineer has to make. A failure to make the right assessment can and does seriously affect the obtaining of contracts. Interchangeability requirements need to be considered alike for civil and military aircraft. If interchangeability is required then the tools must be good enough to ensure it and to maintain it throughout the life of the contract. Interchangeability inside the factory also has to be considered as assembly breakdowns to facilitate production need to be made if assembly costs are to be kept within reasonable limits. All these considerations call for very close co-operation between the design and planning departments and often materially affect the initial design.

Reverting to the desirability of keeping the number of parts to a minimum this point was very well illustrated in the production of the Seabee by the Republic Aviation Corporation. The original design was produced in the drawing office on conventional lines and although an excellent job, viewed purely as an aeroplane, its selling price was more than twice what the manufacturers had envisaged. A radical re-design of the structure was undertaken involving a considerable amount of structural tests, as the final structure was not amenable to ordinary methods of strength computation and it was finally found possible to reduce very considerably the number of structural parts with an immense saving in cost.

It is true that, owing to the amount of tooling involved, a certain minimum number of aeroplanes must be built to achieve this result but there can be no doubt that if aeroplanes are to be produced at a reasonable cost, far greater consideration must be given in the future to this vital point of reducing to the absolute minimum the number of parts making up the structure.

The Drawing Office can also assist the organisation effectively in a careful watch on material specifications. No special material should be employed for which a cheaper substitute can be used.

It is certainly very tempting to take advantage of the many new materials that are continually being developed and often considerable benefits are obtained by their use, but there is always another side to the picture.

The job of the Chief Buyer is becoming increasingly difficult and responsible, controlling as he does probably about 20 per cent. of the total cost of making the product. Not only is he responsible for the first cost of all material, but he is also responsible for its usage and distribution after it has been received. He must also ensure that the material is received at a time to fit into the production programme. Materials which are difficult to obtain constitute for him a severe headache and their use needs to be amply justified by the Drawing Office. The position can be eased somewhat by giving the buyer, if possible, a range of alternative materials.

Perhaps the most controversial subject of all in the production of aeroplanes is the paper work.

Prior to the last war with production limited, the need for an extensive system of paper work was not very vital. Prototype and pre-production aeroplanes were produced reasonably cheaply and quickly with a minimum of system, the work mainly being controlled by the Works Managers and foremen. With the need for rapid expansion the defects of this system at once became apparent and it became necessary to devise production systems which ensured the supply of the numbers of aeroplanes required. This led to the appointment of production managers whose function it was to initiate the necessary paper work to get the job started and to keep it moving until the aeroplanes left the factory.

In aircraft work it is fair comment to say that the larger the aircraft the greater is the number of parts and, in proportion, the greater the number of bottle necks, for the simple reason that every individual piece is a potential bottle neck.

The Production Manager is responsible for the smooth passage of orders to the shops and also for the maintenance and representation of data which will give an up-to-date picture of the state of the work. This duty is important as things which go wrong need to be caught up with before the production machine comes to a standstill. The one thing which upsets all his work is inaccuracy, particularly when it relates to the drawings, schedules and method sheets. The results of such inaccuracies may prove extremely costly both in time and money. In this sphere of the work the Drawing Office undoubtedly can make a great contribution towards the success of an organisation by reducing initial errors to the absolute minimum. During the late war with the many inexperienced draughtsmen employed it was not always easy

to give the correct information to the shops, but to my mind it pays handsomely to employ in the Drawing Office an efficient checking system. This may slow up, initially, the output of work from the Drawing Office as the checking staff would normally consist of experienced men who, the Chief Draughtsman often thinks, should be employed on fundamental design. Therefore, it comes back to availability of staff and many of us have looked with envy on the wealth of fully qualified men that all American companies appear to obtain.

Any system of paper work depends very largely on the accuracy of the information received from the Drawing Office and if this accuracy can be kept at a high level then it is invaluable and without it any Company would soon come to a full stop and control of any sort would quickly disappear.

Production technique also plays an important part in the modern aeroplane and the Drawing Office should keep well abreast of all developments in this direction.

Many of the leading draughtsmen in the Industry to-day have grown up in the business and, if I may say so, are apt to be somewhat conservative in their ideas. It is difficult for them to visualise that the production of the modern aeroplane must be tackled somewhat on the lines of car production inasmuch that improved methods of manufacture must continually be investigated.

Two examples of these which have become almost universal are automatic riveting and the welding of aluminium alloys. There are many others but to me these two seem outstanding and they have done an excellent job on high quantity production during the war years. To take full advantage of these techniques, components must be designed initially to suit them and that imposes on the designer the necessity of knowing what can or what cannot be done with the equipment concerned. It is always extremely difficult without a complete re-design to adopt a particular production technique to a component for which it was not originally designed.

It would be advantageous if draughtsmen could be given refresher courses in the development of production aids, although this could be achieved to a certain degree by advice from the planning engineers attached to the Drawing Office.

Electric welding, in particular, was boldly used on both German and American aircraft where it was suitable and the saving in man hours must have been considerable. Continual development in the technique of welding may yet see a wider application of this technique even to the extent of complete aircraft being assembled by welding. In the construction of flying boats the chief problem to overcome will be the prevention of corrosion at the welds and the seams, a problem which is still far from being solved.

Perhaps the chief obstacle to rapid production, and one which to a certain degree can be controlled by the Drawing Office, is that of modifications.

Many, however, such as those arising from Drawing Office errors, are avoidable and I have already expressed the opinion that they can and should be avoided by the introduction of a thorough checking system.

Modifications caused as a result of operational experience or the introduction of new equipment are in a different category and there is a lot to be said for the introduction of modification centres such as were introduced in America during the war.

A contribution towards smoother relations between the shops and the Drawing Office over this difficult question of modifications, would be for the Drawing Office to discuss fully with the shops the reason for, and the necessity of, incorporating at an early stage many of these essential modifications.

Before proceeding to more specific problems I should like to make the point that the late war has radically changed the relations between the designer and producer. The modern aeroplane is so complex that it

requires a scale of tooling and equipment far exceeding that of any other engineering industry. It therefore follows that the efficiency of any design depends on the possibility of producing it quickly and cheaply.

In the past the individual efforts of the designer have produced excellent and even brilliant results and to a degree this will still remain so, but it is team work which will count in the end and in this team the designer must play an important part.

SPECIFIC PROBLEMS

Experience on Shetland Aircraft

In the design of the Shetland many constructional problems from the production side had to be faced, mainly because of its size and also the possibility of extended production being required.

As a result a method of construction was adopted that had not before been attempted. Normally a flying boat hull is built in a similar manner to a ship, *i.e.*, the keel is laid, the frames are erected, stiffeners fitted and the complete skeleton sheeted. This method calls for relatively skilled labour and the amount of labour which can be used on the hull is somewhat restricted.

In the case of the Shetland the hull was 113 ft. long and 22 ft. deep and several discussions were held as to the best method of undertaking its construction. Finally it was decided to divide the hull into ten complete and separate units. This was achieved by a horizontal joint approximately midway up the hull, the upper section being split into four units and the lower section into five units. The tenth unit comprised the complete tail section and included the spar frames for the tailplane and fin. It was comparatively easy to plan and design assembly rigs for the upper section of the hull as this section resolved itself into more or less conventional aircraft sections. The rigs themselves were designed using heavy mild steel angle and channel iron for the general support structure, with cast iron frame references fore and aft at the breakframes and heavy cast iron inverted channels forming face plates for the horizontal joint.

The assembly rigs for the lower sections were made on similar lines with the exception that the sections were constructed bottom upwards in order to facilitate the riveting of the planing bottom sheeting and to obtain good water-tight joints.

Special rigs were designed for turning over the bottom sections and for the assembly of the separate components into a complete hull. No difficulties were experienced in the method of construction adopted, the only disadvantage being the extra weight of the joint which, for the Shetland, was estimated to be of the order of 300 lb.

It is still an open question as to whether this method of construction shows any appreciable advantage over the more conventional method.

For limited production it is doubtful whether the increased cost of rigs, etc., is justified and the probable number off for any particular design would need to be known before a decision could be made.

Another feature of the Shetland which merits consideration in the design of large aeroplanes was the attachment of the wings to the hull. Because of the necessity of erecting the aeroplane in the open air it was desirable to design the wing connection as simply as possible to avoid elaborate and accurate fitting work when the wings were lifted into position. This was achieved by the use of a single pin attachment which could be jigged accurately in the shop before erection. The actual spars through the hull were forged from S.65 steel with the female end fittings incorporated and the aluminium alloy spars in the wing had an S.65 male steel forging spliced into the end. The pins through the top and bottom flanges were 3 in. dia. and were of the expanding type. This attachment proved extremely satisfactory and the time required to lift and attach the wings to the hull was only a matter of a few hours.

TOLERANCES—CONTROLS, ETC.

It should be a guiding principle to all designers that, other things being equal, the best design is the simplest. By "simplest" I do not mean only that it should necessarily be made of few parts but that it should be easy to make and, therefore, the manufacturing tolerances should be adequate. With this wider interpretation of the principle we sometimes find that what appears to be additional complication does in fact simplify the design.

Let us consider the flying controls on a large flying boat. Experience of quantity production during the war has shown the weakness of the simple set-back hinge type of balance, for it proved impossible to obtain "repeatability," in the sense of an equal standard of control heaviness, on all aircraft. This was so in spite of determined efforts to reduce the production variables between individual components. Valuable time and money were spent in tightening up the production limits and in repeated test flying to bring each aircraft up to the desired standard.

The basic reason for this is not far to seek. With a set-back hinge, the contribution of the nose is to subtract from the hinge moment of the basic control. On large aircraft the degree of balance is so close that the load which comes on the pilot's hand is the difference of two relatively large quantities, and variation in either quantity will have a disproportionate effect on the resultant load. There are two ways of overcoming this. One is to introduce an adjustment, preferably one which can be made in flight; the other is to reduce the loads by factoring instead of subtraction. This can be done either by power assistance or by a spring tab system.

The Seaford flying boat has an infinitely variable adjustment to the elevator servo tab which can be set to the pilot's requirements during flight and which is then locked. This is proving very satisfactory and is saving much time in test flying and irritation to the production department.

The Shetland has electrical power assisters on the aileron and rudder. I think that this system, or its equivalent with hydraulic operation, will be necessary on all large boats of the future. It has the merit of factoring not only the aerodynamic loads but also the friction and inertia loads.

Both systems add complication to the design but simplify production.

There are other instances, however, where no amount of careful design can alter the fact that accuracy is essential. We cannot achieve laminar flow in the boundary layer without exceptional accuracy in manufacture. What success has been achieved is the result of close co-operation between the design and production departments whereby the final design is such that the best possible chance is given to the production men.

On the question of tolerances, generally, more discrimination should be shown by the Drawing Office in the tolerances called for on the drawings. An immense amount of labour in the shops could be saved if close tolerances were used only where essential.

PRESSURISATION

There can now be little doubt that the demands of passenger comfort and of regularity of service in the face of all weather conditions will result in pressure cabins being in general use on the majority of long-range passenger aircraft.

The advent of turbine engines further supports the argument for high altitude operation and so for pressurisation.

Structurally the best cross-sectional shape for a pressure cabin is undoubtedly the circle. In large sizes, however, the circle is a very uneconomical shape into which to fit passenger accommodation. As a result, the cottage loaf or figure of eight section is finding more favour as the best compromise. This is true of landplanes and flying boats alike.

By good fortune this cottage loaf section is ideally suited to the high wing flying boat in which it is already standard practice to provide two decks.

With propeller turbine engines, the operating altitude is likely to be 25,000-30,000 ft., and the pressure differential will be about 6 lb./sq. in. The question then arises as to whether the pressure shell need be continued below the floor, whether the planing bottom itself should be made to withstand the pressure or whether the floor, resting as it will on diaphragms extending to the planing bottom, can be made pressure tight.

The answer can only be given after careful consideration of the production aspects and may well depend on the parallel problems of manufacturing breakdown and provision of water-tight bulkheads. I would hazard a guess, however, that for pressure differentials of no more than 6 lb./sq. in. the pressure-tight floor will be the most popular solution. Water-tight bulkheads will then extend only to the floor level and the manufacturing breakdown will be made at the floor, thus allowing the planing bottom to be constructed in an inverted position, as on the Shetland.

With this goes the problem of air conditioning including the control of pressure, temperature and humidity. Insulation of the cabin from engine noise and silencing of the cabin blower will also be important. I think, however, that it is still too early to discuss these matters in relation to production. Many ideas have been put forward but few have yet been tried in service. I would rather see the technical solution established before committing myself on the production aspects.

I think we have started right in getting certain firms to specialise in research and development concerning pressurisation and when the technical problems are solved concentration can then be given to the more practical problem of installation.

Jet Engines

With regard to the installation of jet engines in flying boats, this should present few problems to the production organisation as it is likely to be simpler than with reciprocating engines, because of the far fewer controls required. It would be preferable, from the operator's point of view, that the engines were not completely buried in the wing in order to facilitate maintenance and change of engine. This would, however, need to be carefully weighed against any probable loss of performance due to external nacelles. Care would also need to be taken in the case of the flying boat that sea water could not enter the air intakes, as this may prove disastrous to the engine. The air intakes would, therefore, need to be placed as far forward as possible.

In what form the jet engine will be fitted, whether pure jet, propeller turbine, ducted fan or the compound engine, it is probably too early to state, but each will have its own application depending on the requirements of the specification.

So far we have had very limited experience with the jet engine and I feel that we should obtain, as early as possible, a great deal more experience under actual commercial operating conditions because of our future programme of installing such engines in our largest aircraft.

Progress cannot be halted and with the present incessant demand for more and more speed, designers are compelled, sometimes against their will, to meet this demand.

Safety, however, must come first and I should like to feel sure that our problems with combustion chambers, turbine blades, etc., are satisfactorily solved before these engines are installed in our largest aircraft. The far-reaching effects of a crash of a large aeroplane because of engine failure may well set back development for a number of years and I feel that the failure of a jet engine may be more catastrophic than that of a reciprocating engine.

Conclusion.

This paper has dealt almost entirely with the relations between the design and production departments and although it was intended to apply primarily to flying boats, yet, as mentioned earlier, it applies with equal force to the landplane. As aircraft

become larger the difference between the two types becomes smaller, *e.g.*, it is probable that two or even three decks will become universal because of the necessity of providing adequate space for the load to be carried. The only marked difference will be the type of undercarriage fitted and it may well be, in the larger sizes, that the provision of a planing bottom with the necessary lateral stabilisers will be an easier proposition for both the design and production organisations.

The development of the flying boat is outside the scope of this paper but I firmly believe, having had experience of both types, that there is a place in the future for the flying boat on many of the world's routes and we should not neglect its development in our future programme of research.

The main problem to be solved is to provide efficient handling and maintenance facilities at the base from which they operate. In the past, work has been concentrated almost entirely on the provision of suitable land aerodromes but it is now becoming recognised that serious consideration must also be given to the seaplane base. When a satisfactory solution is found there is no reason why the large flying boat and the landplane should not be developed on parallel lines.

DISCUSSION

Mr. H. Roberts (Imperial College, Associate Fellow): The author had said that the incorporation of new material generally gave someone a headache to start with, but surely it was wrong to reject the utilisation of new materials merely because that would involve a little extra work on the part of one member, or a few members, of the staff. The one object was the production of the best machine, and even if the advantage derived from the use of the new materials was small, it was none the less justifiable, irrespective of the amount of extra work involved.

When planners and production engineers got together they always tended to complain that the design produced by the draughtsman or the technical man was difficult to make. It was their job to make a success of whatever they were asked to do. It was not the designers' job merely to produce a design which it was easy for the production men to make, or which was suitable from the planners' point of view. The designers were concerned to turn out a good job, and if the methods which the production engineers were accustomed to use were not up to the designers' demands, surely the production engineers should modify their ideas rather than expect the designer to come down to their level.

Mr. Lipscomb: Where special materials were necessary they should be used. But on many occasions he had found that the draughtsman called for the use of special materials where simpler and cheaper materials should be used. The cheaper and well known materials should be explored first. He did not decry the use of special materials where they were necessary or desirable.

The point with regard to production engineers and planners was a good one. The Production Engineer should, however, know the capabilities of his factory probably better than would the man in the Drawing Office. Therefore, when the draughtsman drew up his first design he should consult the Production Manager as to how it should be made; the latter might have a technique of which the draughtsman was not aware, and the draughtsman should listen to his suggestions. In many cases the draughtsman thought that his design was the best, whereas it might be open to argument. It was up to the production man to produce the job as quickly and as cheaply as possible, and if his suggestions could lead to an improvement in the design he should have a say in the matter.

Sqdn. Ldr. D. H. Grundy (Assoc. Fellow): Was there any information which would give

some idea of the profit and loss involved in the use of modification centres such as were introduced in America during the war and to which the author had referred? Was there anything to show the saving in money effected by using a system which produced an aircraft to a certain standard, and then took it away and tore it to pieces in order to bring it to a more advanced standard. Again, would a saving of man-hours result from the use of modification centres, or would a loss be incurred? In America during the war unlimited man-power was available, so the question was of little account, whereas in this country man-power had to be severely rationed and they had to consider always the best way of using that which was available.

Mr. Lipscomb: In peacetime it would probably be an extravagance to have a modification centre. But those who were working on the design side during wartime would be aware of how they were restricted as the result of aeroplanes being kept in the shops because modifications were not fitted. It might be that advantage was derived if there were a modification centre, in that there would be a certain amount of extra floor space in which to do the work; but also, he believed that the production men would be more pleased to get the aeroplanes in their own factory to a certain standard and to dispose of them, rather than to have them hanging around, not knowing from day to day what modifications had to be put into them. During the war, modifications appeared every week on every job, and sometimes it was difficult to get rid of the aeroplanes; on occasions he had felt that if they could send the aeroplanes elsewhere, where somebody else could put in the modifications, it would be a great advantage.

Mr. W. F. Spurr (Ministry of Supply, Fellow): He was disappointed that there was no reference in Mr. Lipscomb's paper to designing for maintenance, because one of the most important features in connection with the flying boat was that it should be easy, not only to produce, but to maintain in service. Ease of maintenance by the operator was as important in peacetime as the ability to produce the aircraft in a short time. Improved handling facilities were also required.

As Mr. Lipscomb had had experience of large flying boats and large landplanes, he would like his views about the relative merits of the two in respect of design effort and production effort.

Mr. Lipscomb: He had overlooked the question of designing for maintenance, a matter, which, for the operator, was almost as important as primary design. If an operator were supplied with an aeroplane which was difficult to maintain in service, he would never be happy with it, and would always complain. It was essential that, both for military and civil flying, the aircraft should be designed with a view to the easiest possible maintenance.

He would prefer to build a flying boat rather than a landplane every time because as aeroplanes became bigger and bigger the difficulties with undercarriages would become more and more difficult. He shuddered to contemplate what undercarriages would be like when the 250,000 lb. and 300,000 lb. aeroplanes were exceeded. The flying boat was fitted with a planing bottom, and that was that! Undoubtedly, it was the easier of the two to construct. It was still an open question whether or not the wing-tip floats should be retractable. He believed the Germans and the Americans were going away from this principle, but he felt that in this country they should try to do it. In view of the requirements in respect to wing loadings, and the increasing demands in the way of lateral stability, the matter was difficult.

Probably the flying boat did cost a few more man-hours in actual construction than did the landplane, simply because, in connection with the landplane the aircraft people invariably put out their designs for someone else to do!

Mr. H. Knowler (Chief Designer, Saunders-Roe Ltd., Fellow): His Company was

attempting to retract wing-tip floats completely into the wing; but it was most difficult with a high wing loading to get enough volume, and it was necessary to go to the rather drastic measure of splitting the float into two sections.

The cottage-loaf design of hull — or the double-bubble section as it applied to the landplane—was a very good form for the pressurised cabin. As indicated by Mr. Lipscomb, it enabled the wing of the flying boat to be raised well above the water for a given peripheral measurement; and it enabled two floors to be provided, giving a bigger floor area than that of the circular form because the floor area in the cottage-loaf form was completely usable right up to the sides, as shown by his Company's investigations. Furthermore, in the flying boat he had mentioned, the lower deck was pressurised, and his Company had found that there was little increase of weight because they could reinforce it from the planing bottom of the boat, which already had to be fairly strong. As a rough idea, the planing bottom of the boat would stand about 40 lb. per sq. inch, whereas the pressurised floor would withstand, say, 12 lb. per sq. inch.

Tests had shown that large quantities of salt water entering the turbine encrusted the blades; but the effect of flying through a rain-storm was to remove this deposit. Small quantities of spray, such as would be anticipated in the take-off of a flying boat in normal conditions, would have a negligible effect.

He would not say that the production engineer was the "enemy" of the designer, but there were many subjects of disagreement between the two and, unfortunately, what the production engineer called for always involved more weight.

The Shetland hull, which was split up into ten sections for production purposes, was carried out extremely lightly for 300 lb. additional weight, but, bearing in mind the joints involved in the sub-division of the wings and tail junctions to the body, and so on, it cost anything between 1 and 2 per cent. of the total weight for joints; furthermore, those joints involved a great deal of labour, so that the total monetary cost of the aircraft was considerably increased. Could the production engineers justify this additional labour and cost?

Mr. Lipscomb: He agreed that the production engineers made demands which did result in increase of weight. He also agreed with the remarks on the splitting up of the Shetland hull into sections; it was done against his wish, for he was over-ruled. Although much unskilled labour was employed at that time, he had felt that it was not justifiable to split the hull into so many sections. It was done to suit the production people, because they wanted to employ unskilled and girl labour on the work, each piece being made separately and joined up afterwards. Left to themselves, he and his colleagues would have built on conventional lines.

Mr. Marples (Miles Aircraft Ltd.): Did Mr. Lipscomb advocate a combined design drawing office and jig and tool drawing office, or did he prefer separate departments, the design draughtsmen having had experience of jig and tool drawing office work? Also, did he consider that design draughtsmen should spend a long or a short period in the jig and tool drawing office before going to the design office?

There seemed to be rather an extraordinary attitude towards what was called "good design." One speaker appeared to assume that a good design was one which the designer produced, and not one which the production engineer produced; that a design for a small weight was the acme of design. Perhaps, because of the two wars, they had had a lot of experience of designing aircraft to give the last pitch of performance, without regard to cost—that was the case also with racing machines — and designers had become a sort of aristocracy. But now they had to compete in an economic world instead of in a pugilistic world, and

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Mr. H. Giddings (Bristol Aeroplane Co. Ltd., Associate Fellow): Would Mr. Lipscomb agree that the size of the aircraft was a considerable factor in the problem of the relative importance of ease and cost of production and technical design refinement? Several authorities on the subject had shown that as aircraft became larger, the first cost of a machine had a lesser bearing, whereas the weight and the aerodynamical performance became increasingly important. With increase of size it would appear that technical design considerations should have preference. For example, the adoption of some new material or new process of manufacture would increase the first cost considerably, but it would probably reduce the weight and improve the performance and the earning capacity of the machine sufficiently to override this.

He was surprised to hear that the chief flying boat problem was the provision of the equivalent of the aerodrome. He had thought it had always been claimed to be the chief advantage of the flying boat that it needed only any suitable stretch of water on which to alight, whereas for landplanes it was necessary to build elaborate and expensive aerodromes. Was the complaint not that the provision of alighting places was particularly difficult, but that they had not been provided? If alighting places for flying boats were more easy to provide than aerodromes, could Mr. Lipscomb offer any reason why they had not been provided in the past, and why they suffered from lack of "sea-dromes," whereas there were plenty of very costly aerodromes for landplanes?

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attempting to retract wing-tip floats completely into the wing; but it was most difficult with a high wing loading to get enough volume, and it was necessary to go to the rather drastic measure of splitting the float into two sections.

The cottage-loaf design of hull — or the double-bubble section as it applied to the landplane—was a very good form for the pressurised cabin. As indicated by Mr. Lipscomb, it enabled the wing of the flying boat to be raised well above the water for a given peripheral measurement; and it enabled two floors to be provided, giving a bigger floor area than that of the circular form because the floor area in the cottage-loaf form was completely usable right up to the sides, as shown by his Company's investigations. Furthermore, in the flying boat he had mentioned, the lower deck was pressurised, and his Company had found that there was little increase of weight because they could reinforce it from the planing bottom of the boat, which already had to be fairly strong. As a rough idea, the planing bottom of the boat would stand about 40 lb. per sq. inch, whereas the pressurised floor would withstand, say, 12 lb. per sq. inch.

Tests had shown that large quantities of salt water entering the turbine encrusted the blades; but the effect of flying through a rain-storm was to remove this deposit. Small quantities of spray, such as would be anticipated in the take-off of a flying boat in normal conditions, would have a negligible effect.

He would not say that the production engineer was the "enemy" of the designer, but there were many subjects of disagreement between the two and, unfortunately, what the production engineer called for always involved more weight.

The Shetland hull, which was split up into ten sections for production purposes, was carried out extremely lightly for 300 lb. additional weight, but, bearing in mind the joints involved in the sub-division of the wings and tail junctions to the body, and so on, it cost anything between 1 and 2 per cent. of the total weight for joints; furthermore, those joints involved a great deal of labour, so that the total monetary cost of the aircraft was considerably increased. Could the production engineers justify this additional labour and cost?

Mr. Lipscomb: He agreed that the production engineers made demands which did result in increase of weight. He also agreed with the remarks on the splitting up of the Shetland hull into sections; it was done against his wish, for he was over-ruled. Although much unskilled labour was employed at that time, he had felt that it was not justifiable to split the hull into so many sections. It was done to suit the production people, because they wanted to employ unskilled and girl labour on the work, each piece being made separately and joined up afterwards. Left to themselves, he and his colleagues would have built on conventional lines.

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ture, which would probably result in reducing both production problems and costs.

He supposed it was true that "sea-dromes," as such, existed. The operators of landplanes were provided with aerodromes and all services and facilities, whereas a flying boat had to be moored to a buoy, and a motor boat had to take passengers and stores out to the flying boat. A floating pontoon should be provided, so that passengers could step from the flying boats on to the pontoon connected to the shore, and from which they could board the flying boats. Flying boat bases could be provided in various parts of the world for perhaps not much more than £100,000 per base; they would be far cheaper than aerodromes for landplanes. But it seemed that nothing was being done except to provide a few buoys, and he did not regard that as providing a sea-drome.

THE ROYAL AERONAUTICAL SOCIETY

SOME ENGINEERING PROBLEMS OF LARGE AIRCRAFT

by

A. E. RUSSELL, B.Sc., F.R.Ae.S.

Mr. Russell took his degree at Bristol University and joined the Bristol Aeroplane Co. Ltd. in 1926. *He was appointed Chief Technician in* 1931, *Technical Designer in* 1938 *and in* 1944, *Chief Designer, Aircraft Division.*

Introduction

THE scope of this paper is necessarily restricted. A large aeroplane, within the new definition, has not yet flown in this country. This event, however, is not very far off and the preparatory work that makes it possible has brought to light many novel problems. In some cases these could be predicted with normal anticipation, whereas others have only emerged as analysis progressed. These latter are frequently good examples of the dangers of too facile extrapolation.

Some of the reasons for having large aeroplanes are examined and subsequently, the chief technical problems arising are described. These descriptions avoid mathematics, as in many cases this approach is extremely long and involved and merits individual publication.

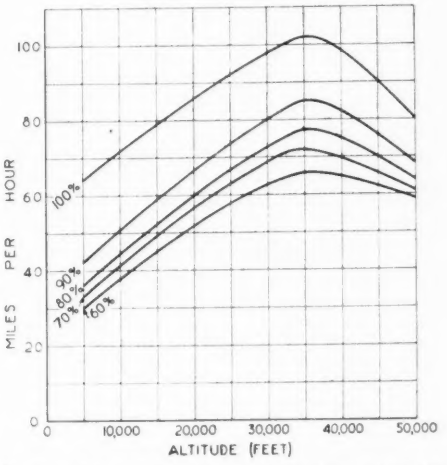
In the first place, the greater the distance to be flown non-stop so the aeroplane suitable to operate a scheduled service tends to increase in size. A brief analysis will be made of the London-New York route, one of the most difficult but also one of the most important of the long hops. On this route prestige value is high, a high volume of traffic is assured and the competition will be fierce.

London-New York

The distance between London and New York along the great circle is approximately 3,450 statute miles, but allowing for small navigational errors and the use of an alternative airport in case of need, the total geographical range required is about 3,750 statute miles. An aeroplane required to operate this route non-stop, with reasonable regularity, is determined entirely by the westbound crossing as the prevailing winds in this direction are consistently unfavourable.

Figure 1 shows the best available information on the magnitude of the effective head wind at various altitudes during the winter months of December, January and February, when flying from London to New York by way of Foynes and Newfoundland. The percentages attached to the curves indicate the frequency of occasions on which the effective head winds are less than the

Fig. 1.
LONDON-NEW YORK
Average effective head winds during the winter months



Percentages give frequency of occasions when the effective head wind is less than the value given by the curves.

values given. Thus, a head wind of 100 m.p.h. must be catered for if 100 per cent. regularity of operation at 35,000 feet altitude is required. The other seasons of the year are only slightly less severe, the equivalent values for spring, summer and autumn being 90 m.p.h., 70 m.p.h. and 90 m.p.h. respectively.

If a regularity of 85 per cent. is accepted during the winter period, then a head wind of 80 m.p.h. must be allowed for. With the same allowance the non-stop flight should be possible on 98 per cent. of the occasions in spring and autumn and 100 per cent. in summer. The effect of this head wind is to increase the required still air cruising range to approximately 5,000 statute miles. When fuel for take-off, climb and circuits in stand-off periods is added and an allowance made for engine and aircraft depreciation, without being pessimistic it is easy to arrive at an equivalent still air range requirement exceeding 5,500 miles.

Obviously it is not possible for an aeroplane to make more than one crossing in 24 hours and there is a strong argument for making the journey overnight. Under these conditions, frequency of service, other than daily, does not arise and it is more economical to operate one large aeroplane rather than two or more smaller ones.

The payload is, however, inevitably a small proportion of the take-off weight and the weight breakdown, in broad categories, for an aeroplane of approximately 300,000 lb. illustrates this.

Structure	32%
Power plant and services ...	14%
Fuel and oil	37%
Operating equipment and crew	5%
Passengers' accommodation and stewards	4%
Passengers and luggage ...	8%

In this particular case the appropriate number of passengers can be accommodated in an orthodox single deck circular body with provision for bunks for all passengers.

It is interesting to examine the possibilities of an alternative solution. With available stopping places at Rineanna in Ireland, and Gander in Newfoundland, the smaller weight of fuel required leaves more load available for a greater number of passengers and their accommodation.

If a stop at Gander is scheduled, 76 per cent. of the non-stop fuel is sufficient and the reduced weight of fuel is equivalent to about 9 per cent. of the all-up weight. If a stop is made at Rineanna as well, then 66 per cent. of the non-stop fuel is sufficient and the saving is equivalent to 12½ per cent. of the all-up weight.

It would appear that a double deck body of figure eight cross-section could provide additional floor space for the accommodation of the extra payload. There are, however, two quite important effects arising from such a change. Firstly, the weight of the body structure, floors, furnishings, pressurising and other equipment is increased in keeping with the increased accommodation. Secondly, the wing structure weight is greater due to the increased bending moments resulting from the transfer of load from the fuel tanks to the body. In a particular case the sum of these two changes was estimated to be equivalent to about 6 per cent. of the all-up weight.

Thus the actual payload was only increased from 8 to 11 per cent. for the single stop and from 8 to 13 per cent. for the double stop.

If special sleeping accommodation is not provided this same number of passengers can be carried in the single deck body. So the choice for the Atlantic appears to be:—

(*a*) A single deck body capable of doing the non-stop westbound crossing with sleeping accommodation for all passengers and with a payload amounting to about 8 per cent. of the all-up weight or, seating accommodation for 50 per cent. more passengers, probably on a stopping service.

(*b*) A double deck body unsuitable for the non-stop westbound crossing but with sleeping accommodation appropriate to a passenger payload equivalent to 12-13 per cent. of the all-up weight on the stopping service and with considerably more seating capacity on alternative shorter runs.

The conclusion is that for aeroplanes of about 300,000 lb. all-up weight, a double deck body is not the best arrangement if a range exceeding 4,000 miles is a primary requirement and that a single deck body gives greater flexibility for alternative passenger arrangements until conditions of operation across the Atlantic become established.

STRUCTURE

We have already seen that both the weight of the structure and the weight of the fuel may be some four times as much as the payload. In these circumstances it is even more important than usual to achieve the greatest possible efficiency.

Now a light and safe structure cannot be achieved without an accurate knowledge of external applied forces and the resulting internal loads in the structure. It is extremely difficult to gain this knowledge for large aeroplanes on account of the great importance of the dynamic effects arising from vibrations. The natural frequencies of oscillation of an aeroplane are roughly inversely proportional to the square root of its dimensions and in the case of the Brabazon I the fundamental vibration has a period of about half a second.

On the other hand, speed rather than size is the most important factor, determining the rate of application and the severity of gust loads. Also one of the features of under-

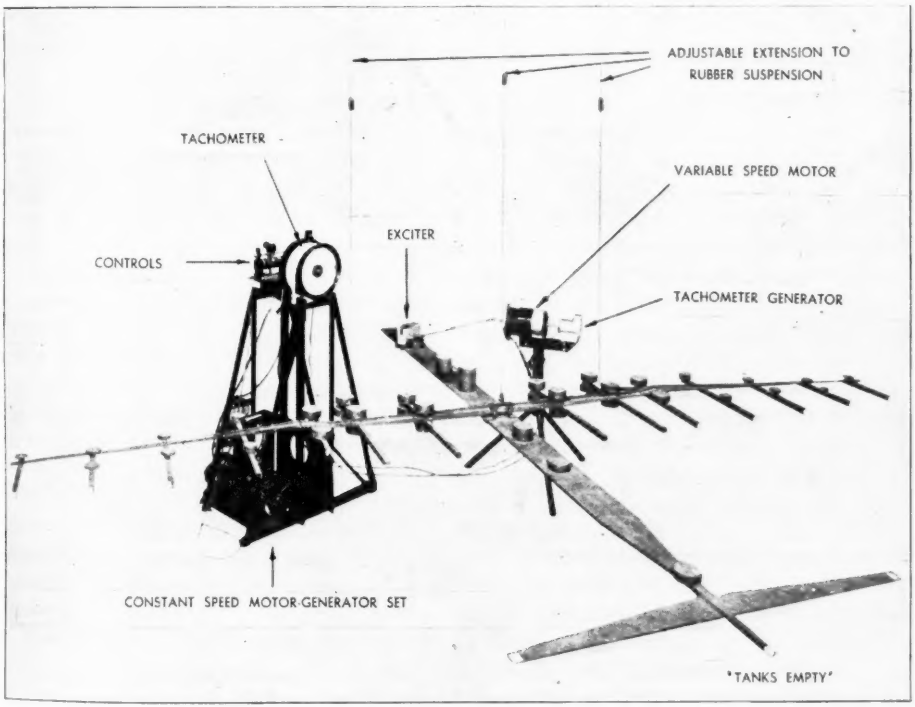


Fig. 2.

carriage design is that the time to develop maximum reactions is more or less independent of size. Both these times are of the order of half a second. The result is that both gust and undercarriage loads induce vibrations in the structure and the maximum stresses are increased by these vibrations.

The solution of the problem determining the response of the aeroplane which has complicated elastic and inertial properties demands considerable mathematical skill. It is solved by analysing the response in terms of the natural modes and frequencies of vibration.

The standard way of determining the modes and frequencies of vibration is to resonance-test the finished aeroplane. This method is clearly not very helpful and can be classified as an expensive method of successive approximation. A theoretical method had to be devised for providing the information, the aeroplane being idealised as an elastic body with 42 degrees of freedom. These calculations were checked by making a 1/20th scale vibration model designed to represent the elastic and inertia characteristics of the aeroplane and good agreement was obtained. This model is shown in Fig. 2 and the measured nodal lines in Fig. 3.

Similar models were also made of two

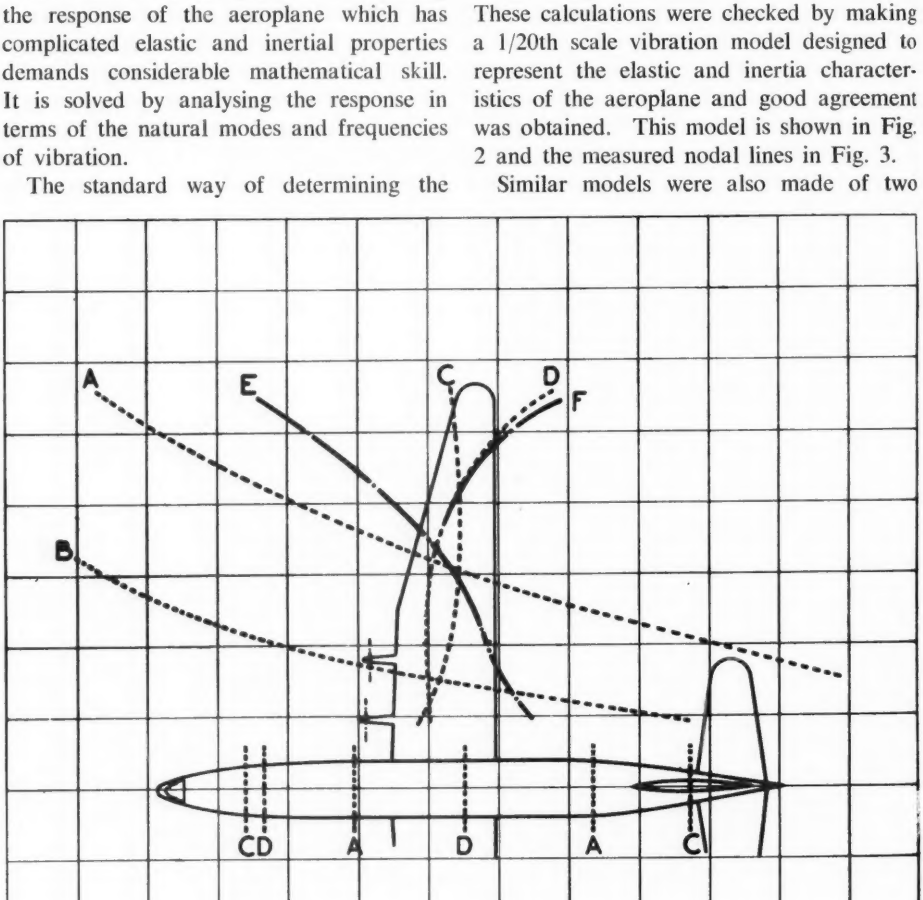


Fig. 3.

Key { A 1st Symmetric mode 1.96/sec. B 2nd Symmetric mode 3.25/sec. C 3rd Symmetric mode 4.91/sec. D 4th Symmetric mode 6.40/sec. E 1st Antisymmetric mode 3.29/sec. F 2nd Antisymmetric mode 5.91/sec. } Corrected to full scale

Nodal lines experimentally determined.

other aeroplanes and the check between calculation, model and full scale resonance tests was very good.

With the principal modes of vibration now available the time history of generalised applied forces in these modes may be calculated. The equations of motion are then set up and as, *a priori*, there is no inertia or elastic coupling between the modes, the analysis for each mode is independent. The equations can, therefore, be solved and will yield the time histories of the amplitudes of vibration on substitution of the appropriate boundary conditions.

Unfortunately, however, the vibrations themselves produce aerodynamic forces on the aircraft, and quite large effects can result from relatively small deflections. It has thus been found impossible to ignore aerodynamic damping, at any rate of the lower modes, and this introduces aerodynamic coupling between the modes which can now no longer be treated independently. The mathematics become extremely complex and it is necessary to have recourse to step-by-step methods, themselves very voluminous and lengthy.

Since the set of model amplitudes define the deflected shape, it is theoretically possible to deduce the stresses, knowing all the elastic properties of the aeroplane. This method was, however, found to be impracticable, and recourse had to be made to calculating the stresses for each individual principal mode from the inertia forces in free vibration and the total stress distribution obtained by addition.

This analysis showed that the response is the same as that of a rigid aeroplane together with the dynamic effects associated with the modes whose natural periods are more than about 20 per cent. of the time taken to develop maximum load, in this particular case the first five.

The complexity of stressing large aeroplanes is greatly increased by these dynamic effects. Contour maps must be drawn showing peak accelerations at all points in the aeroplane and even these peak accelerations do not occur simultaneously.

The local accelerations on the Brabazon I were found to be very high. The up-gust case gave a peak acceleration of 16 g at the wing tip and the root bending moment was increased by 15 per cent. by the vibrations. In the landing case accelerations of 9 g occur at the outboard propeller stalk and the maximum bending moment is 30 per cent. higher than that given by static analysis.

The up-gust case is a very severe design condition for large aeroplanes when the cruising speed is fairly high.

It was found that the acceleration of the aeroplane in a vertical gust of 50 ft./sec. reached the acceleration of the normal design envelope (2.5 g) at a speed of 175 ASI. At a (cruising) speed of 220 ASI the net wing root bending moment is approximately 50 per cent. higher than any other design condition. This is equivalent to a design static load factor of about 6 g. If no alleviating device is incorporated this result would involve a considerable weight penalty amounting to approximately half the payload. There are, however, possibilities of avoiding these effects and I shall have more to say about this later.

The modes and frequencies of vibration were also made use of in the analysis of flutter speeds. The flutter speed of a large aeroplane is liable to be lower than that of smaller types because of the lower frequencies. Another factor becoming important is that flutter involving modes higher than the first two becomes possible.

Fortunately the demand for high torsional stiffness appropriate to large size and high flying speeds, smooth surfaces for low drag and high average developed stresses are not inconsistent. All can be satisfied by using a thick structural covering suitably reinforced.

Where such surfaces are in tension no great difficulty arises. The efficiency in tension is determined purely by the efficiency of the joints which can be as high as 90 per

cent. and is more or less independent of the relative proportion of the skin and stringers.

Skin and Stringer Combinations

The real problem is met on the compression side. The failure of a reinforced skin in compression is invariably due to some form of elastic instability among which the following are the most important.

(1) Flexural or Euler instability which may occur alone or be precipitated by one of the other forms of instability.
(2) Local instability of the stringer.
(3) Skin buckling.
(4) Torsional instability of the stringer in which the stringer rotates about the line of its attachment to the skin over a fairly long wavelength.

In order to preserve a smooth wing surface it is necessary to delay skin buckling until the proof load so that no residual buckles form. With this in mind several types of stringer were examined theoretically, including angle, Z and top hat sections. The most promising when limitations of manufacture were taken into account was thought to be the Z section.

Accordingly, a structure incorporating rolled Z stringers was designed and compression panels representative of this design were tested.

The results of these panel tests were disappointing, since failure occured 10-15 per cent. below the theoretical flexural instability stress. This discrepancy was entirely due to the limitations of the theory at that time: as the tests progressed so did our theoretical knowledge, and the reasons for the discrepancy were found. In the main part, the decrease was due to the interaction of the various types of instability, the mode of failure being a mixture of flexural, torsional and local. This interaction was accentuated by the local flexibility at the heel of the rolled stringer where a large bend radius had been necessary for manufacture.

Further development work showed that it was possible to achieve a lighter structure by delaying skin buckling until failure occurred.

It is a well-known fact that the stress at which a skin buckles initially varies considerably with the nature of the support at its edges. In practice, the stringers used in stressed-skin construction effectively prevent displacement normal to the skin. If the buckling stress is very low, as it has usually been in the past, it has been the custom to assume the panels to be simply supported at their edges. When it is desired to delay buckling to a very late stage, the need for studying the edge conditions is obvious, as the possible variation between "simple support" and "full clamping" corresponds to a 70 per cent. increase in buckling stress.

In the case of Z section stringers there are two kinds of torsional stiffness, the basic Saint-Venant stiffness and a stiffness which arises from a constrained flexure of the free flange. It is this second kind which is the more important at the relatively short wavelengths associated with skin buckling. The introduction of this stiffness into a skin buckling calculation yields the result that in most practical cases extruded stringers can completely clamp the skin. To permit this the stringer web must be able to transmit the concentrated moments from the skin into the free flanges where they appear as shears. The buckling of skin panels supported by Z section stringers is thus a problem of two connected plates, a stringer web and a pair of asymmetrically buckling skin panels. Accordingly a modified combination of skin and stringers was developed using extruded Z section stringers and a skin which did not buckle. Further panel tests showed that this type of construction was, in fact, as efficient as theory predicted.

A general investigation into the problem indicated that the mean stress, which can be realised by a skin-stringer combination, depends on the load per inch run P, the frame spacing l and the relative proportions of the skin and stringers.

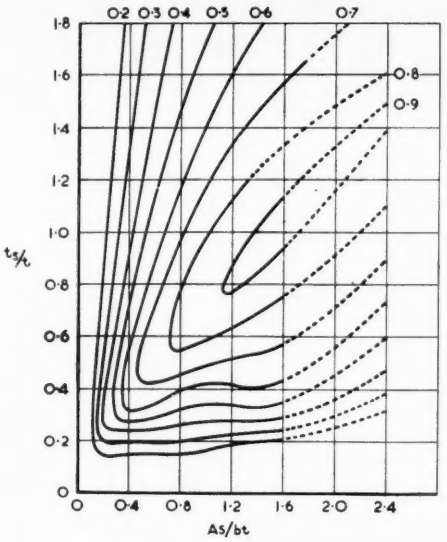


Fig. 4.

The family of curves is plotted for the stated values of $K\left(=\frac{f2l}{PE_3}\right)$

Contours of failing stress with no buckling before failure.

Figure 4 illustrates the general solution when initial buckling coincides with failure. The results for earlier buckling are similar in form. The compressive stress which can be attained is given by the relation

$$f = k\sqrt{PE}/l$$

Any point on the chart corresponds to precise dimensions for skin and stringer thicknesses, stringer dimensions and stringer pitch. This chart is based on the work of Mr. J. Hadji-Argyris and his colleagues of the Royal Aeronautical Society, which constituted an essential forerunner to our theory of optimum design.

The results attainable with optimum design are shown in Fig. 5. Unfortunately, the optimum may require stringers too closely spaced to be practical.

It may also be desirable to compromise on rib spacing. Experience shows that it is very necessary to keep a close watch on rib weight on large aeroplanes. The problem is not complete until the combined weight of ribs, stringers and skin is a minimum.

Similar problems are found with the tailplane and body skin covering, although in these cases a considerable departure from the optimum is necessary. Fairly thick skin is needed to provide adequate torsional stiffness, but it is likely to be impossible to prevent skin buckling. Also, the optimum stringers would be a size impossible to extrude and they would have to be impractically close together, the flange also would not be adequate for normal riveting. The scale may, however, still be sufficient for Z section stiffeners to be more efficient than bulb angles.

The design of the fuselage stringers falls roughly midway between those of the wing and the tailplane, but the practical limitations of frame spacing and stringer pitch still make it impossible to avoid skin buckling except when pressurisation is applied. Where the stresses are highest Z extruded stringers are most suitable. By starting with a thick free flange, taper in strength and therefore weight can be achieved by progressively reducing its thickness by machining.

Structural Testing

Unless a new aeroplane structure is a legitimate descendant of a well-proved family, it is prudent to check the design for

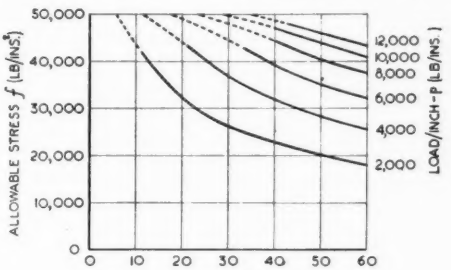


Fig. 5.

Approximate allowable stress. Optimum design of Z section stringers.

stiffness and strength by mechanical tests, preferably to failure. In the case of large aeroplanes practical difficulties are encountered, the most acute difficulty arising in strength tests. The components, *e.g.*, wing or fuselage, are liable to be too large to be tested in existing equipment, both from the point of view of size and the magnitude of the test loads to be applied.

These difficulties can be overcome by using scale models geometrically similar to the aircraft structure. The choice of scale can be made by consideration of the following simple relationships.

TABLE I

Quantity	Units	Model/Full-Scale
Length	L	n
Load	F	n^2
Stiffness	F/L	n
Bending moment	FL	n^3
Stress	F/L	1.0
Strain		1.0
Mass	FT^2/L	m
Frequency ...	1/T	n/m

It will be noted that the mass ratio is tabled as m, which is defined as the ratio of the total mass (model structure + added weights) in any section of the model, to the total mass in the geometrically similar section of the aircraft. The value of m may be arbitrarily chosen to give any desired frequency ratio, or for other reasons. The mass of the model is, of course, n^3 that of the corresponding part of the aircraft component, but, following the above definition, m may be chosen independently of n^3. Nevertheless, if m is made equal to n^3 the added weights can be made geometrically similar to the full scale masses they represent.

For strength tests, the weight of the model is removed from the computation of the loads to be applied by holding it in such a manner as to approximate to the condition in which every element of mass is individually supported.

The actual scale of the model selected is likely to be in the basis of the thinnest sheet available or the smallest convenient size of rivets.

The disadvantages of building a scale model, which may still mean a wing exceeding 100 ft. span, are the amount of additional drawing work required and the manufacture of special tools and jigs. Against this, however, must be offset the elimination of the need for providing special test equipment.

Not least among the advantages is the probability that a model will be made and tested in time for the results to be incorporated in the design of the first aeroplane, and any modification required introduced in the most effective manner. It also gives time for the "second thoughts" which few of us are able to resist.

The use of large scale models can be illustrated by a brief description of the half-scale wing representing the Brabazon I. As you probably know this model was recently tested in the "Cathedral" test frame at the Royal Aircraft Establishment.

The test wing represented the wing structure between front and rear spars from tip to tip and also the propeller stalks and engine mountings. In order that root attachments of the wing to the fuselage could be correctly represented a 40 ft. length of fuselage was provided. Those parts not contributary to strength were omitted, but consideration was given to their ability to accommodate the bending strains of the structure to which they were attached.

A general view of the specimen in the test frame is given in Fig. 6. The loading linkages are not in position, but the loading lugs on front and rear spars are clearly visible. About 500 electrical strain gauges were mounted on the starboard side to determine the stress distribution across various sections of the wing.

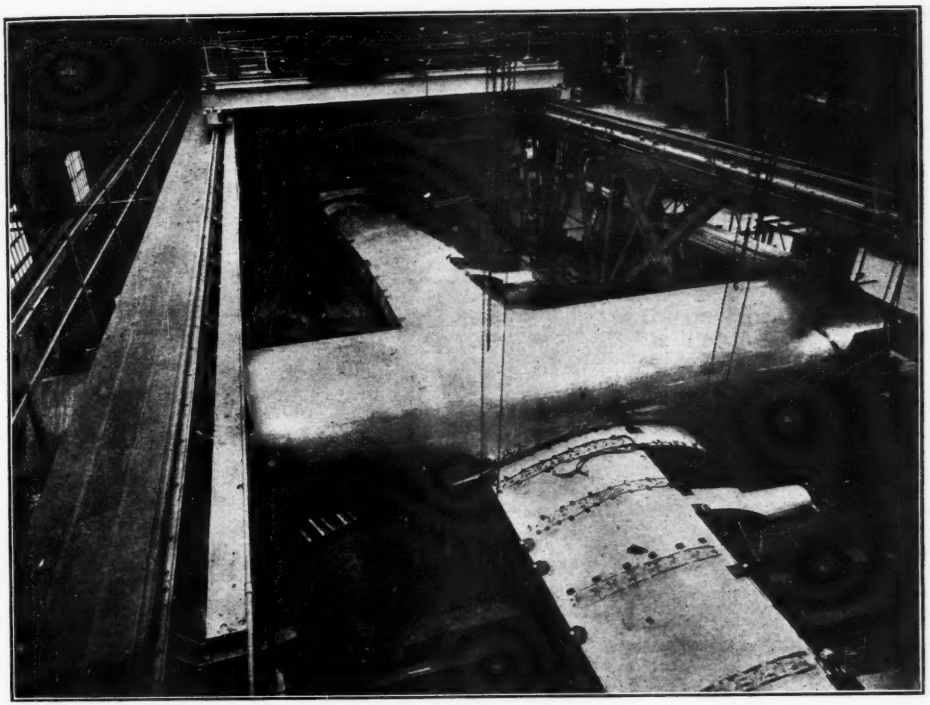


Fig. 6.

Stiffness Tests

Stiffness tests were undertaken to check the calculated stiffnesses used in dynamic stressing cases and in flutter calculations. They comprised flexural and torsional stiffness checks at four stations along the wing.

The shear deflections were found to be quite important, particularly in the inner wing. This effect has an important effect in the calculation of the higher frequency modes of vibration.

Resonance Tests

The low frequency modes of vibration of an isolated component are of little value. It was hoped, however, that light could be thrown on the vibrational characteristics of the propeller stalks and engine mountings. Accordingly the engine, gearbox and propeller masses were correctly reproduced and the propeller stalk was excited vertically and laterally. A mass scale of 1/8 was chosen so that the test frequencies were twice the corresponding full scale frequencies. The arrangement is shown in Fig. 7.

Frequencies of excitation extended up to the maximum propeller running speed. It was found that, at the lower vertical resonant frequencies, the mode was very much involved with wing twist, and in fact the fundamental mode for propeller stalk vertical vibrations was any wing mode involving twist and in which the propeller stalk bending moment did not change sign. However, these modes were below the range of propeller running speed.

The first vertical overtone was found to lie close to the cruising conditions, and although the structural damping was estimated to be high, an examination of the curve of amplitudes indicated the regions where stiffening would have the maximum

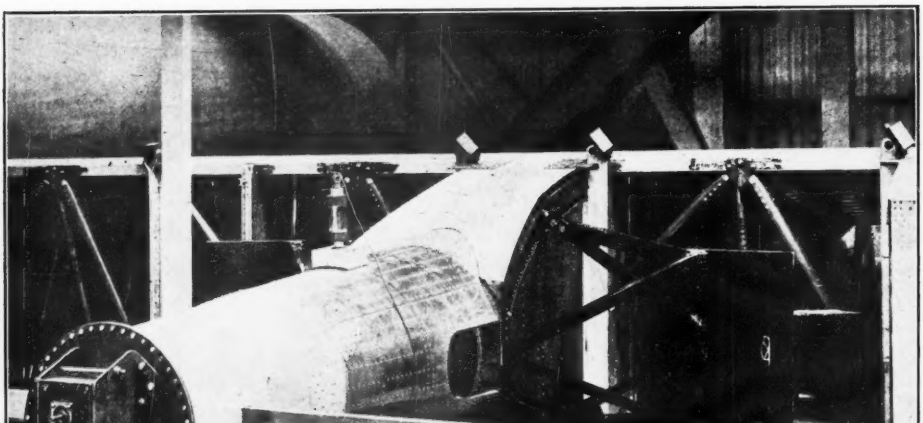


Fig. 7.

effect, and also where it was a waste of time to increase stiffness. The position of the modal lines also indicated the manner in which the full scale engine test rig could be flexibly mounted to represent running conditions in the aeroplane.

Lateral resonant frequencies were not found in or near the running range.

Strength Tests

The strength tests were orthodox and other than the fact that exceptionally high rates of shear diffusion were shown up, do not call for comment.

Materials of Construction

There is no need to emphasise the extremely large numbers of different specifications of materials available for use in the manufacture of aircraft. At the inception of any new design it is necessary to select a relatively small range from those technically suitable, and this choice is governed by the broad strategic conception of the design. The selected range of materials must be small and the individual materials inter-related, if possible, in order to keep the production engineering problems to the minimum.

In the case of civil aircraft, of which relatively large numbers are expected to be built under such conditions that the delivery of the materials of construction will spread over a number of years, a compromise between optimum efficiency and routine procedure during manufacture will be found essential. This is based upon the need to adopt standards ruling at the time for sheet sizes and thicknesses and for specification strength values, in order that stores and

inspection procedure can be covered by normal works organisation and also that continuous availability of the requisite materials may be assured.

For military aeroplanes an additional watch must be maintained to ensure that alternative sources of supply can be made available in time of war, and this emphasises the necessity for skilful selection of materials, and the use of existing standards of size and so forth.

For a large civil aeroplane, where the numbers to be produced are relatively small, and the technical problems great, the designer is in a position to write his own materials specification, taking advantage of the best sources of supply, using non-standard dimensions and tolerances, calling for special manufacturing and inspectional technique and accepting a proportion of scrap higher than normal. An examination into the economic value of a unit of payload shows ample justification for buying structural efficiency by increased expenditure of time and effort on specialised manufacture and inspection of the materials of fabrication.

After the best available materials have been selected for the major structural parts on these and other relevant considerations, the choice of materials for the detail parts will be almost automatic. Special duty requirements on the one hand and the desirability of inter-relation (*e.g.*, common heat-treatment temperatures) with the main structural materials on the other condition the choice; the field can be restricted to a family (or families) having the same chemical composition for all the various manufactured forms.

Before examining the characteristics to be specified for the selected materials it should be remembered that for an aeroplane of about 300,000 lb. all-up weight, the weight of the structure may be some four times as much as the payload.

The greatest scope for weight saving by material control is to be found in the light alloy sheets used, and only a little less exists in light alloy extrusions. This is illustrated by an example where light alloy amounts to 75 per cent. of the structure weight, and about 60 per cent. of this weight is accounted for by sheets and 40 per cent. by extrusions. Of the structure weight only 7.5 per cent. is steel. Considering that most of the steel used is in the form of fittings machined to close tolerances and that a very small proportion of all steel used is of the highest strength, it is hardly worth while to attempt weight or strength control of steels by statistical adjustment.

Now the fundamental thing to realise in choosing a particular material for use in a new aeroplane is that the physical characteristics quoted in the specification are a very elementary first approximation, and one only intended to provide a basis for equally elementary inspection. They are based on the least satisfactory product of all the different firms producing material to that specification.

Other sources of information must be found to give the average strength and the average dimensions, together with the scatter about the mean. Without this information the true appraisal of a material is impossible and, until methods of inspection are based on "quality control," and the appropriate information is made available to designers, no material can be used to its best advantage in aircraft structures.

Under these circumstances, the proper thing to do when selecting materials and their effective strengths for use in large aeroplanes is to compile a private specification conjointly with a material supplier. Unfortunately there are few materials for which there is sufficient statistical background, so that a logical assessment of properties cannot always be found.

Such information that is available shows that light alloys produced by different manufacturers to the same specifications differ more widely in strength from each other and

from the specification than is the case with steels. It is also apparent that a high average strength, relative to the specification minimum, in any one manufacturer's product, tends to be accompanied by a high scatter, signifying that the rejection rate is low and that the material manufacturer reaches his target strength without much difficulty.

The quantities most significant in this discussion are the mean of observed results, the standard deviation σ and the quotient MEAN − SPECIFICATION MIN./σ; the latter is a measure of the frequency of expected rejects —a value of three is equivalent to an expectation that one in 740 observed values would be lower than the specification minimum and a value of two equivalent to one rejection in 44 values.

Table II gives a summary of some information available on four light alloy speci-

TABLE II

			Ultimate Tensile Stress Tons/in.²					0.1% Proof Stress Tons/in.²			
Material	Specification	Manufacturer	No. of tests	Mean	σ	Min. spec.	Mean spec./σ	Mean	σ	Min. spec.	Mean spec./σ
		A	70	26.8	0.69	25	2.6	16.0	0.52	15	2.0
		A	75	26.7	0.59	25	2.9	15.3	0.41	15	0.7
Aluminium Coated	D.T.D.	A	91	26.5	0.79	25	3.0	16.0	0.53	15	1.9
Aluminium Alloy Sheets	390	A	261	27.9	0.75	25	3.9	18.4	1.82	15	1.9
		Total of above	497	27.3	0.96	25	2.4	17.2	1.90	15	1.2
		Various A+B+C	261	27.4	0.73	25	3.3	19.3	1.87	15	2.3
	D.T.D. 546	A	301	28.8	0.93	27	1.97	25.5	1.15	21	3.9
		A	50	32.8	1.88	30	1.5	29.0	1.76	26	1.7
	D.T.D.										
Aluminium	364A	A	74	32.9	1.71	30	1.7				
Alloy		A	48					28.1	1.86	26	1.1
Extruded		A	49	32.7	2.08	27	2.7	28.6	2.56	21	3.0
Bars		A	47	32.5	2.17	27	2.5	28.4	2.47	21	3.0
		A	75	31.1	2.33	27	1.7				
		A	63					26.3	2.77	21	1.9
		Total of	171 ult								
	L.40	above	15° proof	31.9	2.25	27	2.2	27.6	2.85	21	2.3
		C	33	28.6	2.0	27	0.8				
			29					24.5	1.71	21	2.0
		B	44	27.3	0.80	27	0.4	21.9	0.80	21	1.1
		D	251	27.7	1.01	27	0.6	21.4	0.40	21	1.0
Steel Bars	S.11	24 firms	100	60.0	2.7	55–65	+1.8	—	—	—	—
Stainless Steel Sheet	D.T.D. 171	5 firms	96	45.6	2.5	35	4.2	18.3	1.9	15	1.7
Steel Tube	T.45	—	74	49.1	3.9	45	1.1				

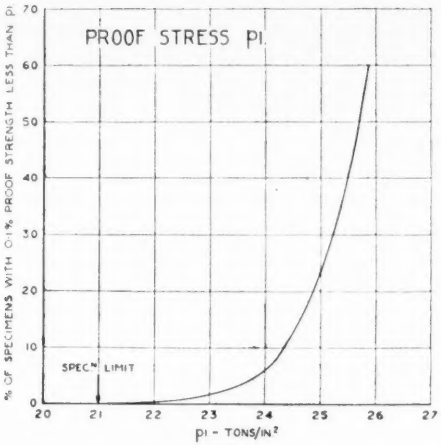
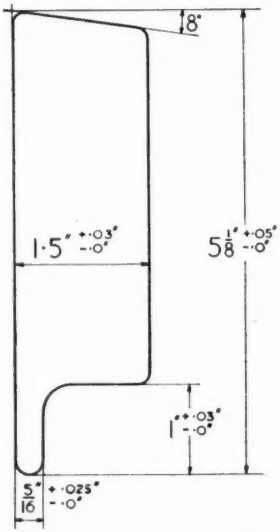


Fig. 8.

fications and three steels, and the most significant feature is the large difference in characteristics of the material L.40 produced by different manufacturers—the difference can be explained by compositional difference and manufacturing technique. The results of some thousands of other tests on L.40, plotted in Fig. 8, show that if the design stress (proof) had been 23 tons/in.² instead of 21 tons/in.² only 1¼ per cent. of the total specimens would have been rejected, but the saving in weight—and, of course, in material—would have been 10 per cent. Obviously this result can have important effect on the saving of structure weight. Other trends reflected in Table II are the high mean ultimate and to a lesser extent proof stresses for D.T.D.390, common to three manufacturers, and particularly the high mean proof for D.T.D.546 from one particular maker. In the latter case the proof stress could be raised without causing any rejects.

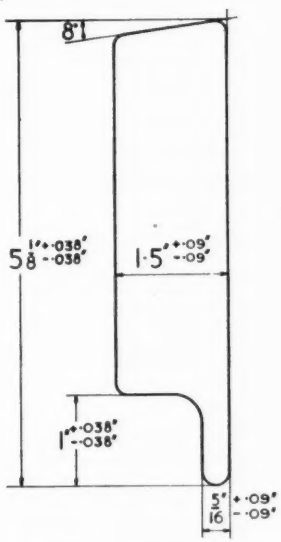
No broad conclusions should be drawn from the values quoted for D.T.D.364A because the ruling thickness of the specimens is not known; nevertheless from other evidence on parts of specific aircraft the

Extruded Sections "H."



AREAS □"				MEAN-MIN MAX.-MIN. %	MEAN-MIN MIN. %
MIN.	NOM.	MAX.	MEAN OBSERVED		
6·09	6·18	6·48	6·28	47·7	3·10

Fine limits.



AREAS □"				MEAN-MIN MAX.-MIN. %	MEAN-MIN MIN. %
MIN.	NOM.	MAX.	MEAN OBSERVED		
5·62	6·18	6·78	6·28	56·4	11·66

Normal Specification limits.

Fig. 9.

specification values have been raised to 28 and 32 tons/in.^{2} for all sections 1 in. and thicker.

A designer anxious to use the highest design stresses possible will naturally select the manufacturer whose product gives the highest value of MEAN - SPECIFICATION/σ, and I would say that a value as low as 2.0, that is a rejection rate of $2\frac{1}{4}$ per cent., is a small price to pay for the corresponding increase in structural efficiency.

A closer hold on dimensional tolerances can also be taken by the designer. Fig. 9 shows a typical extrusion used for the front spar booms of a military aircraft. Specification tolerances are shown, together with mean dimensions, and basing the design as one legally should, on the minimum permitted dimensions, it can be seen that the total weight penalty would be 11.7 per cent.

This question of extrusion tolerances has always been a vexed one, so the building of the half-scale wing was made the subject of a large scale experiment in achieving close tolerances. Tolerances approximately half the standard specification values were set, and the materials supplier made great efforts to comply with them. Fig. 10 shows the set tolerances and the mean results achieved, and the effort was completely successful. Tolerances of the same proportion have been set for the full scale sections.

Since specification sheet tolerances for light alloy appear to have been based traditionally on the assumption that any sheet manufactured must fall within the thickness tolerances of one gauge thickness or the next, there is great scope for closer control in this field. Table III gives the results of measurements on five sheets; the first three were 10′ × 6′ of 14 s.w.g. and 16 s.w.g. D.T.D.546 and although produced on equipment considered unsuitable by the supplier, they are representative of manufacturing technique ruling at the time; the last two sheets 10′ × 4′ × 0.140″ and 0.090″ show the best that can possibly be achieved with present equipment, but the fine control could not be maintained for production.

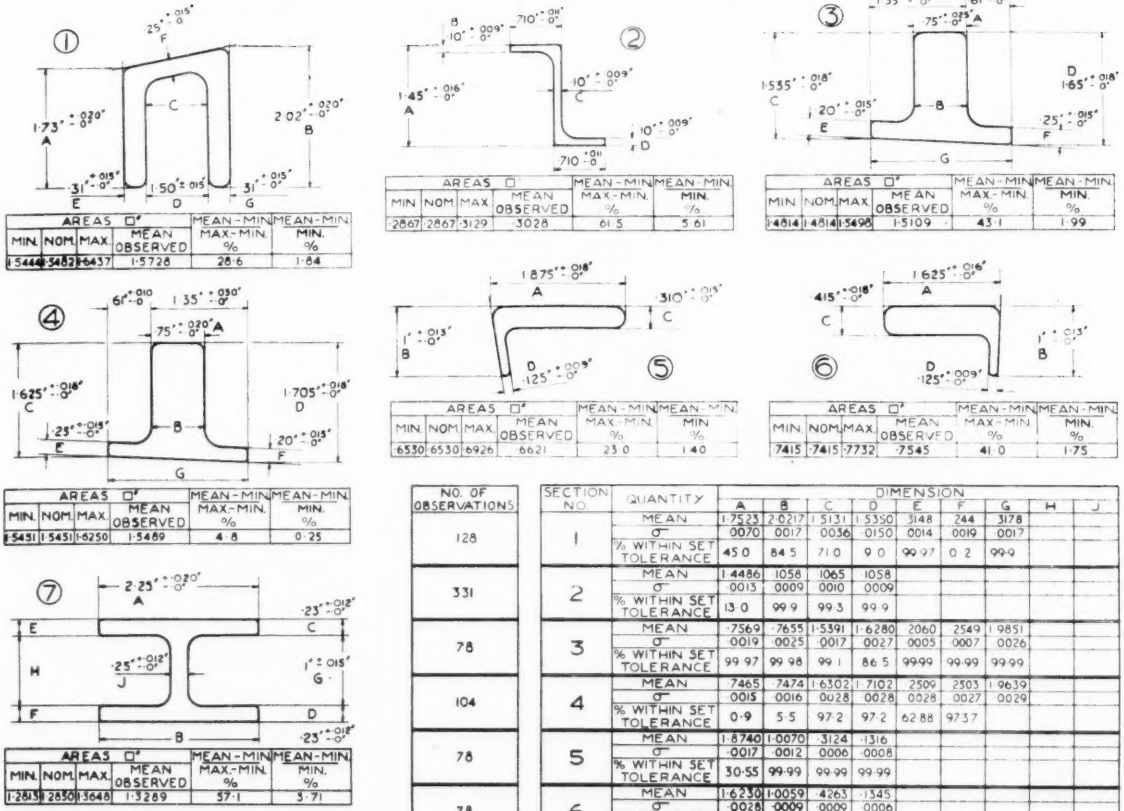
When the sheet thickness required exceeds that normally produced, there is no point in keeping to standard gauge values. In a particular instance the nearest standard gauge thicknesses suitable for the inner wing skin covering were 0.192″ (6), 0.176″ (7) and 0.160″ (8). Instead of 0.185″, 0.175″, 0.165″ and 0.155″ were chosen and this resulted in a more even graduation in strength and a saving of nearly four per cent. on the skin covering weight.

Similarly with standard sizes of sheets; in this changing world one hesitates to quote what is a standard size, but let us take the old pre-war maximum size of 8′ × 4′, which gives 0.75 feet of joint per square foot of area; a sheet 16′ × 6′ would give only 0.46 feet of joint per square foot of area—another saving worth having.

The conclusion is that for large aeroplanes it is often possible to work to higher stresses, lower tolerances, non-standard thicknesses and larger sizes than the standard specifica-

TABLE III

SHEET NO.	SIZE FT.	NOMINAL THICKNESS IN.	MEAN THICKNESS IN.	MAX. RANGE ABOUT NOMINAL IN.	RANGE WITHOUT 6-IN. BORDER IN.	SPECIFICATION LIMITS IN.	MEAN NOMINAL %
1	10 × 6	0.080	0.0859	−0+0.008	+.002+.008	−.005+.006	7.4
2	10 × 6	0.080	0.0861	−0+0.008	+.003+.008	−.005+.006	7.6
3	10 × 6	0.064	0.0684	−0+0.006	+.003+.006	−.004+.006	6.9
4	10 × 4	0.140	0.1418	−0.0015 +0.0025	−0+.0025	±0.007	1.3
5	10 × 4	0.090	0.0914	−0+0.002	+.001+.002	±0.006	1.6



AREAS □"			MEAN OBSERVED	MEAN-MIN MAX-MIN %	MEAN-MIN MIN. %
MIN.	NOM.	MAX.			
1·5444	1·5482	1·6437	1·5726	28·6	1·84

AREAS □"			MEAN OBSERVED	MEAN-MIN MAX-MIN %	MEAN-MIN MIN. %
MIN.	NOM.	MAX.			
·2867	·2867	·3129	·3028	61·5	5·61

AREAS □"			MEAN OBSERVED	MEAN-MIN MAX-MIN %	MEAN-MIN MIN. %
MIN.	NOM.	MAX.			
1·4814	1·4814	1·5498	1·5109	43·1	1·99

AREAS □"			MEAN OBSERVED	MEAN-MIN MAX-MIN %	MEAN-MIN MIN. %
MIN.	NOM.	MAX.			
·5451	·5451	1·6250	1·5489	4·8	0·25

AREAS □"			MEAN OBSERVED	MEAN-MIN MAX-MIN %	MEAN-MIN MIN. %
MIN.	NOM.	MAX.			
·6530	·6530	·6926	·6621	23·0	1·40

AREAS □"			MEAN OBSERVED	MEAN-MIN MAX-MIN %	MEAN-MIN MIN. %
MIN.	NOM.	MAX.			
·7415	·7415	·7732	·7545	41·0	1·75

AREAS □"			MEAN OBSERVED	MEAN-MIN MAX-MIN %	MEAN-MIN MIN. %
MIN.	NOM.	MAX.			
1·2813	1·2850	1·3648	1·3289	57·1	3·71

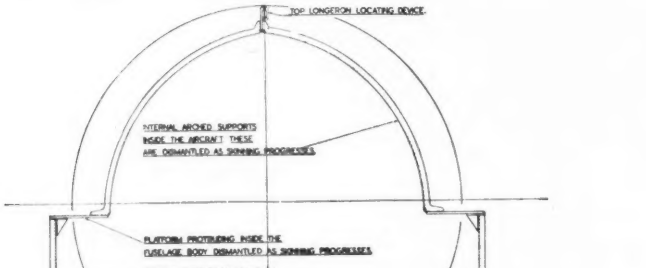
NO. OF OBSERVATIONS	SECTION NO.	QUANTITY	DIMENSION A	B	C	D	E	F	G	H	J
128	1	MEAN	1·7523	2·0217	1·5131	1·5350	·3148	·244	·3178		
		σ	·0070	·0017	·0036	·0150	·0014	·0019	·0017		
		% WITHIN SET TOLERANCE	45·0	84·5	71·0	9·0	99·97	0·2	99·9		
331	2	MEAN	1·4486	1058	1065	1058					
		σ	·0013	·0009	·0010	·0009					
		% WITHIN SET TOLERANCE	13·0	99·9	99·3	99·9					
78	3	MEAN	·7569	·7655	1·5391	1·6280	·2060	·2549	1·9851		
		σ	·0019	·0025	·0017	·0027	·0005	·0007	·0026		
		% WITHIN SET TOLERANCE	99·97	99·98	99·1	86·5	99·99	99·99	99·99		
104	4	MEAN	·7465	·7474	1·6302	1·7102	·2509	·2503	1·9639		
		σ	·0015	·0016	·0028	·0028	·0028	·0027	·0029		
		% WITHIN SET TOLERANCE	0·9	5·5	97·2	97·2	62·88	97·37			
78	5	MEAN	1·8740	1·0070	·3124	·1316					
		σ	·0017	·0012	·0006	·0008					
		% WITHIN SET TOLERANCE	30·55	99·99	99·99	99·99					
78	6	MEAN	1·6230	1·0059	·4263	·1345					
		σ	·0028	·0009	·0009	·0006					
		% WITHIN SET TOLERANCE	24·9	99·99	99·99	20·3					
32	7	MEAN	2·2670	2·2660	·2360	·2350	·2380	·2380	·9986	·9946	·2566
		σ	·0018	·0017	·0020	·0012	·0012	·0008	·0041	·0031	·0007
		% WITHIN SET TOLERANCE	93·67	99·97	99·76	99·99	99·95	99·99	99·97	99·92	99·99

Fig. 10.
Extruded Sections ½-scale.

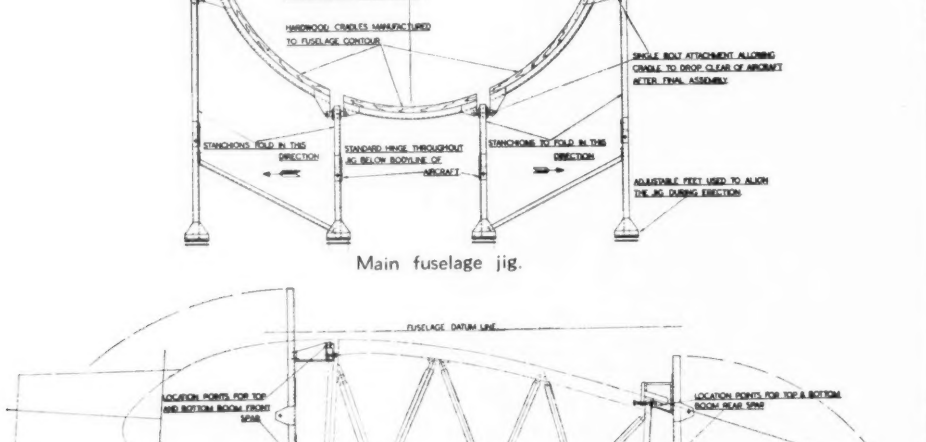
tion allows and that, once the necessary effort has been expended to establish the variability, a handsome saving in weight will be the reward.

STRUCTURE—MAIN ASSEMBLY.

It had become the general practice during the war to split the structure up into relatively small sub-assemblies. This was done to aid quantity production and to ease transport by road or sea. The method is only possible when the number to be produced justifies the heavy expense of elaborate jigs. In a large aeroplane if an attempt is made to break the structure down into components, the size of which permits them to be carried by road transport, the penalties of additional weight and complication in manufacture are


TOP LONGERON LOCATING DEVICE.
INTERNAL ARCHED SUPPORTS INSIDE THE AIRCRAFT THESE ARE DISMANTLED AS SKINNING PROGRESSES.
PLATFORM PROTRUDING INSIDE THE FUSELAGE BODY DISMANTLED AS SKINNING PROGRESSES.
FORMER ATTACHMENT BRACKETS POSITIONING THE FRAME STATIONS
HARDWOOD CRADLES MANUFACTURED TO FUSELAGE CONTOUR
SINGLE BOLT ATTACHMENT ALLOWING CRADLE TO DROP CLEAR OF AIRCRAFT AFTER FINAL ASSEMBLY
STANCHIONS FOLD IN THIS DIRECTION
STANDARD HINGE THROUGHOUT JIG BELOW BODYLINE OF AIRCRAFT
STANCHIONS TO FOLD IN THIS DIRECTION
ADJUSTABLE FEET USED TO ALIGN THE JIG DURING ERECTION.

Main fuselage jig.


FUSELAGE DATUM LINE.
LOCATION POINTS FOR TOP AND BOTTOM BOOM FRONT SPAR.
LOCATION POINTS FOR TOP & BOTTOM BOOM REAR SPAR
WING INCIDENCE.
HARDWOOD BEARER CRADLES WING CONTOUR OBTAINED FROM FULL SIZE LOFTED LAYOUTS.
SINGLE BOLT ATTACHMENT FOR EASY DISMANTLING.
STANCHION HINGED DOWNWARD FORMING PLATFORM FOR ASST OF LEADING EDGE
STANCHION HINGED DOWNWARD FORMING PLATFORM FOR ASSEMBLY OF TRAILING EDGE
STANDARD HINGE.
STANDARD HINGE FOR DISMANTLING JIG AFTER ASSEMBLY

Inner wing jig.

Fig. 11.

very considerable. Furthermore a pressurised body is difficult enough to seal without having to overcome any imperfect mating between two sections.

The methods best suited to assembly are akin to the old fashioned methods of ship building with all, or nearly all the major components built together in a comprehensive jig. Such a jig in addition to fulfilling its functional duties must be capable of breaking down to permit the removal of the aeroplane when ready and its reassembly to take the next aeroplane to be built.

One such jig, illustrated in Fig. 11, has proved very successful.

For the body, the frames and longerons are held by simple brackets with cradles shaped to the outside skin contour positioned a few inches aft of each frame station.

For the wings and tailplane the jig accurately locates the spars with cradles supporting the under surface. In locating the spars a novel feature was used in setting the flange location blocks. Tripartite angular blocks controlled the incidence, sweepback and chordal dimensions. In place of accurately machining each block, a system of jacking bolts was devised, the gap between the locating pad and the fixed bracket was filled with molten Cerro-bend, which when solidified formed a solid joint.

The special feature of the main jig, which contains the whole body, inner wing and tailplane, is the method employed for freeing the aeroplane from the location points and cradle structure after final assembly is completed. Every upright stanchion, in 6″ × 3″ channel iron, which forms the skeleton framework of the jig, is hinged. The hinge points are below the level of the wing and body. The tailplane jig structure owing to its height is jointed in two places. The skin locating cradles have a single swivel bolt method of attachment for easy removal. Additional to the purpose of freeing a completed aeroplane, the design of the jig permits rapid reassembly with the minimum of adjustment.

Another feature is the relatively small amount of constructional steel used. For example, the location of the main longeron at the top of the body is achieved by supports inside the body above the floor level, instead of enclosing the aeroplane in goalpost type of location. As construction proceeds, the internal supports are dismantled and stored in readiness for the next machine.

For the wing and tailplane the hinge principle is carried a stage further, for after completion of the main box structure, the outer stanchions folding outwards form platforms for assembling the leading edges, trailing edges, ailerons and flaps.

In this particular example a nice balance has been struck in the economy of material used and the amount of location required to produce an accurate airframe structure.

Riveting

Riveting is really an unsatisfactory method of fabrication for use in an aeroplane structure. Unfortunately alternative methods of jointing are even more unsatisfactory. The snags appear when in addition to transmitting load between adjacent members of the structure, it is necessary to produce a smooth external surface and also maybe prevent fuel or air leaking through the joint. Experience has shown that in the case of countersunk rivets the heads are oval, eccentric with the shank and the countersunk angle varies over wide limits. The design conditions cannot be met without the use of special precision rivets, these can be produced by either machining or forging. When rivets up to $\frac{3}{8}$″ diameter are used a difficulty is also introduced in forming the heads. A riveting gun weighing up to 20 lb. is required as compared with 6 lb. for that in general use.

Even allowing for the use of L.37 rivets driven in the freshly solutionised conditions, it is highly desirable that the amount of rivet protrusion to form the head should be a minimum in order,

(*a*) to keep the setting loads low and to minimise fatigue of the operators.

FAILING LOAD (LBS.) RIVET IN TENSION
STRESS (LBS./INS.²) DEVELOPED AT FAILURE
HEAD DEPTH (INS.)
SNAP HEAD - TENSILE STRESS ACROSS SHANK
1/4" DIA. RIVETS.
MATERIAL - DURAL.
SPECIFICATION - L.37.
COUNTERSUNK HEAD - TENSILE STRESS ACROSS SHANK
SNAP HEAD - FAILING LOAD
COUNTERSUNK HEAD - SHEAR STRESS THROUGH HEAD.
COUNTERSUNK HD. - FAILING LOAD.
SNAP HD. - SHEAR STRESS THROUGH HEAD.
REACTION HD. - SHEAR STRESS THRO' HEAD
REACTION HEAD - TENSILE STRESS ACROSS SHANK
REACTION HD. - FAILING LOAD
D/2 PROTRUSION (REACTION HD. DIA. 3/2 D)
D/2 PROTRUSION (REACTION HD. DIA. 3/4 D)
D PROTRUSION (REACTION HEAD DIA. 3/2 D)
D PROTRUSION (REACTION HEAD DIA. 3/4 D)
3/2 D PROTRUSION (REACTION HEAD DIA. 3/2 D)
2D PROTRUSION (REACTION HD. DIA 3/2 D)
3/2 D PROTRUSION (REACTION HEAD DIA 3/4 D)

Fig. 12.

(*b*) to keep distortion of the skin low
(*c*) to save weight.

As shown by Fig. 12, giving the values for $\frac{1}{4}$" dia. rivets, the full strength in tension of both countersunk and snap head rivets can be provided by a flat reaction head made from a volume of material equal to 1—1½ D protrusion, and flattened to 3/2D. Lesser values for the formed head diameter produced undesirable rivet-joint proof deformation characteristics. Thus an appreciable saving—of about 200 lb.—can be effected by comparison with the orthodox allowance of 1½—2 D generally used hitherto for protrusion on the more normal sizes of rivets.

Another major issue is the sealing to provide pressure-tightness of joints in the external skin of the body.

A convenient method of examining potential sources of leaks in joints is to test small panels reproducing various structural arrangements. The results of a series of such tests are illustrated in Fig. 13. The arrangements include various types of lap and butt joints and stiffener attachment.

The flat panels with precision countersunk rivets showed much leakage even when sealed, soap-bubble tests confirmed that the leakage was actually at the rivets. In further tests, panels to a typical curvature were used

Index:
(F) Flat panel : 120° countersunk head rivet.
(C) Curved panel : mushroom head rivet.
(1) Joint with no sealing
(2) Rubber sealing compound applied externally.
(3) Rubber sealing compound interlayer.

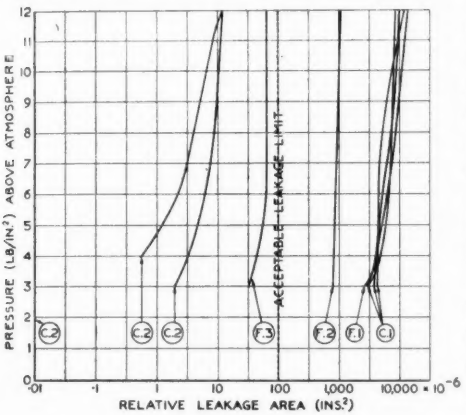


Fig. 13.
Panel incorporating lap and single butt joints.

with mushroom headed rivets and it was now found that a bead of special rubber compound applied at the inside edges of the seams was sufficient to achieve the target. No sealing at the rivets was necessary, and only isolated rivets leaked. This represents quite an appreciable weight saving on an interlayer or overlag type of sealing, possibly 200 lb. or more for the whole body.

Design of windscreen panels for pressurised aircraft is rendered difficult by the characteristics of the material utilised. One trouble is the scatter in results of identical panels tested under identical conditions, but one can contend with scatter alone, as one does with other forms of material, *e.g.*, light alloy castings. The second, and probably the major, difficulty is the effect of sustained and/or repeated loading on built-up panels with cellulose interlayers, from the viewpoint of loss of transparency—so that the ultimate strength of the panel becomes a secondary issue.

It has been demonstrated, by tests in an automatic pressure-maintaining deflection-recording apparatus, that rebated panels of toughened glass, with one lamination cut back in order to achieve a flush finish with the windscreen framing, are prone to failure by detachment of the outer layer, consequent upon failure of the interlayer. The outer lamination scarcely does any share of the work. Untoughened glass, made up into panels with interlayers does not attain a high enough stress to be worthy of serious consideration, and failure into large sharp pieces ensues. With toughened glasses one encounters the development of opaque areas in the interlayers under repeated and/or sustained loadings.

Acrylic resin panels show very good promise and are free from these troubles. Alternatively, the introduction of vinyl interlayers or the use of solid, semi-toughened glass panels may lead to further developments.

Flying Controls

The problem of producing completely satisfactory flying controls, properly co-ordinated one with the other, is always fairly difficult even on small aeroplanes. The larger the aeroplane the more difficult does the problem become as beyond a certain point the pilot is incapable of exerting more effort. Thus as size increases hinge moments must come down, so tending to approach even more critical conditions. Although there have been exponents of aerodynamic balancing for the largest aeroplanes envisaged, I believe that the uncertainty of over-balance or under-balance on the first flight alone demands the use of power operation.

The main advantages of power operation are as follows:—

1. The power control unit can be fully developed on the ground before the aeroplane flies, thus eliminating the often tiresome period of trial and error before aerodynamic balances are satisfactory.
2. If an irreversible system is used
   (*a*) The weight of mass balancing saved is greater than the weight added by the power units.
   (*b*) Trim tabs can be eliminated.

3. Aerodynamic nose balance can be eliminated with the advantages of
   (*a*) A saving in weight.
   (*b*) A simpler construction of flap and associated aerofoil.
   (*c*) A more rearward position of the rear spar giving more space for the torsion box and for inter-spar fuel stowage.
4. Artificial feel can be adjusted to any desired values.
5. The use of electrical controls between the pilot and control is simpler and lighter to install than mechanical controls.
6. The ailerons may be operated automatically to relieve the loads due to severe gusts.

The objections to the use of power are based entirely on the question of reliability. Any power system is inevitably more fallible than manual operation in that it introduces fairly complex mechanisms. Since the safety of the aeroplane depends on satisfactory operation many feel that a powered system should be designed so that in the event of a failure an emergency manual system can save the worst catastrophe. Much sympathy must be felt for this point of view but so many advantages are lost that serious consideration must be given to the alternative solution of providing complete duplication.

In this case each control flap must be operated by two completely independent units each deriving its power from separate engines and through separate transmission systems. Also each of the units separately should be adequate for the full control loads though possibly at a reduced rate of operation. In the event of one unit failing it should be automatically decoupled leaving the second unit in sole operation.

The most favourable conditions are fulfilled if the power control mechanism can be installed as a complete unit leaving only the minimum number of electrical, or mechanical connections to be made *in situ*. This permits thorough testing and checking before installation. Also in the event of a defect developing in service, a proper examination into the nature and cause of the defect can be made. Power units should not be serviced *in situ*, but only on the bench with the aid of any special equipment needed for a complete examination.

Needless to say the development of power control units must be thorough and exhaustive covering all possible conditions of atmospheric changes and structural distortions.

Desirable Characteristics.

The following characteristics are put forward as operational targets for power-operated flying controls.

1. The link between the pilot's controls and the control flap, *i.e.*, the transmission and the servo system, should be such that the pilot's control and the flap should be tied within 1°; this lost motion should be enough to operate the servo.
2. Reversal of pilot's effort must produce an immediate control response. This is essential to facilitate precise trimming.
3. The load on the pilot's control should be produced artificially, *e.g.*, by a spring. It must be zero for the trimmed position and increase uniformly on each side of trim so that when the control is released from a deflected position, control and flap return to trimmed position.
4. The maximum value of the pilot's effort should be

for ailerons	20-40 lb.
for elevators	40-80 lb.
for rudders	80-160 lb.

5. Friction loads should be less than 1/20th of the maximum loads.
6. The feel loads should not be affected by rates of operation,
7. It is probably desirable that the maximum pilot's force should correspond to maximum power loads, *i.e.*, proportional to speed squared as well as deflection.
8. The maximum rate of flap operation should be about 30°/second. This rate is

most likely to be used in landing conditions. With the slow response of a large aeroplane there is no point in exceeding this value unless the ailerons are used to relieve gust loads.

Figure 14 shows alternative forms of response of flap movement for an arbitrary control movement which illustrates 2. above.

If the advantage of eliminating mass balance is taken, the full loads on the pilot's controls must not be produced by feeding back a proportion of the operating load, as under flutter conditions the pilot's control must not respond to flap or wing accelerations. The servo unit must be effectively irreversible. Also both servos on each control flap should be in action together so that in the event of failure of one, the other maintains flap irreversibility.

Damping has been suggested as an alternative safeguard against flutter, the amount required, however, would impose excessive loads on the servos for high rates of operation.

Another form of interaction between the wing and control flap may arise. With large aeroplanes the natural frequencies of the wing and control flaps may be as low as 2 and 5 cycles per second respectively. Thus with rapid movement of the controls the time taken to build up the maximum load may be of the same order as the natural frequency of the wing, in which case vibrations would be excited. It is therefore necessary to limit either the maximum rate of increase of load or the maximum acceleration of the flap. One method of achieving this is the introduction of a damping device into the pilot's control system.

GUST RELIEF.

As stated earlier, the rate of build up of loads arising from gusts also can induce wing vibrations. This effect combined with a high cruising speed produces very severe stresses in the wings which may be 50 per cent. higher than any other design case. The additional structure weight required may be equal

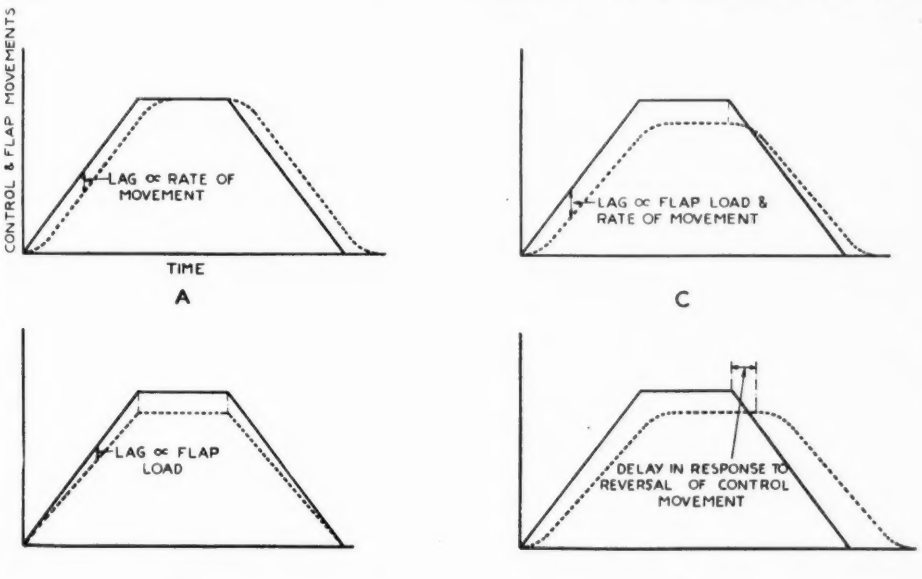


Fig. 14.

A B C satisfactory. D unsatisfactory. Control movement —— Flap movement - - - - -
Forms of response of servo controls.

to about 4 per cent. of the weight of the aeroplane.

Power operation of the ailerons makes it possible for them to be operated symmetrically for gust relief. Thus when the aeroplane enters an up gust if both ailerons are deflected upwards, simultaneously with and proportional to, the gust intensity the loads arising from the gusts are considerably reduced. Since the time to build up the gust loads and the time to over bend the wing are both of the order of one quarter of a second, the ailerons must be operated automatically by the gust. This requires a very short time lag after the gust is detected and a high rate of operation of the order of 60 to 80 degrees a second, *i.e.*, about double the rate of normal operation.

A gust can be detected by a pitch-meter which produces a differential pressure on a diaphragm with change of vertical component of wind. The deflection of the diaphragm can be made to operate the aileron servo through an electrical circuit such that the aileron deflection is proportional to gust intensity. Any lag in the response of the power controls can be countered by mounting the pitch-meter ahead of the wing in the nose of the body.

The gust relief device will operate with vertical acceleration in normal manœuvres and this would have an adverse effect on longitudinal stability. It is, however, permissible and advisable to switch the device out of action at low speeds, but it should be switched on automatically as the critical speed is approached.

To prevent unnecessary action in moderately bumpy weather, it is simple to arrange the device to ignore gusts of low intensity.

TYPES OF POWER SERVO UNITS.

There is a wide range of different types of power servo units to choose from, the following is a selection:—

1. Electro-hydraulic variable delivery pump.

A sensibly constant speed electric motor drives a variable stroke piston pump which transfers hydraulic oil to a hydraulic jack which operates the control flap. The pilot controls the stroke of the pump to obtain varying amounts and rates of operation.

This system uses the simplest and most reliable type of electric motor but a relatively complex hydraulic pump. The large gear reduction from the motor to the relatively slow control rotation is obtained conveniently with a hydraulic jack. This system is efficient as the pump only delivers the quantity and pressure of oil required to operate the flap.

2. Electro-hydraulic-accumulator system.

An electric motor drives a constant delivery pump to charge an accumulator at a pressure rather greater than the maximum operating pressure and the accumulator supplies oil to the operating jack as required. The pump delivery operates intermittently and is allowed to run idle by means of a by-pass valve when the accumulator is fully charged. This system employs the simplest components, but is wasteful of power as maximum pressure is maintained and irrespective of the power required.

3. Electro-hydraulic-gear pump system.

An electric motor drives two constant delivery gear pumps which deliver oil to a control valve, the valve delivers any required proportion of the fluid to the operating jack and spills the remainder of the pump delivery to low pressure. If the pumps give one-third and two-thirds of the total delivery required, the control valve allows the larger pump to idle at low pressure when the fluid demand is less than one-third of the maximum rate.

This system reduces the wastage of fuel by the use of two pumps.

4. Ward-Leonard electric system.

A variable speed D.C. motor operates the control flap through gearing. The motor is supplied by a generator with

direct current at a voltage which can be varied in either direction to obtain varying speeds in either direction. The generator output voltage is controlled by the pilot through a potentiometer to vary the field excitation.

The system is economical, but the D.C. generator and motor are more complicated and heavier than the induction motors used in the electro-hydraulic systems, also the large gear reductions tend to be heavy. The maximum torque transmitted to the gears must be limited to avoid overstressing the mechanism.

The Metadyne and Amplidyne systems are elaborations of the Ward-Leonard.

Many other variables of servo systems are possible, for example those using on-off motor switching, or a motor driving through slipping clutches of electric, hydraulic or mechanical type. No system is ideally simple and only the experience of extensive endurance testing will prove which system combines the essential qualities of reliability and satisfactory control characteristics with reasonable weight.

ELECTRICS—GENERAL.

A relatively short time ago Aircraft Electrics consisted of a small battery sufficient to cater for navigation lights and cockpit lighting. During the course of time the battery capacity had to be increased to cope with engine starting, and this in turn necessitated the installation of a generator to keep the battery charged. The load gradually built up as the advantages of electrical operation for various devices became apparent, until the stage was reached when generator capacity was greatly in excess of battery capacity and, in fact, of the whole load, except for engine starting conditions. Now it appears that the total generator capacity may have to be in excess of one kilowatt for every ton gross weight of the aircraft for civil machines, but that the capacity of the battery need not be much greater than to cope with the starting of one engine.

It is possible to employ a variety of different services in an aircraft, such as hydraulic, high and low pressure, pneumatic, vacuum and electrics, but for the sake of simplicity it is clearly necessary to limit the variety as much as possible. This leads to the conclusion that there are certain functions which can *only* be performed by electrics and few, if any, which *cannot* be carried out by some electrical method. The relative advantages of each means of providing power need not be detailed here, but it will be safe to say that there is a tendency to increase the use of electrically operated devices. As a consequence the safety of the aircraft may therefore depend on electrics, and for this reason increased attention must be paid to reliability, both from the point of view of the individual component and the installation as a whole.

In order that commercial airlines may operate economically it is clearly necessary to reduce as far as possible the time spent on inspection and maintenance, thus allowing the maximum possible time to be spent in the air earning revenue, this perhaps is one of the many reasons why the requirements for the electrical installation in civil and military aircraft are so widely different.

CHOICE OF SYSTEM.

Where physical distance between components become relatively large, the hitherto standard D.C. system at a pressure to 28 volts is quite inadequate, as in order to maintain the voltage drop within the maximum value, cable sizes and weights are excessive.

In the case of a D.C. system it would seem that a pressure greater than about 100 volts is undesirable at the present stage in development of switchgear due to the difficulty of quenching a D.C. arc.

In the case of A.C., however, as the current passes through zero it is comparatively easy to quench the arc which is set up when the switch opens, this allows of a higher voltage, and the limiting factor becomes mainly one of safety to personnel. It is thought that

A.C. systems operating at a pressure of about 200 volts between phases are quite suitable in this respect.

The choice (for the main service), then, appears to lie between 115 volts D.C. and 208 volts, 3-phase, A.C., and it would be as well to review the virtues of each alternative under the three main headings of " Power Supply," " Distribution " and " Utilisation." Considering first the question of " Power Supply," the main advantages claimed for D.C. are that (*a*) the various generators may be conveniently operated in parallel, (*b*) they may be driven at varying speeds, and (*c*) that for emergency or standby purposes a battery can take the place of generators. As regards simplicity and reliability there is little to choose, as it is generally agreed that it is better to replace the conventional D.C. generator by a simple alternator feeding through a metal rectifier to give the D.C. output, and that with these arrangements the weight is no greater than with straightforward D.C. generators. Superficially, it would appear that D.C. has the advantage for power supply, but when other factors are taken into account it will be shown later that this is not necessarily correct.

Turning to the question of " distribution," it is hard to say whether D.C. or A.C. is preferable for a given system pressure, as although 3-phase A.C. shows a saving in copper weight against an insulated D.C. system, it is offset by increased insulation and larger distribution boxes, plugs, etc. An earth return D.C. system is lighter for the same voltage, but with 115 volts D.C. compared with 208 volts 3-phase there is little to choose.

It is under the heading of " utilisation " that 3-phase A.C. is in many respects desirable in preference to D.C., the main advantages being:—

1. Motors that run continuously, such as flying control hydraulic pump motors, air conditioning motors, gyroscopes and fuel pumps are more reliable and require very much less maintenance if they are of the solid rotor squirrel cage type instead of being commutator D.C. machines.
2. Where low voltage is preferable, such as for lamps, it is a simple matter to employ a step-down transformer, whereas with D.C. it requires relatively heavy rotating apparatus requiring maintenance to give any different voltage.
3. Heating apparatus if required to be controlled thermo-statically is preferable on A.C.
4. On the whole A.C. offers better facilities for remote indicating instruments and remote servo mechanisms.
5. D.C. can be conveniently provided at any voltage from an A.C. system by means of static metal rectifiers, whereas if A.C. is required from any D.C. system it is necessary again to resort to rotating plant.
6. The radio interference caused by D.C. commutators usually necessitates the installation of additional weight in the form of suppressors; these, of course, are not required in the case of A.C. machines.

There are certain pieces of equipment which are better operated off a D.C. supply and in particular the various small actuators giving linear or rotary movement, these invariably embody limit and position switches which in the case of A.C. would involve relays, but in the case of D.C. can be (for small powers) of the direct operating type; furthermore, in the case of actuators, they are in the main operating infrequently and thus the maintenance of commutators does not present a serious problem. Likewise, in the case of solenoids either for operating switches or other purposes it is far better to use D.C. preferably at about 28 volts. There are also certain important items such as remote throttle control units which as they embody the " M " type motor can only be operated off D.C.

On reviewing the foregoing points it will be clear that certain apparatus is best suited to

A.C., whereas other devices require D.C., thus the logical conclusion is that a combined D.C.-A.C. system should be adopted, and only a detailed investigation will settle the ratio between the two.

Considering the operation of A.C. apparatus, some items require a frequency constant within close limits, others will operate successfully over a wider range of frequency whereas certain loads such as heating are quite unaffected by any change in frequency. If electrical heating for propeller de-icing is adopted, this load will represent a very large part of the total, probably at least 50 per cent.; there is, therefore, a clear case for taking the supply direct from alternators which may run at any speed, *i.e.*, without the necessity of a constant speed drive. Installing rectifiers for this type of load would mean adding quite unnecessary weight, and there is no case for operating this relatively large and important service on D.C.

It would be as well at this stage, to reconsider in detail the three reasons previously given in favour of D.C. from the point of view of generation. Firstly, the ability to operate D.C. generators in parallel, this is a virtue inasmuch as advantage can be taken of "load diversity" and also that a faulty generator can be tripped out without causing disturbance. Parallel operation means, in effect, a common bus bar with the grave disadvantage that in the case of a bus fault the whole supply is lost. The bus bar can be split with a coupling circuit breaker or fuses in circuit but it is doubtful if discrimination can be afforded as against the main generator protection. It is better, therefore, to split the generation into two separate distribution systems, whether using A.C. or D.C., and in the case of A.C., if it is found that 4 or 6 self-contained systems are involved to avoid parallel operation there is no particular advantage, providing of course that suitable automatic load transfer switches are installed.

The second consideration is that of speed variation of the main engines which does not greatly affect D.C. generators but is undesirable for A.C. except in the case of heating loads as mentioned earlier. With an A.C. system it is, therefore, necessary to interpose some form of constant speed drive between the main engines and those alternators which are supplying frequency sensitive loads. Considerable development work is in hand on these constant speed drives, and it is safe to say that satisfactory types will be available. The weight will depend upon the range of speed variation on the input shaft, and if this is to be as wide as 5-1 it is probable that the additional weight of the drive will be in the order of 4/5 lb. per kilowatt, which will more than outweigh the saving in weight of A.C. generators against D.C.

This leads one on to the third consideration and that is the ability to use batteries with a D.C. system for supplying load during the time that engines are not running.

In a large aircraft with a completely D.C. system the batteries would be no lighter than an auxiliary generating set, and it is clear that as the auxiliary generating set is not limited in duration of output it would be preferable.

If, as seems likely, it is generally agreed that auxiliary generating sets should be carried on very large aircraft, they can just as well be arranged to supply A.C. as D.C. Unfortunately, there has been very little development on suitable A.G.P.'s in this country particularly of the type embodying a supercharger. If auxiliary generating plants are a necessity for use on the ground and at low altitudes they should "pay their way" by providing power throughout the duration of the flight which necessitates supercharging.

It is probable that large aircraft of the future will have a generating capacity of about one watt per pound of all-up weight and that this will be equally divided between auxiliary generating sets and main engine-driven generators.

On reviewing the case for A.C. or D.C., it is clear that the most satisfactory arrangement is to have a combination of the two, with the main supply at 400 cycles, 3-phase,

at a pressure of 200 volts and with a secondary system at 28 volts D.C. supplied through metal rectifiers off the main A.C. bus bars.

At least two auxiliary generating sets will be required, one of which will carry all the loads vital to the safety of the aircraft, and the other will act as standby while carrying non-vital loads; this will probably mean that each unit will have an output of about 0.25 watts per pound of aircraft weight.

By careful attention to layout and providing duplication where necessary it should be possible to comply with the essential requirement. "*Any* type of fault on *any* part of the system must not result in loss of supply to vital components."

CONCLUSION.

A large aeroplane is a complex job of engineering. It is an undertaking that demands the closest possible co-operation between research, design, manufacture and operation. Owing to the varied nature of the problems encountered the degree of success that may be achieved depends on the co-ordinated efforts of a strong team of specialists, each having a vital contribution to make. This aspect cannot be emphasised too strongly and I welcome the opportunity of putting before you the results of some of this work.

I would also like to thank those members of the Bristol design staff who have helped me to prepare this paper.

DISCUSSION

Mr. L. W. Rosenthal: His Company had made an extensive series of tests on plate gauges, although not to any special tolerance. The general tendency was to be down rather than up, but there was practically nothing in it. The plates were selected from the normal batches.

With regard to the plating weight in the wing, he had found that when the wing was complete, the weight was the original basic area times the specific plating weight appropriate to the gauge with an additional 20 per cent. for bolts and rivets, doubling and so forth. Was that excessive?

Could Mr. Russell give some information as to the background from which the figure of £50 per lb. weight was deduced?

He had seen various figures quoted for sealing compound weight. Mr. Russell had stated that he had just a fillet running along the joints. What sort of figure did he expect for his large fuselage?

Mr. Lipscomb favoured the double bubble section, whereas Mr. Russell preferred the circular section fuselage; he would like further comments on the matter.

Mr. Russell: His Company, he believed, had taken up the matter of plate tolerances before the S.B.A.C. had done so, and had found that plates were within normal tolerance around the edges, but were thicker in the middle. They had checked some 2,000 pieces of Alclad. Of those, 20 per cent. were within .001 in. of specification, 30 per cent. were within .003 in., and 50 per cent. were on the top limit or over; that meant that 50 per cent. of the plates sold under nominal gauge were actually in the next higher gauge, except for small regions around the edges.

The figure of £50 per lb. which he had mentioned was really fictitious. But if it were assumed that 200 lb. could be saved and that, in consequence, an extra passenger could be carried, then the revenue provided by the extra passenger over the life of the aeroplane would be equivalent to 200 times £50.

His experience with countersunk rivets was that it was necessary to apply the rubber sealing compound, not only on the rivet heads, but also along the edge of each stringer or frame which happened to be riveted to the stringer, whereas with mushroom rivets they need only seal the edges of the plates, where one plate was joined to

the next, so that obviously a much smaller amount per plate was needed.

The weight per plate that was mentioned by Mr. Rosenthal was unreasonable, but it was not unusual!

Mr. Thomson (Saunders-Roe Limited): What gust gradient distances had been considered when determining the dynamic effect due to gust loads? He agreed with the acceleration of 6G under gust conditions for the large machine at light load, but, at normal load, thought this acceleration would be less.

Was the 50 per cent. increase in wing bending moment associated with the same loading condition?

He was surprised that plate stringers were being used in compression, because he would have thought that, in order to develop high compressive stresses and to overcome the torsional instability of the section, the top hat section would have been preferable. He would like further explanation of the matter.

With regard to the condition of the edge members of panels, it had been pointed out that with the plate section the edge conditions were simply supported, whereas with the extruded section they were fixed. It seemed that the degree of fixation or clamping would be effected not only by the torsional stiffness of the stiffener, but also by the stiffness of the rivets.

Mr. Russell: Little was known about gust loads, but the R.A.E. was doing its best to fill that particular gap. When he had referred to 6G, that did not mean 6G acceleration. The effect of the gust was to set up wing vibrations. Virtually what happened was that the wing would overswing or over-bend; so that the bending moment was appropriate to 6G, but it was not a 6G acceleration. The acceleration, of course, was higher than the normal design case, and if the aeroplane travelled still faster, then the acceleration would continue to go higher. When aeroplanes travelled very very fast, he did not know what they should do about it.

In the paper he had referred to gust-relieving devices, and his Company had attempted to introduce a device to reduce the effect of gust. The ailerons were power-operated. When the gust hit the nose of the aeroplane a signal was sent through to an electric motor, which then got going and, through a differential gear, operated the aileron power controls and in one-fifth of a second after receiving the signal it moved the aileron through 15 deg. in the direction opposite to the gust.

With regard to stringers and clamping, etc., in their first experience of rolled sections, whatever the proportions of the rolled sections, they could not develop more than an average stress of 35,000 lb. per sq. in. When they went to extrusions, they could develop without difficulty stresses of the order of 45,000 lb. per sq. in., but at a reduced rib pitch. With a combination of rolled sections and extrusions, a rib pitch 50 per cent. greater was possible than with extrusions alone. In those conditions they could now develop 45,000 lb. per sq. in. stress, which meant considerable saving of weight.

Mr. F. W. David (Chief Designer, Commonwealth Aircraft Corporation, Melbourne, Australia, Associate Fellow): Would Mr Russell explain the gust-relieving device? He understood that if an aircraft experienced a gust it might well happen, if the aircraft were of large dimensions, that the gust would affect one wing only, and the other wing would be un-symmetrically loaded. Would not that require a gust indicator which was symmetrically arranged?

Gust seemed to play an important role, and he wondered whether that would not require flight at greater heights? Would it not also mean that, the lower the altitude, the lower would have to be the speed of flight?

Finally, had shear forces or stresses in the compression panels been considered?

Mr. Russell: The gust produced severe stresses on the airframe only when it was

symmetrical. If the gust hit one wing and not the other, it rolled the aeroplane and did not produce high bending moments. The actual bending moments and, therefore, the stresses, were not high for asymmetrical conditions; they were relieved by the roll induced.

They knew little about gusts at high altitudes, but there was some evidence that gust increased in velocity with increasing altitude. It was thought that gusts might not continue to increase in intensity at about 30,000 ft.; but very likely, up to 40,000 ft., it was a mistake to think that gusts could be avoided by flying high.

As to shear in panels, they had not reproduced the shear in the simple compression panels, but the complete half-scale wing was tested. The results of the tests provided sufficient answer in themselves.

Mr. J. Hadji-Argyris (Royal Aeronautical Society): It was interesting to see how important theoretical work could be in modern aeroplane design. The breadth of scientific and practical imagination required of the present day aircraft designer was shown forcibly in Mr. Russell's lecture. In the past there had been a tendency to discount the scientific approach and it was refreshing to see in Mr. Russell a designer who was not afraid to use mathematics.

In the name of his colleagues at the Royal Aeronautical Society he wished to thank Mr. Russell for the kind acknowledgment of the work done at the R.Ae.S. in connection with the buckling of skin-stringer combinations (panels) under compression. It was particularly gratifying to hear that the theoretical analysis of these complicated instability phenomena showed such remarkable agreement with the experimental results. He could wish that this were more often the case. He thought that Mr. Russell would agree that in skin-stringer combinations no single mode of instability would occur except in some exceptional cases of symmetry.

Mr. Russell's remarks on the calculation of the loads on the wing and the great importance of dynamic effects in large aircraft were very interesting. He would be interested to hear if, parallel to the undoubtedly more accurate methods of calculating the actual loads on the wing, his company was using more refined methods of wing stressing than they knew to be commonly used. The engineer's theory of bending and the Bredt-Batho distribution of torque were not reliable enough to estimate the stress distribution in a stressed skin structure.

Mr. Russell mentioned the high rate of shear diffusion effect in the strength tests. He would be interested to hear what methods the Bristol Aeroplane Co. was using to analyse these effects.

Mr. Russell: Mr. Argyris himself had made some extremely valuable contributions to the theories, and he was well in touch with the design staff of the Bristol Aeroplane Co., who, he must say with all humility, had done some extremely fine work scientifically, mathematically and practically. Some of them were present at the meeting, and it would probably be best if Mr. Argyris were to put his questions direct to them.

Evening Session

GENERAL DISCUSSION

In the chair the President, **Sir Frederick Handley Page, C.B.E.**

Mr. S. Scott Hall (Ministry of Supply, Fellow): They must not lose sight of the fact that it was the jet turbine which had really opened up all the vast possibilities of aviation which lay ahead. Much had been said about the propeller gas turbine; but he would emphasise what Mr. Owner had said about the propeller gas turbine being a bridge, and would add that they should not spend too much time building the bridge, forgetting meanwhile the goal beyond. They had a tremendous advantage over the rest of the world in the present stage of development of the jet engine, and it should be exploited.

As to the development of the jet engine from the point of view of noise and the comfort of aircraft passengers, anybody who had flown in an aircraft with jet engines would bear him out that it gave a great increase of comfort. The noise was like that of the engine room of a ship.

The development of the jet engine involved two major problems. One was the specific fuel consumption, and they should bend all their energies to improving that. The other problem, and one of the most difficult, was that of anti-icing.

Mr. Lipscomb had suggested that a failure of a jet power plant — and presumably he included all turbine plants—would be more catastrophic than the failure of a piston engine. He believed the information available to date showed that that was not the case.

He was filled with despondency by Mr. Owner's statement that another ten years would elapse before they could fly at 40,000 ft. If they were to use the jet turbine economically they must fly high and fast. They needed more industrial and design effort on compressors and blowers, particularly in relation to problems such as pressurisation. It might not be one of the most spectacular aspects of development, but he believed it to be of supreme importance. He strongly supported Mr. Owner in his plea for some progressive effort, surmounting obstacles stage by stage.

During the discussion that afternoon on gust relief and power-operated controls he had wondered how he would feel, if he were a prospective airline passenger, about Mr. Russell's remark that, if the control worked, there would be a saving of 13,000 lb. of wing weight. He thought that unless he knew of a long history of development and test, he would much prefer that the 13,000 lb. were in the structure, even though the air liner might not be making quite so much profit for the operators.

Thus he made a plea for research, development and test of all these new developments in aeronautics. It was absolutely essential to put them into the air and test them; Mr. Russell himself had indicated that everything depended on reliability.

Finally, on the question of effort; never before had they been faced with so much to do in aeronautics, and yet with so few qualified people available; they must find more.

Mr. R. Hadekel (Associate Fellow): An appreciable amount of work had already been done on both the hydraulic and the electrical versions of flying controls which had been applied mostly to either small high-speed machines or medium-sized aircraft, at any rate according to Mr. Russell's standards. It seemed that when they came to the larger sizes the problem of power for those controls became a major one. For instance, in a small aeroplane it might be permissible for a good deal of the power from the prime mover for the flying controls

to be wasted, whereas in a plant such as that of the Brabazon it was obviously worth while economising. Some peculiar figures were mentioned in regard to the Northrop Flying Wing; the power required for the servo control was 92 h.p., which was a staggering figure. Presumably the controls were completely unbalanced. Was Mr. Russell permitted to divulge data concerning the actual power required in the Brabazon for controls?

To what extent was "feel" in flying controls necessary from the functional point of view, or was it just a question of complying with present habits? It would seem that if "feel" could be entirely eliminated, they would simplify the problem.

Mr. G. P. Hebden (Associate Fellow): Mr. Lipscomb had said that the double-bubble type of construction fitted more conveniently into the flying boat than did the circular form; he suggested that Mr. Lipscomb was really making a virtue of necessity. Inherently the flying boat must have an enormously large fuselage in order to obtain propeller clearance over the large waves which large flying boats were expected to meet. The lecturer had stated that jet engines would also require a large clearance, so that the great depth of fuselage must be maintained, whereas the landplane designer expected to shorten and lighten the undercarriage when employing jet propulsion. Thus the landplane seemed likely to increase its advantage over the flying boat.

Did Mr. Lipscomb consider that it was simpler to build a double-deck body rather than a single-deck body having the same floor area? Inasmuch as simplicity was so desirable, some consideration should be given to that matter of the number of decks to be provided. It might be necessary to use a number of decks, for many reasons besides that of the floor space required.

Mr. C. P. Wadsworth (Bristol Aeroplane Co., Associate Fellow): Would Mr. Lipscomb give some reasons for his view, or at any rate his implication, that the buried engine would be more difficult to maintain than the engine in a nacelle? It would appear that the aircraft designer was in an excellent position so to accommodate the engines within the wing that even better maintenance facilities were possible than in the case of engines in nacelles. That seemed to apply particularly to flying boats, where the maintenance of engines placed well forward of the leading edge was a horrible problem. If the engines were accommodated in the wing, it would seem possible to provide space within the wing so that the engines would be readily accessible; the engineers could step off the surface of the wing into cockpits alongside the engines and would be able to deal with those engines more easily than was possible in other forms of installation.

Mr. L. W. Rosenthal (Saunders Roe, Ltd., Associate Fellow): He discussed the merits of the double-deck fuselage as compared with those of the circular form. If the length, depth and beam were plotted against the cube root of the gross weight (the linear scale), it would be found that from very small sizes up to the order of 400,000 lb. weight, all the dimensions followed a linear relationship, within reasonable limits; the line wandered a little from the straight locally, but it straightened out eventually. From the figures he had produced he had found that, although the fuselage was shallower for a given gross weight, it was longer than the flying boat hull; the beam was much the same in both sizes. The flying boat hull might have been expected to be bigger; but the Brabazon fuselage was a 16-ft. circle, and the Saunders-Roe boat had a beam of approximately 16 ft. The total surface areas compared very well. The vices of the double-bubble fuselage did not then seem immediately apparent.

It might be a good thing to have engines housed in the wing so that the maintenance people could get all round them. There must be covers over the engines, and those covers must be easily removed. One's experience

of putting holes in wings, however, was unfortunate, and the placing of the engines in the wings might involve very severe weight penalties.

There were pleas for aeroplanes which were safe and which were easy to make and maintain; but it seemed that if they bowed down to all the various requests they were in danger of finding that they had no payload left at all. It would be no use making an aeroplane easy to maintain if no one wished to buy. He pleaded for a light aeroplane. A considerable amount of time at the Bristol Aeroplane Company appeared to have been devoted to reducing weight; and Mr. Russell had mentioned a reduction of 13,000 lb. in wing weight. They heard a lot about wing weight reduction from time to time, but he had yet to see a report on how to save fuselage weight. It seemed very difficult even to compute, much less reduce, the weight of a flying boat hull.

Mr. P. G. Masefield (Director-General, Long-Range Planning and Projects, Ministry of Civil Aviation, Fellow): One of the most significant indications in Mr. Owner's paper was that they were likely to have in the future a wider choice of power plants than they had ever had before in the history of Aviation. He suggested that they would be able to afford the luxury of determining specialist types of power plant for specialist purposes. In that connection he had hoped that for really long ranges the compound engine might prove attractive, and he was rather sorry to hear Mr. Owner pour cold water on that possibility. He hoped that the problem might not prove so difficult as Mr. Owner seemed to think. The possible advantages of the low fuel consumption of that engine appeared very great for long-stage routes.

It seemed at the moment that all their eggs tended to be in one basket — the jet basket; like all eggs these days, they were in rather short supply.

The discussion had served to emphasise what a tremendous gamble they were taking. He considered it a bold but justifiable gamble. Almost without exception, their new civil aircraft depended for their success on the gas turbine—with or without propellers. About four years hence they were hoping to see new civil aircraft operating with engines which had not run, even experimentally, yet. He did not think the British Aircraft Industry as a whole had ever taken such a gamble before. He believed it to be justified, because the rewards were enormous if it came off, although he would not like to think what might happen if they should fail, by reason of, for instance, really serious icing troubles such as they had not yet encountered, but could envisage.

The time factor was of enormous importance. A good aeroplane was no good unless it came out at the right time, when its performance was up-to-date in relation to competitive types. In that connection it was worth noting that the Industry in America had not gone—or had not been able to go—so far as they had in this country towards turbine engines; they had concentrated much more on piston engines with turbo blowers. If they succeeded with their civil turbines in Great Britain they would make tremendous advances over the Americans.

He was glad Mr. Scott Hall had emphasised particularly the pressurisation aspect. Some of their new plain jets might depend for their success upon being able to cruise at heights above 40,000 ft.

It would appear, with engine developments going the way they were, that they would need some form of assisted take-off. Personally he felt that it was better to go off in a cloud of glory when half way down the runway than to "go to glory" at the end of the runway if one engine failed!

He believed there was going to be a very important place for the flying boat. It was entirely a question of economics, and the economics were bound up more with adequate bases than with anything else.

The paper by Mr. Russell showed that one of the great aeronautical adventures of

the day was the Bristol Brabazon I. There again an enormous amount depended on lot of unknown factors. They might see in the Brabazon one of the great successes of British Aviation. They must go to the big aeroplanes to gain enough payload at long ranges, and also to overcome the congestion caused by unworkably high frequencies at the airports. They could not increase frequency beyond the capacity of an airport, so that as traffic increased they must go to big aircraft. Therefore, the courage of the Bristol Company might help very much in the years ahead.

Mr. F. H. Pollicutt (Bristol Aeroplane Co. Ltd., Fellow): Two very important ancillary services—anti-icing and pressurisation—took considerable power from the engine. On the previous day he had seen one of the 80 lb. blowers intended for the Brabazon at work; and it was really a shocking experience. He had felt that it would be a good thing if they could do away with them. How soon were they likely to be able to take a direct tapping from the gas turbine or jet to give power for anti-icing, pressurisation and cabin ventilation?

Would Mr. Lee enlarge on the Prandtl rule; he had not heard of it before, and wondered whether anyone had had any practical experience of replacing the section for compressible flow by a section for incompressible flow, with particular regard to the accuracy of results so obtained.

He disagreed with Mr. Lipscomb's plea for a checking system in the drawing office; they started with an assembly layout and a detail layout, and then detail draughtsmen made detail drawings from the detail layout, and an assembly was made from that. They tried to arrange that one set of men did the detail work and another set did the assembly, so that there was a check. Then there was a further check of a somewhat clerical nature to ensure that all notes, etc., were included. A check-up on the drawing office, such as Mr. Lipscomb had suggested, would probably occupy 10 per cent. of the manpower of the drawing office. On the Brabazon the actual drawing error was small, and so far they had not had a major error. The alteration of drawings, from the points of view of drawing errors and shop queries, was handled by one man and a boy—and the man was not really a senior layout draughtsman. If Mr. Lipscomb's suggestion were adopted, there would be a staff of 15 or so engaged in checking, and it would probably put back the flying date by two months. They would not take kindly to that.

It seemed almost as though one of the speakers in the discussion had been trying to start a war between the various departments, in discussing whether the design office should design for light weight or whether the production engineer should engineer for low costs. The departments should all get together and consider the aeroplane that was to be produced; as Dr. Joad would say, "It all depends." In one case, for instance, they would have to design primarily for lightness, and dispense with every piece of structure that was not really necessary, and so on.

It would not be fair for him to heckle Mr. Russell; but he endorsed the plea made by a previous speaker for more and more research. Some of them were trying to do a very big job with insufficient staff, and the better their background of test and development, the happier would be their lives.

Air Commodore D. F. Lucking (Fellow): He was inspired by parts of Mr. Russell's paper to plead that members of the Society should come forward with more papers of technological interest, papers dealing with economy in methods of manufacture and production. Large numbers of papers were presented on general questions, and on aerodynamics, stability, and similar subjects; indeed, in this country they were in a very good position, in that they were well advanced in the pure science of airframe and engine design. But there seemed to be some reluctance on the part of contractors to share their technological secrets. In the interests

of the nation and of the Industry more of those technological secrets should be revealed.

Mr. G. F. Webb (Short Bros. (Rochester and Bedford), Ltd., Student): Would Mr. Russell comment on undercarriages for large aircraft to balance Mr. Lipscomb's comments on flying boats?

Mr. M. Mitchell (College of Aeronautics, Student): Could any of the authors say anything about the future of the auxiliary plant, such as was used in the Shetland, and whether or not it would have any place in the jet air liner of the future?

Mr. Quin (Bristol Aeroplane Co. Ltd.): Could Mr. Lee say whether any considerable work had been done anywhere on the design of the slots for boundary layer suction, and whether information was available of the suction quantities and pressures required?

Mr. H. Roberts (Imperial College): He would like some indication of what was to be regarded as "a large aeroplane"; he appreciated the difficulty of giving a figure, but it was necessary, especially from the point of view of controls. For example, there were many kinds of ships — rowing boats, tugs, lighters and so on, up to the smaller liners and eventually to a vessel such as the "Queen Elizabeth"—and essentially different problems applied to each. There was much discussion as to whether aeroplanes should have power controls or spring tabs; the basis of differentiation between the two was tied up with the size of the aeroplanes, and as they became larger the tendency should be to dispense with spring tabs in favour of power controls. Where was the limit?

Some time ago a member of the Bristol Aeroplane Company's senior design staff had said that there were certain advantages in eliminating spring tabs, particularly because weight was saved by so doing. He was rather doubtful about it at the time, but presumed the statement was based on the experience of that Company with present-day large aeroplanes. He had the ambition to see an aeroplane in which they would dispense completely with the control column and all conventional controls, and operate by means of a set of buttons and knobs. After all, a liner such as the "Queen Elizabeth" was operated by proper engineering methods, and they must reach that stage eventually in the case of a very large aeroplane. He believed it would involve the complete elimination of "feel." Did Mr. Russell envisage that it would be eliminated, and that the operation of large aeroplanes would depend on buttons and knobs, with the necessary dials?

Dr. H. Roxbee Cox (Vice-President, Fellow): The ideal of operating an aeroplane by means of a set of buttons was very proper; and it was interesting that, in the old Graf Zeppelin in which he had flown in 1931, that was all the pilot had normally for its control. A wheel was provided, and it spun round at great velocity when the pilot pressed the appropriate button; the wheel was there so that, if the pilot lost his button, or the power failed, he had manual control.

In Mr. Lee's paper reference was made to the annoying habit of the swept-back wing whereby, if there were imposed upon it pure bending, there was automatically an aerodynamic twist. It was well to point out, however, that while that was always probable, it was not necessary; and it was worth elaborating the point, as Professor G. T. R. Hill had done at a recent meeting of the Society. If one imagined the wing as a sort of plank swept-back, then obviously on bending there would be aerodynamic twist. On the other hand, if one imagined a swept-back wing with two parallel spars of equal length fixed to a fuselage the side of which was parallel with the axis of the aircraft, and if the wing deformed simply by the bending of those two spars, then if a load were placed along the flexural axis between the spars, there would be no twist. Thus, the amount of aerodynamic twist would depend very much on the internal structure of the wing.

Mr. Scott Hall had referred to the necessity to reduce the fuel consumption of gas turbines; that was the main thing to be done in the gas turbine field during the next few years. It was a hopeful line of endeavour, and he believed the fuel consumption would be so reduced as to render the jet propulsion gas turbine competitive with other forms of gas turbine over very long ranges.

The danger of the failure of power-operated controls was met by Mr. Russell, he believed, by duplicating the power-operation system. But was there not the possibility of using manual control when things did go wrong? Of course, they could not use manual power to replace hydraulic or other power entirely, but conceivably some part of the control system could be operated manually. If, for example, an aileron were made in two pieces, one piece being small and the other large, the pilot could achieve some control by operating the small piece directly if the power failed.

He hoped that designers would not maintain the same wing clearance with jets as with propellers on flying boats, because he believed that great advantage could be derived from bringing the wing nearer to the water. It should not be beyond their ingenuity to design some device for use during the take-off which would bring in the air from above and so prevent water getting straight into the jets; such a device could be retracted once the flying boat was in the air.

As to the gust-relieving device, Mr. Russell had made it quite clear that, when the wing was about to pick up load, the pilot put the ailerons up, producing a counteracting down load. But would not the down load act at such a point as to give the wing further incidence? Obviously the effect must be there; but what was its extent, and had it been taken into account in the 13,000 lb. saving of wing weight which Mr. Russell had mentioned?

On the question of the unsymmetrical gust, he had gathered that there was no serious consequence because the gust would roll the aircraft. But what about the aircraft inertia? He was surprised that the aircraft would roll fast enough to relieve the load.

He felt that the problem of gust was one of the most serious that had to be faced. If they built a small aeroplane it could perhaps be made to travel safely very fast, but he felt they should not try to achieve great size and high speed together for a little while yet.

He did not quite like the idea of pyrotechnics for assisted take-off; a modification of the gas turbine engine could be made which would give extra power when it was required for take-off or other purposes.

He was glad to hear speakers in favour of the flying boat, and to hear Mr. Masefield refer to the importance of providing adequate bases. With proper airport facilities, they should find flying boats a highly economical means of transport.

Mr. E. J. Richards (Vickers-Armstrongs Ltd., Associate Fellow): Was the large aircraft the aeroplane of the future and were all those engineering problems associated with very large aircraft worth solving? When crossing the Atlantic they would travel at somewhat lower speeds than would a jet machine, with the result that the time occupied on the journey would be longer and the allowances for wind, and so forth, had to be very much greater. The standard of comfort and the need for bunks would be greater so that in turn the size of aeroplane would be increased. Had Mr. Russell any strong views as to where the jet air liner came in and where the large propeller turbine type came in? It seemed that they would think about three types of aeroplanes (1) the large landplane, (2) the large seaplane, and (3) the very much smaller jet air liner which would do the journey much more rapidly and would be just as sound economically, except for very large ranges.

With regard to the comfort of jet air liners, he wondered whether people were being misled concerning the comfort of the experimental Lancastrian. The jets on that

machine were right outboard. On walking around a jet machine such as the Meteor or Vampire the low-frequency noise came on suddenly at an angle of about 45° to the jet. He doubted whether the Lancastrian engines would be really quiet for the passengers if the jets were fitted inboard. That was a major point which would need to be taken into account at the design stage of an aircraft.

As to the engines cutting, he had been interested to obtain from America some data on the windmilling drag of jet engines, and to note that, as the speed increased, the windmilling drag increased as a percentage of the total thrust. Could Mr. Owner say if that were really true, and whether severe asymmetric loads were likely to arise when flying at high speed if the engines did cut?

Mr. Rant (Handley Page Ltd., Student): Had Mr. Russell any idea of what the maximum economic size of a large aircraft could be?

Mr. P. G. Masefield (Fellow): He discussed the size of aircraft in relation to the demands on the services. He had been much impressed recently by the achievements at airports in America. The fact was that the airports of La Guardia, Washington, Chicago and Los Angeles were handling at the moment more than 750 aircraft movements per 24 hours, and peak loads were at a higher rate than that.

It was obvious that the number of aircraft using the individual airports could not be increased *ad lib*. As traffic demands increased — and they were increasing very fast—the ceiling of frequency of operation into and out of an airport was reached; and the bad weather criterion was possibly the one on which calculations must be based. Then they had merely to divide the maximum number of operations that could be put on to a runway, or parallel or tangential runways, into the number of passengers wishing to travel. The answer was coming out, in congested countries such as the United States (and it would inevitably be the same here later), to indicate the need for much bigger aeroplanes than they had thought of recently. The 21-seat DC-3 was much too small where there were heavy traffic demands. The present economic size — which had nothing to do with range or engineering or structures or speed — was working out to be somewhere around a 60-seater for the major routes in America to-day. The figure was still increasing rapidly, and he believed it might be 100 seats before long.

Thus, from that point of view, he was not at all sure that the Bristol 167 might not turn out to be too small, and that they would not have to think, from the engineering point of view, of something bigger, more particularly for long journeys non-stop. It was not just a question of whether they should use small aeroplanes at high frequency or larger aeroplanes at lower frequency; it depended on the number of passengers wishing to travel and the number of aircraft that could be operated through an airport in the average weather which was likely to prevail.

The President: Many problems had been raised in the papers and in the discussions; those engaged in the development of aviation were very enthusiastic and willing to overcome the difficulties, but they might be appalled by all that they would have to deal with in the future.

For example, there were psychological and physiological problems. If they were to fly at great speeds and heights they would undoubtedly have to experience extreme "bumps," which might cause passengers to be subjected to anything up to, perhaps, 3G; there were few people who would wish to travel by any form of locomotion in which they would experience 3G, unless they were of the type who liked spending their lives on a switchback at a fun fair! Most people preferred to travel in comfort. Then there were the problems of maintaining correct pressure and suitable air conditioning at altitudes.

Those problems had nothing to do with aerodynamics as such.

On the aeronautical side one might think, from the reading of certain papers or the popular press, that with the advent of jets and special forms of wings, as they generally understood laminar flow wings, aviation was going to experience a complete change and that that change was just around the corner. He felt sometimes that the position was very much over-stated. Various things seemed to be so simple when they were introduced, but they involved a great many difficulties of a practical character which must be ironed out and problems which must be solved, and all that would not be done in a day.

They must be careful that, by seeming to give acquiescence, they did not lead the general public to expect that they would be able to travel in commercial airliners at 500 m.p.h. to New York, in beautiful air-conditioned cabins, and that by the time they had ordered a drink and had consumed it they would have reached their destination. Those things would not all be brought about in a moment, but only after several years.

Meanwhile, there was a good deal of life left in the old piston engine and in the ordinary form of aircraft. Whether aircraft would be large or small would depend on the definition of "large" and "small." They must not again be led away by always thinking that what was large was necessarily the best; if that were so, the elephant would be the lord of creation on the land and the whale would be the biggest thing in the sea.

They must remember the great many engineering problems that must be solved in the future. It was only by meetings such as they had had that day, at which those problems were fully discussed so that everybody gained a little knowledge from others, that they would gradually find the solutions of those problems.

Air Commodore F. R. Banks (Technical Manager and Chief Engineer, The Associated Ethyl Co. Ltd., Fellow), *contributed*: He would not comment on any particular point in Mr. Owner's paper because he agreed with most of it. But he wished to make a few remarks on engines in general and the future in civil aviation in particular; because it was the safe and speedy transport of the peoples of the world over the world that was going to matter to them all.

The piston engine which had served them well in aviation for some forty years was on its way out, but it would be about ten years or more before it faded from all important uses in aviation—if for no other reason than that it would take this time for an appreciable number of turbine engined aircraft to have established themselves.

They had the jet turbine in its simplest form now, but his guess was that a jet-propelled aircraft, cleared for the carriage of fare-paying passengers, would not be flying the air routes sooner than 1951. This would also apply to aircraft fitted with propeller turbines. The aerodynamic problems in high-speed jet aircraft would be considerable, to say nothing of the mechanical and physical difficulties attending the safe and proper pressurising and air conditioning of cabins for flight operation at 40,000 ft. Passenger comfort, when flying in aircraft at speeds which were some 350 m.p.h. greater than those at present, would be difficult to achieve. The effect of bumps might be more than disturbing. And although it was thought, wrongly, that bumps did not occur at heights of 40,000 feet the climbing speed of this type of aircraft would not be less than 300 m.p.h., at which speed it would be uncomfortable in rough weather.

The aircraft with the propeller turbine would take almost as long to evolve because the engine was more complex than the jet turbine. The aircraft, also, would not be ready much earlier; even though the aerodynamic problems were probably less than those of the faster jet-propelled machine.

It might be that aircraft with propeller turbines would fly as fast as jet-engined versions, up to a maximum speed of, say, 550 and perhaps 600 m.p.h., if the results of recent experimental work on propellers was confirmed in practice, where efficiencies of 80 per cent. at 550 m.p.h. appeared feasible. The advantages of better fuel economy, superior take-off and climb performance would be considerable with a propeller turbine, should such propeller efficiencies be realised, even allowing for the greater weight and complication of this type of power unit.

It was not generally realised that one of the most ambitious large aircraft in the world, if not the most ambitious yet, had piston engines. This was the Republic Rainbow, and it was outstanding. Although it was still on test, he was sure it would be regarded as a classic of its type. Its designed performance which, he understood, was being met in flight-tests was as follows:—

All-up weight = 113,250 lb.
Cruising speed = 400 m.p.h. at 40,000 feet.
Range = 4,000 miles, with 10,000 lb. of payload.

He quoted the case of the Rainbow to emphasise two points. One, that the unit power of the piston engine, together with the proper utilisation of its exhaust gases, was still suitable for fairly large multi-engined high-performance aircraft. The second point was equally, if not more, important than the first, because it largely controlled the shape of aircraft to come. This concerned the framing of the requirement, which must be the designer's guide before he could put a line on paper. A poor requirement made for a poor or mediocre aircraft. If in this country they were going to equal the Americans in the quality and performance of civil aircraft in five or seven years from now, they must learn how to frame a requirement which was going to make the aircraft designer stretch himself to meet it.

He would prefer their aircraft manufacturers and designers to think ahead and formulate their own advanced requirements; to lead the Government instead of awaiting Government enquiries before doing anything. The latter procedure was too often the case to-day.

The original idea that the gas turbine should be tailormade for the aircraft must be modified, because although a particular engine might have been built in the first place to suit a certain aircraft requirement, the development of engines was too expensive and protracted to permit of an entirely new engine for each and every new aircraft type. This particularly concerned the propeller turbine.

What would happen in practice was that the arithmetic would be reversed. Instead of stating the power requirement for a certain design of aircraft the available engines and their power would be known beforehand and it would then be up to the aircraft man to see how best he could design the machine around them.

This was inevitable because, in a sense, the airframe was basically a static structure, whereas the engine was a dynamic one which demanded considerable cut and try before it was suitable for its job; this took time. Engine design and development was still very much of an art, although the gas turbine did need and lend itself to an appreciable amount of calculated design in the early stages.

The gas turbine would establish itself in its good time and all the ballyhoo and publicising, which had at times been almost nauseating, would not bring it forward any quicker. It was now up to the engine manufacturer to put all his experience at the disposal of the turbine and do some solid development.

In his view this country would make a success both of the jet and propeller turbine for civil transport aircraft, particularly since they had not an established piston engine for this work and a good deal had to be

done to develop and modify military type engines to meet their airline needs. They were starting with a clean sheet and a new prime mover, whereas the Americans, who were behind in the development of the gas turbine, had one or two well-developed piston engines which were most suitable for airline operation. They also had the exhaust turbo-supercharger and could combine it in the best manner with the piston engine. He expected the piston engine to have a longer life in the United States in civil aircraft and then they would see a more complete jump there to the jet turbine, leaving the propeller turbine, metaphorically, in mid-air and little used.

All this must of course be conjecture, but the rushed days of war were over and safety came first in airline operation, so—let them not fall up the steps, but walk them.

W. S. Farren, C.B.E. (Blackburn Aircraft Ltd., Fellow), *contributed*: The wide range of subjects covered by the four papers should serve to convince those who sometimes felt that the Society did not provide enough variety in its discussions, that its interests covered the whole field of aeronautical engineering. It was true that most of their discussions were concerned with the scientific and technological aspects of their profession—the reasons underlying developments, and the methods by which they were put into effect. But they certainly did not neglect the economic aspect—whether these developments would give the world the service it needed at a price which it could afford, and would pay.

Mr. Lipscomb dealt with the influence of manufacturing techniques on what Mr. Russell called the strategy of design. The remarks of some speakers suggested that it was not clearly recognised that this influence depended very much on the circumstances of the times and on those of the problem under consideration. They were still very much aware of the wartime situation, when the manufacture of large numbers of units and the shortage of materials and of skilled labour dominated their methods. For the future, it seemed unlikely that these factors would govern the design of civil aircraft of substantial size. Small civil aircraft, for the large numbers of private owners whom they hoped would use them for personal transport, must be designed so that they could be sold at a low price. For military aircraft they must plan design so that the lessons of the recent past had their due influence.

But for all kinds of aircraft—and indeed he believed for every product of human skill—the true economic criterion was *the cost of the service rendered in terms of the currency available at the time when the service was required*. This depended on many factors besides manufacturing costs, and to assess it was an extremely important part of an engineer's job.

Mr. Russell had mentioned examples where, in his view, the circumstances warranted what might seem extravagance. By reducing tolerances on sheet and extrusions he expected a handsome saving in a structure whose weight was about 100,000 lb. It would be interesting to know what percentage he expected to save—each 1 per cent. being equivalent to a 4 per cent. gain in payload. His gust-relieving device, which would clearly be very costly in the associated research and development charges, might even save him more than 10 per cent. of his structure weight. This was a surprising figure. He was not clear from his remarks whether he was actually planning on this basis.

He was glad that Mr. Owner did not disguise the present high cost of turbines. The cost of blades would no doubt fall as methods improved and design became stabilised. But although the number of blades in a turbine was large, the number of different types seemed likely to be considerable also, and it would need great skill to reduce the total cost. The basic materials for rotor forgings seemed likely to remain costly. But he did not think they should be deterred—clearly Mr. Owner was not—by a situation

which had often faced aeronautical engineers, and had been successfully overcome.

He wished Mr. Owner could have been more explicit about fuel consumption. Here they had a factor of overwhelming importance. Taking Mr. Russell's figures, a decrease of one per cent. in fuel was equivalent again to an increase of four per cent. in payload—and *vice versa*. This was the purely technical side of the matter. Probably as important was the character of the fuel which turbines used. They had been led to believe that the price of fuel was what it would fetch—the implication being that there was a possibility that a relatively crude fuel might not in practice cost less than a high grade fuel. If this was likely to be true it was important to know it in good time. But he did not believe that such considerations should be allowed to deprive aviation of what was obviously a better fuel—one which was essentially cheaper than high grade petrol, could be safer, and would be easier to distribute and to store. Were they justified in assuming that the goods would be available when needed, and at a fair price? If so, they could make their calculations of the cost of providing air transport, in the future, with some confidence. If not, a large element in their calculations was open to very serious error.

He would like to see more discussion of the economic aspects of future aircraft and hoped that it would be recognised that engineers could hold their own in a sphere where excessive concentration on purely scientific and technological matters had sometimes left questions which they alone were competent to decide, to those whose interests were wider but had not the experience to make sound assessments of technical questions.

Mr. F. Radcliffe (Handley Page Ltd., Fellow), *contributed*:—Mr. Owner's Paper: The aircraft designer had accepted the gas turbine in its different forms as the power plant of the near future for higher performanced aircraft. In many ways, as several speakers had already remarked, the gas turbine had been rather over-advertised and the fact that it had not become commonplace, while well understood in the Industry, was not appreciated to the same extent by "the man in the street." Until a background of experience had been built up by flying gas turbines on test bed aircraft, the airline operator would be unable to employ gas turbined units on public services. His reputation for efficiency and reliability demanded this since he flew for "hire and reward." It was desired to emphasise the need for putting gas turbine "proving" in the highest priority so that assurance could be given that satisfactory measures had been provided for dealing with icing conditions, fire risks and with prolonged flights. Until this had been done, it would clearly be unfair to expect the gas turbine to come into civil aviation in a big way.

England led at the present time with successful military types fitted with gas turbines, but unless they acted quickly they would lose that valuable lead.

With high-speed aircraft flying at heights of 25,000 ft. and over, the provision of pressurised cabins became a necessity. Steady development was required to make available the specialised components and units which were now known to be essential. More coordination of the pioneer efforts of a considerable group of specialists was required so that skill and ingenuity were used in the most efficient manner.

Air conditioning, it was claimed, must from now on be a regular feature for the comfort and well-being of passengers in all types of civil aircraft, whether pressurised or not. Where the weight involved permitted it would have to embrace the quality of the air, its temperature, humidity and on some routes, its refrigeration. The amount of work involved, as mentioned by Mr. Owner, was considerable and its successful development would cost a great deal in both time and money. On its success would depend much

of the attraction of air travel to the fare-paying passenger.

Mr. Lipscomb's Paper: He had stressed rightly the importance of the production engineer as a member of the design team, but he must be a member and not the captain. The lead for improvement in the engineering structure, in the aerodynamic conception and in the internal layout, should come from the designer. He pointed the way and there should be a sympathetic reaction on the part of the production side to keep in the van of progress so that the designers could keep to the highest standard of efficiency whether measured by payload or performance in its widest interpretation. Aircraft as they knew them to-day were by no means "frozen" as regards design. There was still great scope for development and originality, and pre-eminence in aircraft design would follow vigorous development work following logical steps in both conception and construction.

The value of weight saved in the primary structure had been emphasised by several speakers. What had not been pointed out was that the same vigilance was required in the furnishings of an aircraft. At a slightly higher cost equipment could be materially reduced in weight and the saving placed on the credit side as additional payload—available as a revenue return, appreciably more profitable than the capital cost incurred. Cabin furnishings, seats, galley equipment, electrics and radio were some of the items meriting this closer scrutiny for weight saving.

The growth of passenger traffic had, as some speakers had pointed out, made the case for the bigger aeroplane very real, due entirely to the take-off and landing saturation of terminal aerodromes. This gave weight to Mr. Lipscomb's plea for more attention to be paid to the big flying boat.

Recent analyses by the R.A.E. comparing the large landplane with its flying boat counterpart had revealed that a good case could be made for the flying boat as it should be possible to achieve a 3 per cent. saving in the structure weight of a flying boat of approximately 100 tons A.U.W. over its equivalent landplane. It was further noted that the additional drag of the flying boat would not reduce this gain to a vanishing return. The elimination of the undercarriage problem in big aircraft would be a very real gain and was yet another reason for further thought being paid to the flying boat.

Mr. Russell's Paper: He had made a valuable contribution when he mentioned the lessons to be learned from a study of available materials inspected on a basis of "quality control." For small-scale production it had always yielded good dividends and was used extensively some years ago when aircraft were being designed and constructed for the Schneider Trophy contests. Its continuance to-day seemed highly desirable when the control of the empty weight of an aeroplane was so important—especially for long range types with which the lecturer was so closely concerned. It would mean, undoubtedly, a higher cost for raw materials and it would depend on the ingenuity of the metal producers to keep their costs below that maximum which the aircraft constructor could afford.

It was some time now since mention had been made of the cube square law and it was gratifying to find that the break-down of component weights for an aircraft of 300,000 lb. A.U.W. revealed no appreciable increase in the structure weight. That this was so reflected undoubtedly the skill of, and the care taken by, the Bristol design staff in the evolution of their primary structure.

A full-day meeting such as this, had unquestionably been most refreshing. Some remarks made by speakers were not easily expanded into arguments, for or against, but the personal opinions expressed gave the listener a richer background from which he might draw his ideas or check tentative theories.

J. C. White (Associate), *contributed*: Mr. Owner pointed out that the fuel con-

sumption of a propeller turbine might be improved by the use of a heat exchanger. It was admitted that considerable weight and bulk were added, but they were not told what order of heat transfer efficiency must be attained to compensate for these disadvantages. Since the Theseus was the only aero gas turbine at present equipped with a heat exchanger, could the author give some indication of the efficiency and pressure drops obtained in this component?

The probable use of propellor turbines in all classes of civilian passenger transport was foreseen. For the short-haul class, a simple turbine without heat recuperation was proposed. Since the heat recuperator only appeared to pay for its weight after about 1½ hours in the air, possibly the slight fuel saving on medium-range aircraft might be offset by the additional inspection and periodic maintenance involved. This left the long-range class where the recuperator showed up to advantage on units operating at present compression ratios and turbine inlet temperatures: increase either of the latter and its usefulness even in this class would diminish. This being the case, was the recuperator anything more than an intermediary step?

J. Hadji-Argyris and **P. C. Dunne** (The Royal Aeronautical Society), *contributed*: After studying Mr. Lee's interesting paper, they would like to draw attention to a point affecting the deformation of swept-back wings under load. In his discussion of the change of incidence due to bending and twisting of a swept-back wing, Mr. Lee implicitly assumed that the ribs were normal to the spar locus (Fig. 10) and maintained the wing section in the rib planes. He then proceeded to find the change of incidence of the section in a plane parallel to the main airstream. The method of finding the change of incidence by dividing the difference of the deflections of the points A and B by the distance AB implied in addition that the deformation of the section AB in its own plane was negligible. Was this really the case? They could imagine that the change of wing camber due to such structural distortion might be of importance. If, however, they assumed the ribs to be parallel to the direction of flight so that there was practically no distortion of the aerofoil section, Mr. Lee's analysis was obviously inapplicable, and in fact it was no longer possible to treat the wing in what Mr. Lee called "a perfectly ordinary manner."

Mr. Lee and contributors to the discussion referred to the flexural axis of a swept-back wing and drew certain conclusions regarding the twist of a wing under load. They would point out that except in the simplest structures, such as a two-spar wing with non-load-carrying covers, the flexural axis could not be assessed intuitively or by the methods of the ordinary engineer's theory of bending and torsion. The latter statement applied with particular force in the case of wings with large sweepback.

F. W. David (Chief Designer, Commonwealth Aircraft Corporation, Melbourne, Australia, Associate Fellow), *contributed*: In Mr. Lee's paper they had received an excellent summary of what would happen if they tried to introduce the swept-back wing designed on conventional principles.

What were Mr. Lee's views on the swept-forward wing, which would appear to behave better with regard to tip stall? The inward flow tendency should reach favourably to tip flow and wing-fuselage junction. He appreciated of course C.G. difficulties with regard to weight distribution. But he could not imagine that they could not be overcome on certain types of aircraft.

A. R. Weyl (Associate Fellow), *contributed*: Mr. Lee's painstaking account of more recent research which was applicable to some of the most troublesome problems in the development of tailless aeroplanes, would certainly be of great help to the designer, and the various suggestions he had made would deserve the fullest attention.

The following was intended as an ampli-

fication to certain points the lecturer had made.

(1) The lecturer had referred to four factors as influencing the high-incidence stall peculiar to swept-back wing systems. There was, however, reason to presume that a number of other factors might also contribute one way or another:—

(a) Due to the increase of the aerodynamically effective wing chord towards the tips (as a result of outwards flow), the Reynolds Number was increased; this might affect as well the maximum section lifts as the chordwise lift distribution at high incidences.

(b) At the wing tip, an inflow existed over the upper wing surface, due to an exchange of pressure around the wing tips (origin of the wing-tip vortices of the induced drag). This inflow which was reinforced by the sweep-induced outflow along the lower wing surface, and which should have its greatest intensity where the maximum pressure gradient occurred (*i.e.*, at the region of maximum negative pressure of the tip section), would seem to exert the following effects on the tip stall:—

(i) The outwards flow due to sweep was neutralised, and boundary-layer material was made to accumulate over the wing tip, in the region of the trailing edge.

The well-known phenomenon of a very thick and stagnant boundary layer at the tips was probably one of the predominant causes of the premature tip stall.

(ii) By inducing vorticity at the tip region near to the leading edge, the character of the stall was locally changed.

(c) As the lecturer had stated, the outflow along the upper surface was mainly restricted to the boundary layer and did not extend to the flow above it. This being so —detailed experimental evidence of it would be welcome—there would be reason to presume the existence of a vortex sheet between the two flow strata having different velocity vectors.

At the lower wing surface, tests of G. T. R. Hill (cf. J. Aeron. Soc. 1926, p. 535) established beyond doubt that the outwards flow was not restricted to the boundary layer.

(d) When wing-tip disk fins were fitted to the wing, premature stall might be induced by

(i) the accumulation of boundary-layer material between fin and wing tip;

(ii) the formation of an expanding passage between the wing trailing edge and the fin;

(iii) the mutual interaction between circulations about both components;

(iv) the sweep-induced inflow near to the leading edge of the wing.

With wing systems having low aspect ratios at which the pressure-equalising flow causing the induced drag was predominant, the effect of the inflow at the tip could be expected to be of considerable magnitude and able to prevent a premature tip stall, even at large angles of sweep.

This expectation was confirmed in all tests made on swept-back wings of varying aspect ratios. It was most probably the explanation of the experimentally established fact that low-aspect-ratio wings with large sweep-back were free from tip stall.

The action of the inflow on the outflow along the upper wing surface might also explain the variation of the slope of the pitching-moment curve with incidence which had been observed to occur at rather small lift coefficients, in N.A.C.A. tests (cf. H. A. Soule, N.A.C.A. Tech. Note No. 1088). Remarkably enough, this change in slope occurred at the large sweep-back angles and low aspect ratios, at a characteristic incidence beyond which the lift increment with incidence was suddenly improved. This led to the conclusion that up to this characteristic incidence, the wing tips had been dominated by the outwards flow and hence contributed less or nothing to lift; they even might have been stalled. Beyond the characteristic incidence, the tips became lifting or unstalled, and this was most probably due to the pressure-equalising inflow over

the tips (which should grow with something like the square of the lift coefficient).

Soule, who did not give an explanation, reported that the characteristic incidence of the lift-curve inflexion was accompanied by a "slight ruffling of silk tufts" near the leading edge, at about 40 per cent. of the semi-span from the wing centre line. A small barrier to span-wise flow in the boundary layer attached at this point, affected both the shape of the lift and pitching moment curves in respect to wing incidence. This would seem consistent with the hypothesis put forward.

The addition of a deflected half-span split flap to a wing system of substantial sweep-back and low aspect ratio seemed to express a somewhat more abrupt stall in the lift curve and a decreased inflexion of the lift curve.

The change in the stalling characteristics which was caused by the deflection of partial-span flaps was probably mainly due to the discontinuous distribution of lift along the span; this obviously tended to aggravate the effect of sweep, though the apparent wash-out effect was moderating it. He wondered if the provision of a "point sink," *i.e.*, of a suction orifice locally, would not be a great help to improving the adverse influence. The same consideration was valid for adjustable trimming tips of the Hill type; they might thus be made more effective to remedy premature tip stall. Such "spot-welding" of an unstable boundary layer might be more economical than sucking away of the whole mass of flow particles contained in a thick boundary layer.

(2) In connection with devices which might remedy the tip stall, Mr. Lee had not mentioned the diffuser wing tip. Yet there were reasons to believe that such wing tips might not only remedy tip stall in an economical way, but also would form solutions to some of the most troublesome problems which beset the progress of tailless aeroplanes. Research was, of course, required to investigate the properties of such tip shapes. The experience of Northrop had indicated that there were advantages in such tips, although they were structurally unwelcome and although the prevention of tip stall was apparently not intended with the shapes adopted. He firmly believed that further research in this direction would lead to agreeably surprising results, and that final solutions might be found in the adoption of adequate wing tips in combination with boundary layer control at appropriate regions.

In respect to the suggested suction section of the Lighthill type (shown in Fig. 7a), tests by Schwier at Goettingen had indicated that suction at the leading edge of low-drag aerofoils was rather ineffective.

The main effect of front suction could be the suppression or removal of transient turbulence at the front stagnation point of a rounded section nose. At the pointed nose of the section shown, it was difficult to visualise how a beneficial effect on drag or high-lift could be obtained. For the flattening of the suction peak, the lecturer rightly pointed to the effect of the suction aft. When — in an uneconomical way — it would be intended to accelerate the circulation, it would seem that the suction slot should be located below the pointed leading edge. The slot shown at the nose would seem perfect for the discharge of air (cf. Aircr. Eng., 1945, p. 325), but then the advantage of laminar flow would not be retained.

When the lecturer could give more details on this point, it would form a valuable widening of the prospects he had opened for the application of suction aerofoils to tailless aeroplanes.

(3) According to a statement of the lecturer, tests had shown that tip stall persisted at high Mach Numbers. It would be of great interest to have more details of this point.

Reports relating to experience with the Lippisch Me. 163B seemed to agree in that, when the critical Mach Number was reached the tendency to dive appeared. This would appear inconsistent with tip stall at compressibility speed. Due to the distribution

of supervelocities, shock waves should form first at the centre line of a swept-back aerofoil, and the presence of a nacelle could but only emphasise this tendency. Aft of a shock wave, the boundary layer separated from the surface; hence its effect on the outwards flow was not quite evident.

(4) The Royal Aeronautical Society would earn the gratitude of all who took an interest in problems of tailless aeroplanes (industrially and non-industrially) by making accessible to all its members the British and German research reports which had been quoted by the lecturer; otherwise, the references given to unpublished, restricted and inaccessible reports would seem meaningless.

J. D. North (Boulton Paul Aircraft Ltd), *contributed*: In his discussion on Power Controls Mr. Russell proposed a limiting lag of 1 degree with unit function input velocity of 30 degrees per second, *i.e.*, a time constant of 1/30th second. This figure was probably low enough to bring in a great number of elements of the transmission system in addition to the error control and would seem to him to be unnecessarily low. A time constant of 1/10th second would, he suggested, be quite adequate. This time constant could be significantly related to the time constant of the aircraft which had the value $\frac{2Cc}{\omega n}$ where Cc was the damping ratio and ωn was the natural circular frequency. In the case of the longitudinal control, which was the critical case, the time constant was computed or measured for the short period of oscillation. This time constant varied with speed, and at the low speed end of the scale Cc approached unity; at high velocity Cc was over unity and the motion was aperiodic. The natural frequency varied with scale in the ratio $\frac{1}{\sqrt{L}}$ and a plausible value of $\frac{2Cc}{\omega n}$ for an aircraft of the size of the Brabazon I would probably be about 2. The effect of a .1 time constant in the servo would then be to increase the time constant of control of the aircraft from 2 to 2.1. The influence on phase lag and attenuation of the controlled motion was easily seen to be insignificant.

The desiderata numbered 3, 4 6 and 7 were, he thought, open to argument on psycho-physiological grounds assuming that pilots were initially trained on power controls. These desiderata were aimed to produce to the pilot effects similar to those to which he was inevitably accustomed with direct controls, but this did not imply that they were necessarily the best to enable the pilot to guide the aircraft on a desired course, the word "course" being used in its most general sense.

MR. OWNER'S REPLY

Mr. Scott Hall: He believed that the propeller turbine was perhaps to be regarded as a bridge in the development towards the pure jet. Some bridges were fairly long, and it was perhaps as well to cross them in order to get to the other side. For some time to come the propeller turbine might have a wider application than any other form of turbine.

As regards the advantage or otherwise of the pure jet unit in respect of noise, it would depend largely on the installation and on the measures taken to deal with propeller noise, which in the propeller turbine was the only important source of this nuisance.

He agreed that work on anti-icing was one of the most important things they had to do in connection with turbine development generally.

As to the possible failure of turbines, provided that care was taken in the design and proof testing of the high-speed rotating parts, failures were not catastrophic, in his experience. In the early stages of development there had been failures of various parts: it was no uncommon thing to have quite a major failure—a blade or even a bearing—which was practically undetectable by the man who was running the engine. That had happened, not once, but a number of times.

But, provided care was taken to see that there was no danger of a high-speed part bursting, he considered that the danger of the complete break-up of a turbine engine could be reduced to vanishing point.

He supported Mr. Scott Hall's plea for safety; it was of transcendental importance; and he had endorsed it in the paper.

His forecast with regard to pressurisation was gloomy, but it was designed to provoke the very reaction it had provoked, namely, that there was a great deal of work to be done and that high-speed high-altitude aircraft, which they required, would not arrive until that work had been done. In his opinion it had barely started yet, and the sooner they recognised that and got on with the work, the better.

Mr. Masefield: The compound unit was an interesting project and was the subject of considerable examination by Dr. Ricardo at the all-day conference in March 1946. But he felt that the rate of development of the propeller turbine was such that in economy it would so nearly overtake the compound engine in the fairly near future as to render the latter applicable only for extremely long ranges; so long that, with the exception of Naval reconnaissance aircraft where range rather than speed was important, he could not see that there would be any but restricted applications for it. The size of the world was limited and they could not need a range of more than 12,000 miles.

He emphasised how seriously they must regard Mr. Masefield's warning of how much depended on the rate of development of turbines, both propeller and jet, in this country, and the factors on which the rate of development depended; they would only come as quickly as they wanted them if they grappled at once with the problems involved.

The President: He echoed his remarks that they wanted work on the new units, rather than advertisements of their merits when they were developed.

Mr. Pollicutt: Reasons were given in the paper as to why abstraction of air from the main engine blowers was undesirable, and he believed those reasons would continue to be valid. They did not want a variation of engine operating conditions to upset pressurisation, and there was danger of contamination either by anti-icing fluid or oil; he did not see therefore any likelihood of being able to tap pressurisation air from the main engine blowers. However, it should be possible to get a very convenient source of heat in the turbine engine, whether pure jet or propeller turbine.

Mr. Mitchell: Auxiliary power plants would be very desirable in connection with A.C. supply. Constant A.C. supply could be nicely arranged by means of an auxiliary power plant; but there was the difficulty that the auxiliary unit itself had to be supercharged. If used in large aircraft to supply current for power-operated flying controls, it would not be possible to pressurise the auxiliary power unit from a source which might suffer reduction of pressure and, therefore, cause loss of flying control. Thus, the need for supercharging might complicate the problem of auxiliary power plants.

Dr. Roxbee Cox: The future of the jet air liner would be restricted until they had more experience of the reduction of drag on the aircraft and reduction of fuel consumption. Until they had achieved more in that direction, the long-range jet unit was likely to be expensive.

Mr. Richards: On the question of asymmetric loads, he did not think that it would be serious if one turbine cut; he did not think the magnitude of the drag would be such as to affect the aircraft seriously, if it were a large aircraft.

Reply to WRITTEN CONTRIBUTIONS.

He welcomed the support given to his general conclusions by Messrs. Banks, Farren and Radcliffe, and felt that, in general, little further comment on their remarks was necessary or even possible.

Their emphasis on the economics of air-

craft operation was, in his opinion, timely, and he heartily concurred with Mr. Farren's request for a clarification of the supply position of the various types of aviation fuel now under consideration.

Air Commodore Bank's reiteration of the paramount need for speed and reasonable passenger comfort was also welcome; recent data on the effect of "bumps" on high-speed aircraft were "more than somewhat" disturbing. The matter was considered more exhaustively in Mr. Russell's paper, but it might be pertinent to remark that the gust relieving devices contemplated by Mr. Russell did little to alleviate the effect on the passenger, however much they might reduce the wing stresses, and it was not easy to foresee what remedial action would be possible in the case of some of the very high-speed machines now being contemplated.

Mr. White: The design problems of heat exchangers offered sufficient material for several additional papers which, it was hoped, would be forthcoming in due course. In the meantime, this component was under such an active stage of development that specific figures might prove misleading. Probably the best indication of the present position as regards efficiency was that the heat exchanger had already proved itself capable of saving an amount of fuel equivalent to its own weight in about five hours' flying, but that when allowance was made for the loss of power occasioned by pressure drop, this figure should be increased to seven hours. A material improvement in these figures, as a result of development work, appeared not only possible but probable.

Obviously the heat recuperator was only advantageous on fairly long flights, and it was agreed that it might eventually be supplanted by units of higher compression ratio when the considerable technical problems attendant upon increase of compression ratio had been satisfactorily solved.

MR. LEE'S REPLY

Mr. Pollicutt: As he had tried to indicate in the paper, the Prandtl was a development of what was usually named the linear perturbation theory for compressible flow. It was explained in a paper by Young in Reference 18. There was a further and rather clearer explanation of the theory as applied to swept-back wings by R. Dixon, in the R.A.E. Report, Aero 2146. One could apply the Prandtl rule to cases where the body was slender or where the aerofoil section was of small, but finite, thickness. He believed tests were to be made which would involve trying out a derived wing in incompressible flow and matching up the results with those obtained with the original wing in high-speed tests.

Dr. Roxbee Cox: He emphasised his comments on the importance of the structural details when considering aeroelastic effects on swept-back wings, a matter which had been brought home by Professor G. T. R. Hill at a meeting of the Society recently. The wing model he had drawn really corresponded to the single spar wing. If there were two spars or some other type of structure it was necessary to work very carefully and logically with the correct flexural axis, and so on; if one worked out the real flexural axis carefully, the right answer would be obtained, provided that the resulting torsional and flexural distortions were combined in the proper manner.

Mr. Quin: A considerable amount of work on suction applied to straight wings had been done in this country at the N.P.L. and the R.A.E. In Germany, at Goettingen, a good deal of work had been done, especially during the war, on suction applied to straight and swept wings. He regretted that space did not permit him to go into details on this very large subject. (See also reply to A. R. Weyl.)

Mr. Argyris and **Mr. Dunne:** He agreed that there might be some distortion of the wing section but he would not expect it to be of major importance; so far as he knew there had never been any attempt to calculate it.

Regarding the wing with its ribs parallel to the airflow instead of normal to the spar,

in this case change of slope of the spar (due to bending) would have a component in the plane of the rib (now no longer perpendicular to the spar) and this would produce a change of incidence substantially the same as that calculated by the method given in the paper.

Never having attempted to deal with any but simple single-spar swept-back wings, he did not feel competent to comment in detail on the last paragraph of the contribution. He could well believe, however, that the problem presented some considerable difficulties and was glad to note the suggestions made.

Mr. David: While it was true that the swept-forward wing tended to stall at the root instead of at the tip (as was the case for a swept-back wing) this tendency, from the point of view of pitching stability at the stall, was just as bad as the tip stall on a swept-back wing. With a swept-back wing the tip stall was undesirable from the point of view of both longitudinal and lateral stability, so, although difficult to avoid, was an unmixed disadvantage. With forward sweep, tip stall was beneficial from the point of view of longitudinal stability although detrimental as far as lateral stability was concerned; there was thus an apparently irreconcilable conflict of requirements. Despite this, swept-forward wings seemed to have flown reasonably successfully in the United States of America.

Other disadvantages of the swept-forward wing were:

(1) The centre of gravity of the aircraft would almost certainly lie forward of the centre-line leading edge so that a long body must be cantilevered forward from the wing.

(2) In itself the wing with forward sweep was unstable and this instability was greatly increased by the body shown to be necessary in (1) above. Hence a large amount of weathercock stability must be supplied by a vertical fin at the rear, and since there was no other structure located well aft of the C.G. it followed that either the fin must be very large or else the body must be extended rearwards solely to carry it; either solution clearly involved a considerable penalty in weight and drag.

(3) In order to maintain adequate lateral stability at high lift a considerable positive geometrical dihedral was necessary to offset the large negative dihedral effect of the swept-forward planform at high C_L's. Hence at high speed (low C_L) the aircraft would have excessive positive dihedral (since the negative dihedral from the wing was small) and so might suffer from a "Dutch Roll" at high speed, which could be far more dangerous than the propensity of the swept-back design to show this form of lateral instability at low speed.

In favour of the swept-forward wing it should be noted that the aeroelastic effects of wing bending were likely to be favourable.

Taking all the facts into account, however, there did not seem to him to be much of a case to be made in favour of the swept-forward wing.

Mr. R. Weyl: This was a valuable contribution and he would make the following comments:—

Regarding (1a), since only the boundary layer, and not the main flow, suffered a large outwards deflection it was difficult to say whether there should be an effective increase in Reynolds number or not.

The phenomenon of the tip-vortex inflow might well be of importance in wings of low aspect ratio, although tests in the wind tunnel of tufted swept-back wings of normal aspect ratio (about 6) all showed the outflow over the rear part of the wing persisted right up to the tip (or at least to within a few per cent. of the chord from the tip).

The evidence for his statement that the top surface outflow was confined to the boundary layer was furnished by some German tests and by some tests which he himself had witnessed which showed that whereas the surface flow on the rear part of the wing might be parallel to the trailing edge, at a few per cent. of the chord above the surface, the flow was

practically straight downstream so that, as Mr. Weyl suggested, there must exist a vortex sheet just above the wing.

He found the idea of an outflow on the lower surface of a swept-back wing at high C_L's rather surprising since on all but the thickest sections there would (if the wing was unflapped) be a favourable gradient here near the stall and this should result in an inflow (see Fig. 2B); flaps on the lower surface would naturally produce an adverse gradient ahead of themselves and this, of course, would produce an outflow.

On the subject of tip fins, he agreed with Mr. Weyl's argument in general with the exception of the item under (i). He did not see why there should be an accumulation of boundary layer between the wing and the fin, especially if the fin itself was swept-back (as would be necessary, in any case, for a high-speed machine) since in this case the fin would continue to prolong the outflow tendency (on the top surface) so that the boundary layer could be expected to continue its journey from the wing upper surface along the adjacent surface on the fin (and thereby possibly causing trouble with the fin stalling). Regarding the less subtle forms of wing-fin interference, he believed that if as much thought and experiment were put into the detailed design of the wing-fin junction as in the past had been put into wing-fuselage junctions, the major troubles could be avoided.

Not being familiar with the subject of the low aspect ratio swept wing, he could not discuss this matter in detail. Mr. Weyl's suggestion that the tip-vortex inflow was of major importance seemed to be very reasonable, although it was hard to see how it could explain the change in flow at 40 per cent. of the semi-span observed by Soule.

The idea of applying suction (or presumably, some other form of control) to the boundary layer at the critical point he believed to be quite sound and worth careful consideration; it was, in fact, the principle used by Lighthill in the design of the nose suction aerofoils. What Lighthill had really done was to design a profile such that at high lifts there was, theoretically, an infinitely steep adverse gradient behind the top surface suction peak and this was then followed by a very gentle gradient over the rest of the aerofoil. The sharp leading edge brought this suction peak right to the front of the section, so that, apart from the zero length infinitely steep adverse gradient, the whole upper surface had only a slight pressure rise in going from leading to trailing edge. Clearly there must be separation at the infinity but since it was at the leading edge the boundary layer thickness was vanishingly small so that reattachment could easily be attained by suction on the *upper* surface.

The attraction of the Lighthill section from the swept-back tip stall aspect was that it therefore effectively eliminated the usual steep adverse gradient at the leading edge and so, for the reasons given in 1.2.2, was expected to have a beneficial effect on tip stalling Ordinary profiles all had a steep adverse gradient near the L.E. at high C_L so that suction, at the leading edge or elsewhere, could not produce the elimination of the suction peak that was hoped for with the Lighthill section. The rear suction step shown in Fig. 7 was not intended to reduce the suction peak but merely to reduce still further the overall adverse gradient.

On the subject of tip stall at high Mach number, he regretted his inability to give further details, but at the present time information was very scanty and he could only repeat what he said in the paper, namely that the few tests that were available did show a tip stall in some cases.

It was important to differentiate clearly between the final stall (or break-down of lift) at high Mach numbers and the onset of shock waves. The lift of a wing could continue to increase with incidence after shock waves had appeared. Thus for example a 45° swept-back wing of moderate thickness could

probably attain a maximum lift coefficient of 0.6 or 0.7 at $M=0.85$, although at this M shock waves would probably develop at a C_L of about 0.3 or 0.4. Considering the Me.163 in a dive, here C_L would presumably have been very small (probably <0.1) and it was probably safe to say that the changes in pitching trim occurred when shock waves first developed and that in this case the aircraft was not flying at its maximum lift so that the question of high speed tip stall discussed above did not arise. He had heard it said that in fact the trouble on the Me.163 was caused by the presence of too much washout which resulted in the tips carrying a download in the dive and that the local negative C_L was high enough to cause a negative tip shock stall at the critical M with a consequent diving tendency.

He also regretted the difficulty in getting access to reports, but he put the references in because, while those who could not get them were at least no worse off, he believed that they might be of value to those who were more fortunately situated in this respect and this category did, in fact, embrace most of those actively engaged in the British aircraft industry and research establishments.

MR. LIPSCOMB'S REPLY

He was relieved to hear from Mr. Scott Hall and from Mr. Owner that a failure of a jet engine would not be so catastrophic as he had thought; he had been under the impression that, with nine "blow-lamps" inside a wing, a failure might have very serious consequences. He had been reassured by Mr. Owner, who had said there would be stainless steel all round the combustion chamber, which would probably provide the measure of protection which he would like to see.

On the controversy concerning the double bubble form, it was agreed in the past that flying boats were bigger than was really necessary for carrying the load required. Nowadays it was not quite so. The Shetland, which was about 22 ft. deep, could be divided into two decks, giving roughly 8 ft. headroom in each deck, which was sufficient height for passengers. But there was still not enough room on the Shetland to carry all the load required. They did not confine the accommodation just to seats, but tried to provide also some comfort in the way of a dining room and, if possible, a lounge or promenade. The double decks did provide the extra space that was needed, and that would become more and more the case in the future. He did not know what the payload would be in Mr. Russell's aeroplane, but it might not be more than 5 per cent. On the Empire route, of 2,500 miles, he would probably have considerable difficulty in getting in the payload that he required.

With regard to the maintenance of engines, operators would want to remove the cowlings, but he felt that if they had to remove the stressed skin covering they would object. On the face of it, he would have thought it easier to maintain the engines outside the wing than inside.

One speaker referred to the possible reduction in the weight of flying boat hulls. It must be borne in mind that the planing bottom had to be stressed to withstand a factored water pressure of 25 lb./sq. in. in addition to which the hull sheeting must be robust enough to put up with normal usage. The more stringent operating conditions necessitated slightly heavier sheeting than would be called for purely from strength considerations alone. Even so, the bare weight of the Shetland hull was only just over 10 per cent. of the all-up weight of the aeroplane. In the past he agreed that hulls had of necessity been larger than would be strictly required based on the load carried but, as he had mentioned before, they had now caught up with this position and would like even more accommodation.

Naturally they were all anxious to reduce structural weight but he did think that the margin between a corresponding fuselage and a hull was rapidly being narrowed.

In connection with the auxiliary power plant they fitted in the Shetland duplicate units each capable of supplying an A.C. supply of 20 k.w. at a power factor of .83 in addition to a D.C. supply of 120 amps. at 29 volts. The corresponding weight including the electrical equipment was approximately 4,500 lb. which was a fairly heavy price to pay for independent units.

He felt that the best solution was to fit reduced power A.C. generators to all engines for the main supply and a small auxiliary power unit for use on the water.

Regarding push button control this would undoubtedly come in time but at the moment most pilots were loath to give up "feel" particularly on the elevators. His Company had made a start with the engine controls, having sponsored an all-electric control which was meeting with some success.

MR. RUSSELL'S REPLY

He would not enter into the argument concerning the definition of a large aeroplane. Mr. Masefield had put the case with regard to size of aeroplane in relation to frequency of operation to and from airports when he had discussed conditions in the United States; and the same conditions would apply in England some day, when aeroplanes were made safe. If they were going to remain dangerous, then small aeroplanes would do. But he was not sure that small and fast aeroplanes would do because, with the President, he did not think that fare-paying passengers would be prepared to fly at 500 m.p.h., or perhaps even 400 m.p.h. The experience of flying at such speeds was that, on a moderately bumpy day, there was ample evidence of examples of 6g occurring frequently; and passengers would not fly under those conditions.

It was impossible to say much about power controls in the time available to him. Power controls must be made reliable; but so must everything else in an aeroplane. In his own Company's machines they relied on electrical power as the main source for operation. They had six different alternators, any one of which would serve the power controls; they in their turn drove either electric or hydraulic power controls. If they were hydraulic, there were two completely independent units, each having two electric pumps. So that for each control there were six alternators which had to fail, and four pumps. He believed the power required was about 35 h.p. per aileron; and power control could not be complete unless advantage were taken of irreversible control surfaces.

The maximum power required in his Company's installations was reduced by the use of small flywheels attached to the electric motors; the motors just maintained their revs., and when an excessive demand was made the energy came from the flywheel, and that reduced the demand on the motor system by 30-40 per cent.

Power was essential for gust-relieving devices, and those devices would be necessary on civil aircraft, whether they were large or fast. But power control was not the only line of defence. Gusts of sufficient magnitude to cause damage or harm were experienced only in the neighbourhood of cumulo-nimbus clouds, and those clouds could be detected by radar. There were cloud-detecting radar sets in the aeroplanes which indicated where the regions of high vertical velocities could be expected, and, where those regions were detected, the pilots had to slow down or make a detour.

The question of the double bubble form was entirely a question of range. They were trying to fly over 5,000 miles, and they could carry something of the order of 70 passengers direct to New York. If it were thought desirable to make the aeroplane suitable for a shorter route as well and to have sufficient capacity for, say, 200 passengers by means of the double bubble form, they would have the additional weight of the extra floor, the sound-proofing, the additional blower equipment and the increased wing weight arising from the extra body weight, because the

wings were designed in the tanks-empty case, and in the tanks-empty case every lb. in the body meant more weight on the wing structure. They could just fly from London to New York in a double bubble aeroplane with no passengers at all; it began to be a paying proposition at a range of about 4,000 miles.

With regard to the point that the undercarriage must be as efficient and as light as possible, in his Company's aeroplane, which was the only one he could quote, the weight of the tricycle undercarriage complete was just under 5 per cent., estimated accurately from drawings; that was not in accordance with the forecast made some years ago, that as the aeroplane became larger the undercarriage became heavier. Apparently the undercarriage weight as a percentage of the aeroplane weight was not increasing at all.

Finally, on the question of push-button control, the automatic pilot was a button, and that flew the aeroplane for most of the time in the air. He could see no reason at all why the aeroplane should not be controlled normally through the automatic pilot, which eliminated the need for "feel" and relieved the pilot of worry. If they took all the instruments and controls away from the pilot he had practically nothing left to do.

Mr. Farren: The following were estimates of the savings brought about in this excess structure weight by the use of special tolerances on light alloy sheets and extrusions, for an aircraft in the 300,000 lb. all-up weight category. The structure weight of such an aircraft, the Brabazon I, was made up as follows:—

Total Structure	93,000 lb.	*i.e.*	100.0%
Light Alloy Sheet	40,000 lb.	*i.e.*	42.6%
Light Alloy Extrusions	28,000 lb.	*i.e.*	30.2%
Steel	7,000 lb.	*i.e.*	7.6%
Miscellaneous (excluding furnishings)	18,000 lb.	*i.e.*	19.6%

A statistical analysis of normal production sizes of sheet material in Stores showed that there was an average of about seven per cent. excess weight over nominal weight, which figure was in close agreement with the figures for the extra-wide sheets quoted in the paper, where no special control was exercised in manufacture. An analysis of results for sheets to special tolerances showed a corresponding value of about one per cent. excess weight. Thus a saving in excess weight of approximately six per cent. of the weight of light alloy sheets would be expected by using the special tolerance material.

A similar analysis for light alloy extrusions manufactured to special limits showed approximately two per cent. excess weight over nominal. Given the normal tolerances, it was suggested that approximately five per cent. excess weight would be obtained, so that an estimated three per cent. of the nominal weight of light alloy extrusions would have been saved in the excess weight by using the special tolerance material.

Applying these figures to the structure weight of the Brabazon I, it would be seen that a saving in excess weight of 2,400 lb. for light alloy sheets, and 900 lb. for light alloy extrusions, could be expected, *i.e.*, a total saving of 3,300 lb. in excess weight. This represented 3.55 per cent. of the total structure weight. In relating this figure to the saving in the payload, the ratio being as stated above, the expected payload, when employing these special tolerances, would be 14 per cent. (*i.e.* 3,300 lb.) of the nominal value of the payload in excess of the value expected if no special control of tolerances were applied.

Mr. J. D. North: He agreed that the 1° lag between the pilot's control and the flap was on the low side but 3° (=0.1 sec.) was probably too much. In the case of landing

at 130 m.p.h. and considering a rapid drop of wind velocity, the application of 18° of elevator at 30°/sec. restored the 4° of incidence with a loss of height of about 10 ft. The rotation in pitch was respectively 2.0°, 1.85°, 1.50° in one second with no lag, 1° lag and 3° lag. Thus 3° lag gave an appreciable loss of response.

The fact that the desiderata numbered 3, 4, 6 and 7 gave effects similar to those of ideal direct controls did not make them wrong; they were actually based on other considerations; thus:—

(3) The artificial feel as distinct from feed back was essential with an irreversible system; the self-centring enabled the previously selected trimmed setting to be regained without further trial.

(4) and (7). It was easier to apply a *precise* control deflection when working against a smoothly progressive spring load free from appreciable friction. The pilot's judgment of what he was doing to the aircraft must be helped if he felt that he was applying considerable loads.

(6) Reports of experience with servo-assisted controls indicated that feel loads due to rate of operation were disturbing to the pilot.

Paper received November, 1946.

MODEL AIRCRAFT ILLUSTRATING SIGN CONVENTION USED IN APPLIED AERODYNAMICS

by

F. W. FREE, A.F.R.Ae.S.

Reason for the Development of Model

Much difficulty has been experienced in the past by those who have to apply aerodynamic data to aircraft design problems, in visualising the standard sign convention for controls, movements along and moments about the X, Y and Z axes. This difficulty has been specially pronounced with the staffs of wind tunnels where often, for mechanical reasons, the tunnel model has to be tested in an inverted position.

In order to save unnecessary reference to standards files when the sign (+ ve or − ve) of any direction, control movement or moment is required, the model described here was developed. The basis of the model is to have a colour (say red) to represent all positive moments and directions when that colour leads the direction of movement, and another colour (say blue) to represent all negative moments and directions.

The Model

The model is constructed preferably of sheet metal to enable the components to be conveniently assembled by soldering, and is made to represent the plans and side elevations of an aeroplane. An exploded view of the model is shown in Fig. 1. The model can be of any size convenient to the users, but it is not advisable to have the wing span less than 10 inches, as the necessary wordings which have to be put on become rather small. For normal office use a wing span of 1 foot is recommended, it being found just about the right size for clarity, and not too cumbersome.

The controls, flaps and control tabs are marked on as well as sections to represent the planes in the X, Y and Z axes of the model. These sections are appropriately labelled and it is also convenient to mark on the wing tips port and starboard. The various sections and controls are coloured in any two contrasting colours, say, red (to represent positive) and blue (to represent negative) the colour corresponding to the sign of the section, *e.g.*, Elevators—top surface red, bottom surface blue. A useful addition is to have a section for side force, it being located on each side of the C.G. band on the model. This C.G. band should be marked with a distinctive colour from the other two colours used, say white.

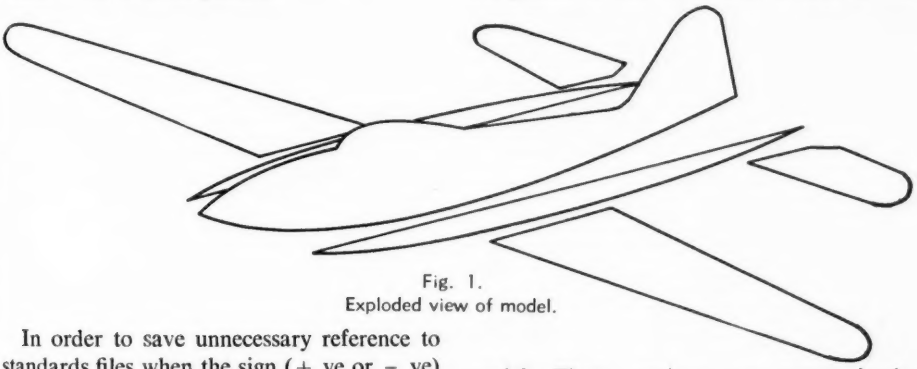


Fig. 1.
Exploded view of model.

The sections and colouring of sections are given in Figs. 2 to 5. The layout shown is for a conventional aeroplane, but the same

system can be applied to a tail-first aeroplane with the necessary reversal of the colours on the elevators.

INSTRUCTIONS ON USE

All controls, tabs and flaps are taken as rotating about their hinge lines, and when the red surface leads rotation the control angle is positive; when blue leads rotation control angle is negative. In the standard sign convention a positive moment tends to increase a positive angle, therefore, the sign of the moment is given automatically.

Directions of pitch roll and yaw are given by rotating the model about the C.G. and the positive direction is given when the red section of whichever plane is being considered leads rotation. If the blue section leads rotation, direction is negative. The sign of the moment is also given, as by standard convention a positive moment tends to increase a positive direction.

As an addition the signs of wing and tail incidence and dihedral angle, as well as forces, may be similarly included, *i.e.*, by taking the wing or tail as rotating about their trailing edges a positive incidence is given when red leads rotation; similarly, by taking the wing as rotating about its root a positive dihedral angle is given when red leads rotation.

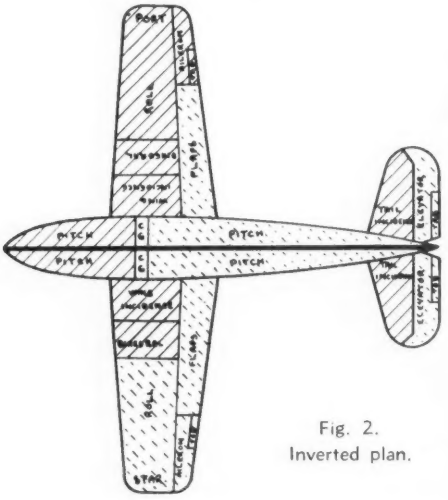


Fig. 2.
Inverted plan.

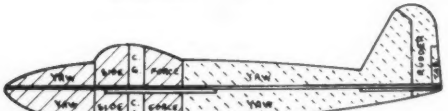


Fig. 3.
Side elevation—Port.

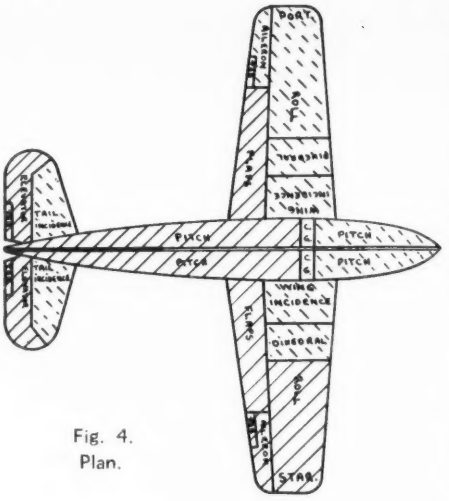
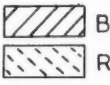


Fig. 4.
Plan.



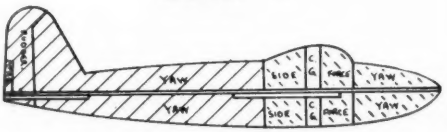


Fig. 5.
Side elevation—Starboard.

USES

This model is particularly useful to wind tunnel staffs, both to the technical staff when discussing results and to the rigging staff when carrying out rigging instructions. It would also be useful in colleges and schools where aerodynamics is taught and should prove a far easier way of teaching the standard sign convention used in aerodynamics.

THE ROYAL AERONAUTICAL SOCIETY

Paper submitted October 1946.

THE GENERAL THEORY OF CYLINDRICAL AND CONICAL TUBES UNDER TORSION AND BENDING LOADS

Single and Many Cell Tubes of Arbitrary Cross-Section with Rigid Diaphragms

by

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Mr. Hadji-Argyris took his M.Sc. degree, with first class honours, in Civil Engineering at Munich University Technical College in 1936, *specialising in the Theory of Structures and Applied Mathematics. He then became Scientific Assistant at this University and later worked in industry on design, theoretical and experimental research on steel and light alloy structures. In* 1941 *and* 1942 *he took a post-graduate course in Aeronautics and Mathematics at Zurich University Technical College. He has been with the Royal Aeronautical Society since* 1943 *and has published a series of papers on the Theory of Structures.*

Mr. Dunne is a graduate of London University (*Honours Mathematics* 1941). *He worked at the Building Research Station from* 1937 *until* 1940 *and from then until* 1945 *was with Boulton Paul Aircraft Ltd. During this time he was mainly engaged on investigations into theoretical stress analysis. He has been with the Royal Aeronautical Society since October* 1945.

SUMMARY

This paper presents a rational method of stressing single or many cell tubes particularly of the type encountered in wing structures. The theory has been developed for conical or cylindrical tubes of arbitrary cross-section the shape of which is maintained by a closely spaced system of diaphragms rigid in their own planes and parallel to the root section. Within the limits of the assumptions the theory is exact for right cylindrical tubes, and is applicable with adequate accuracy to cylindrical or conical tubes in which the inclination of any generator to the normal to the root plane does not exceed 10°.

The analysis given unifies the theories of bending and torsion and shows that the commonly used method of separating the bending and torsion loads by means of a shear centre is in general incorrect.

The formulae have been developed in such forms that attention is concentrated on the necessary corrections to the stresses as indicated by the ordinary engineers' theory. These correction terms include all effects of shear lag, diffusion, and end effects hitherto taken into account only in some very special cases.

Particularly important for practical applications is the structure consisting of a number of direct stress carrying members (booms) connected by walls effective only in shear. The simplest structure of the latter type is the four-boom tube with or without nose and trailing

cells, and in this case explicit formulae are given which are immediately applicable to practical calculations. Formulae are also given for the more complex case of a six-boom tube in which the two extra booms are introduced to represent more accurately the direct stress carrying capacity of the top and bottom panels between the two spars.

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5. General analysis for conical or cylindrical tube under arbitrary shear distribution.
(Detailed contents will appear with publication of this Section).

6. Numerical examples of wing stressing.
(Parts 5 and 6 will appear in later issues of the Journal).

1. INTRODUCTION.

1.1. Necessity for investigation and brief review of past work.

The necessity for a more comprehensive theory of stressed skin wings than is at present available has long been apparent. It is well known that the ordinary engineers' theory of bending and torsion is generally erroneous when applied to stressed skin wing structures. The inadequacy of the latter theory arises mainly from its neglect of the effects which have been variously described as axial constraint, diffusion, or shear lag. Previous work on the subject has not emphasised the essential unity of all these effects, due perhaps to excessive concentration on over-simplified structures and loadings which obscure the true general nature of the problem.

Before proceeding to a discussion of the scope of the theory presented in this paper a brief review of some representative previous work on the subject will be given. No mention will be made of work concerned only with circular cylinders.

The first important paper on the subject of tubes under torsion was presented by H. Reissner, 1926[1]. He investigated the uniform doubly symmetrical rectangular tube under linearly varying torque built-in at one end, with the assumptions that the direct stress across the sides of the tube varied linearly and that the cross-section was maintained by closely spaced rigid diaphragms*. Reissner's results were substantially correct but there was a slight error in the expression for the deflection of the walls due to shear.

Ebner, 1933[2] treated correctly the doubly symmetrical rectangular uniform or tapered tube with finitely spaced deformable frames under torsion with the same assumptions regarding direct stress distribution as Reissner. The taper considered by Ebner was not necessarily conical and he introduced for the first time the conception of representing the direct stress carrying capacity of the sides by adding one sixth of their areas to each boom. Ebner's paper had a great influence on much of the subsequent work on tubes under torsion.

The first attempt to give an exact solution of the problem of a doubly symmetrical uni-

* The phrase, "closely spaced rigid diaphragms," will be denoted henceforth in this section by the abbreviation C.S.R.D.

form rectangular tube under torque was made by Williams, 1934[3]. Williams' solution was inconsistent with his assumption of C.S.R.D. owing to the use of an inappropriate stress function.

Williams' next paper, 1936[4], dealt with uniform and conically tapered singly or doubly symmetrical rectangular tubes under torque. In this paper a system of C.S.R.D. was assumed and the same engineers' theory assumptions regarding the direct stress distribution were made as Ebner[2]. The effect of taper, however, was not correctly treated.

A series of reports by Cox were collected together in R. & M. 1908, 1942[5]. Cox's analysis applied to flat sided polygonal tubes of arbitrary taper in which the booms carry all the direct loads and the sides carry only shear loads. For the particular case of the doubly symmetrical rectangular cylindrical tube he considered the deformability of the frames. Cox developed his analysis in detail for the four-boom singly symmetrical trapezoidal tube with rigid finitely spaced frames. The form of cross-section taper could change at the frames as variations in the cross-section expressed themselves only as changes in the coefficients of a series of simultaneous equations (analagous to the three-moment equations) connecting the boom loads at successive frames. The tube with cut-outs, where one or more sides of the tube are missing, was also considered.

In the papers so far mentioned the tube was considered as consisting of four beams constrained to deflect equally at the corners of the tube, a conception which complicated rather than simplified the problem and at the same time obscured the proper approach to the solution of more general cross-sections (for example, with curved walls or many cells). Among the first writers to treat the tube under torsion as a whole were Fine, 1943[6] and Kuhn, 1942[7]. Fine investigated the doubly symmetrical rectangular four-boom cylindrical or conical tube with C.S.R.D. under arbitrary torque distribution, and was the first to give the correct differential equation for the boom loads in this class of conical tube. Kuhn's report covers the same ground as Cox's work on the doubly symmetrical rectangular cylindrical tube with rigid diaphragms.

Williams and Fine, 1945[8], attempted to depart from the over simplified conception of doubly or singly symmetrical box beams by considering a single cell aerofoil section under torque. Attention was concentrated on the direct stress distribution at the built-in end of the tube, and they assumed that the building-in stresses were proportional to the free warping of the tube. This assumption is examined in paragraph 3.2.5. of the present paper and is shown to be in general unjustifiable.

Nearly all the investigations on shear-lag or diffusion in tubes under bending loads have been restricted to doubly symmetrical cross-sections.

As far as the authors are aware the first paper to investigate the effect of shear-lag in rectangular box beams was given by Schnadel, 1928[9].

In a series of reports by Wagner, Simon, Ebner and Köller doubly symmetrical single cell cylindrical tubes under bending and torsion were investigated. The shear-lag and axial constraint effects were taken into account. In these papers the structure was idealised by a number of concentrated booms and shear carrying walls. The most comprehensive paper in this group was that by Ebner and Köller, 1937[10], which introduced the first systematic use of self-equilibrating load systems. The diaphragms were represented by deformable rings. The method was worked out in detail only for the circular cylinder. Ebner and Köller touched on a generalisation of their method to non-doubly symmetrical tubes.

Williams and Fine, 1940[11], investigated the shear-lag effect in various doubly sym-

metrical cylindrical box beams with C.S.R.D. under bending loads. They showed that in some very special cases the ordinary engineers' theory of bending is strictly true.

Hemp, 1943[12], gave an exact analysis of the stress distribution in a doubly symmetrical cylindrical rectangular box beam with booms and C.S.R.D. under bending and torsion.

Both of the latter papers developed the stresses in a Fourier series along the length of the tube, a method which obscured the existence of the characteristic self-equilibrating end load systems.

In all the doubly symmetrical tubes investigated in the above reports the problem of separating the torsion and bending loads does not arise, the torsion loads being obviously referred to the axis of symmetry. Dunne, 1945[13], gave a rational procedure for separating bending and torsion loads in the case of the four-boom singly symmetrical trapezoidal and uniform tube with shear carrying walls. He showed that it is incorrect to refer the torsion loads to an axis through the shear centre, and that the correct procedure was to calculate the bending stresses with the condition of zero warping rather than zero twist.

It will appear from the above survey that practically all the work on tubes had been concentrated on doubly symmetrical cross-sections and gave no indication of a rational procedure for stressing the more general shapes of tube encountered in practice. Even in the few singly symmetrical tubes investigated for direct stresses due to torsion no indication was given to the stressman as to how he was to calculate the torsion loads. Little attention was paid to the redistribution of shear stresses due to axial constraint, although the modification to the latter stresses as calculated by the engineers' theory is often more important than that to the direct stresses.

1.2. Scope of present investigation.

This report gives a rational method of stressing single or many celled conical and cylindrical tubes of arbitrary cross-section, the shape of which is maintained by a closely spaced system of diaphragms rigid in their own planes and parallel to the root section. Within the limits of these assumptions the theory is exact for right cylindrical tubes and is directly applicable with adequate accuracy to cylindrical and conical tubes in which the inclination of any generator to the normal to the root plane does not exceed 10°.

The assumption of rigid diaphragms makes it possible, in Part 2, to express the direct and shear stresses in the tube in terms of four displacements. Three of these displacements are the translatory and rotary movements of the diaphragms in their planes and thus vary only along the length of the tube. The fourth displacement is the longitudinal displacement normal to the diaphragms and thus varies with position on the periphery as well as along the generator. If the ribs are designed to deform linearly under the loads indicated by the rigid diaphragm assumption, their stiffness will be such that the C.S.R.D. assumption is in general justified, at least in wing structures. The introduction of deformable diaphragms in tubes of arbitrary cross-section enormously complicates the analysis. There are certain special cases, such as the existence of marked discontinuities in structure or loading, where a closer examination of the effect of diaphragm deformability is necessary. The authors intend later to give an analysis indicating the magnitude of the effect of frame deformability in some special cases.

At the end of Part 2 it is shown that the shear stress distribution at a fully axially constrained section can be calculated for any loading by statics. This important result of the C.S.R.D. assumption has, to the authors' knowledge, been overlooked by previous writers.

In Part 3 a complete theory of the conditions for the existence, and the form of the self-equilibrating orthogonal end load systems (eigenloads)*, which vary similarly along all generators, is given. It is shown that the existence of such eigenloads requires that the sheet thicknesses and boom areas should also vary similarly along all generators. In general there is an infinite set of orthogonal eigenloads characterised by a corresponding set of positive eigenvalues* λ, which are physically analagous to the diffusion coefficients in the sheet-stringer analysis of flat panels. The secular* equation determining the eigenvalues λ is in general transcendental, but in the case of the structure idealised by n concentrated booms, and with walls effective only in shear the secular equation is algebraic and has n-3 roots in λ^2. Thus in the n-boom tube there is a finite set of n-3 self-equilibrating end load systems which simplifies enormously the analysis. In particular the four-boom tube has only one eigenload system . At the end of Part 3 are derived simple expressions for the warping of cross-sections corresponding to each eigenload system. It is shown that in each eigenload system the warping is proportional to the direct stress.

Part 4 presents the general theory of conical and cylindrical tubes of the type discussed in Part 3 under any distribution of torque. It is shown that the stress distribution in all cases can be expressed as the sum of the generalised Bredt-Batho distribution and self-equilibrating load systems with the same variations over the walls of the cross-section as those of the eigenloads of Part 3. The typical formulae for the direct stress σ and shear stress τ in the tube are

$$\left.\begin{aligned}\sigma &= \frac{T_o}{2A}\sum_i G_{\sigma i}H_{\sigma i}\\ \tau &= \tau_B + \frac{T_o}{2A}\sum_i G_{\tau i}H_{\tau i}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where τ_B is the shear stress given by the Bredt-Batho theory, T_o is the torque at a fixed section (preferably the root), A is the total area of cross-section at the root. The $G_{\sigma i}$ and $G_{\tau i}$ functions depend solely on the loading, boundary conditions and the lengthwise variation of cross-section. Explicit formulae for these functions are given in certain cases of practical importance. The $H_{\sigma i}$ and $H_{\tau i}$ functions depend only on the geometry and elastic properties of the root cross-section. For the particular case of a four-boom tube, with or without nose and/or trailing cells, the $H_{\sigma i}$ and $H_{\tau i}$ functions have been completely worked out for immediate practical application.

In Part 5 the general theory of conical and cylindrical tubes under any distribution of shear is given. The results for the stress distribution are presented in a similar form to equations (1), as the sum of the engineers' theory stresses and self-equilibrating load systems, as in the pure torque case. In the case of the four-boom tube the bending solution may be, in general, conditioned by zero warping at all sections, the stress distribution then being exactly that given by the engineers' theory. Thus in this case a special analysis of the bending case is not necessary, the appropriate corrections to the engineers' theory being obtained by the method of Part 4 only.

The shear lag effect in the top and bottom panels between the two spars is of major importance in the bending case. To represent this influence more accurately than is possible using a four-boom idealisation, the six-boom tube is fully worked out. The two additional booms are placed in the top and bottom panels.

It is worth noticing that the present theory embraces, as a particular case, the theory of

* For meaning of this term see page 205.

diffusion in flat panels, as given for example by Hildebrand, 1943 [14] and Argyris, 1944-46 [15–17].

Part 6 applies the methods of this paper to some numerical examples of wing stressing.

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NOTE ON THE MATHEMATICAL TERMINOLOGY USED IN THIS PAPER.

The terms eigenvalue, eigenfunction, eigenload, secular equation, and normalisation as used in this paper may not be generally familiar. A brief explanation of their meaning will therefore be given here.

Certain problems in physics and engineering, such as those of elastic stability and free vibration are defined by homogeneous differential equations containing an undetermined parameter λ and with homogeneous boundary conditions. The differential equations have non-zero solutions only for a discrete set, not necessarily finite, of values of the parameter λ, which are called the eigenvalues λ of the problem. The eigenvalues appear as the roots of an equation in λ, which is known as the secular equation. The solutions of the differential equations corresponding to the eigenvalues λ are called the eigenfunctions of the problem and are determined to a constant multiplier only. For example, in the case of a vibrating string the eigenvalues are the natural frequencies and the eigenfunctions are the corresponding normal or principal modes of vibration which are undetermined as to amplitude. The secular equation in such examples is often known as the frequency equation.

A very important property of the eigenfunctions is that they are orthogonal over the region in which the differential equation is satisfied. Thus in a one-dimensional problem there will be relations of the type,

$$\int mh_ih_jds=0 \text{ for } i\neq j,$$

where h_i, h_j are the eigenfunctions corresponding to the distinct eigenvalues λ_i and λ_j and m is the so-called weight function.

It is often convenient in an analysis to give the eigenfunctions definite magnitudes by means of the relation,

$$\int mh_i^2ds=1$$

The eigenfunction h_i is then said to be normalised.

The special term eigenload used in this report refers to the stress-function, representing orthogonal self-equilibrating load systems applied to the ends of the tube, and which varies over all cross sections as the eigenfunctions of a set of differential equations characterising the geometric and elastic properties of the root cross-section.

1.3. NOTATION.

r	Distance along generator from apex 0 of cone.
r_o	Value of r at root.
$r_e=r_o\rho_e$	Value of r at free end of tube.
$r_1=r_o\rho_1$	Value of r at loaded section.

$\rho = r/r_0$	Non-dimensioned co-ordinate.
s	Clockwise distance along periphery at root section measured from a fixed point.
$s' = \rho s$	Clockwise distance along periphery at any section measured from a fixed generator.
OX, OY, OZ	An orthogonal right-handed system of axes. The Z axis is normal to the diaphragm planes and the directions OX and OY are arbitrary. The origin O is at the apex of a conical tube or at an arbitrary point in the plane of the free end of a cylindrical tube.
x', y', z'	Co-ordinates of any point on tube *w.r.t.* X, Y, Z axes. Note that $x' = \rho x$, $y' = \rho y$.
z_1	Value of z' at loaded section.
x, y, z	Co-ordinates of point on root section *w.r.t.* X, Y, Z axes.
l, m, n,	Direction cosines of any generator *w.r.t.* axes X, Y, Z.
ψ	Angle between positive direction of s and Y axis.
p	Normal from Z axis to tangent at root section.
$p' = \rho p$	Normal from Z axis to tangent at any section.
$\hat{p} = p - \xi \cos\psi + \eta \sin\psi$	Normal from centre of rotation S to tangent at root section.
ξ, η	Co-ordinates of S *w.r.t.* axes X, Y.
c	Length of tube.
ζ	$\dfrac{\text{Length of tube}}{\text{Distance of section from root}} = (1-\rho)/(1-\rho_c)$ for conical tubes, or $(c-z')/c$ for cylindrical tubes.
d	A constant length.
$\bar{z} = z' + d$	
t	Effective thickness of sheet in shear at root.
t_s	Effective thickness of sheet under direct stress at root.
$t' = t^* t$	Effective thickness of sheet in shear at any point.
$t_s' = t_s^* t_s$	Effective thickness of sheet under direct stress at any point.
t^*, t_s^*	Functions of r or z' only.
B_m	Area of mth boom at root.
$B_m' = \rho t_s^* B_m$	Area of mth boom at any section.
$s_{m,\,m+1}$	Length of periphery between mth and $(m+1)$th booms at the root.
S	Total periphery of root section.
A	Area of cross-section of tube at root. In a multicell tube it is the total area unless otherwise stated.
A_N	Area enclosed at root by Nth cell of a multi-cell tube.
A_{os}	Area swept by radius vector as its end moves over the distance s.
$A_{m,\,m+1}$	Value of A_{os} between mth and $(m+1)$th booms.
A', A'_{os}, etc.	Values corresponding to A, A_{os}, etc., at any section. Thus $A' = \rho^2 A$, etc.
A^C, $A^C{}_{m,\,m+1}$	Defined in equation (207).
A^m	Defined by equation (210).
A^B	Defined by equation (225).
$A^B{}_{m,\,m+1}$	Defined by equation (248).

A_{123}, etc.	Area of triangle formed by booms 1, 2 and 3 at the root.	
A^*	Area of quadrilateral 1 2 3 4.	
A^*_{12}, etc.	Area of triangle $Q12$, etc., where Q is the meet of diagonals 13 and 24.	
$N_{12}=A_{12}-A^*_{12}$, etc.	Area between s_{12} and line 12.	Applies to four-boom tube only.
$\bar{N}$	Defined by equation (192).	
$\bar{A}_{12}$, etc.	Defined by equation (191).	
$\bar{A}^{B}_{12}$, etc.	Defined by equation (250).	
N^{B}_{12}, etc.	Defined by equation (252).	
$\bar{N}^{B}$	Defined by equation (251).	
u, v	Displacements of rigid diaphragms in X and Y directions.	
θ	Rotation of rigid diaphragms about Z axis.	
v_t	Tangential displacement of point on periphery.	
w	Displacement of point on periphery of tube in direction OZ.	
w'	Displacement of point on periphery in direction of generator through that point.	
w''	Displacement of point on periphery in direction normal to v_t lying in the tangent plane.	
w^*	Warping displacement normal to diaphragms, *i.e.*, the part of w giving out of plane deformation of diaphragm.	
$w_m{}^*$	Warping displacement of mth boom.	
$w_i{}^*$	Warping displacement for ith eigenload system.	
$w_B{}^*$	Warping displacement for generalised Bredt-Batho stress system.	
$w_T{}^*$	Defined by equation (123).	
$W_P{}^*$	Function proportional to magnitude of warping for self-equilibrating end load system.	
$W_B{}^*$	Function proportional to magnitude of warping for generalised Bredt-Batho stress system.	Applies to four-boom tube only.
$W^*=W_P{}^*+W_B{}^*$	Function proportional to magnitude of warping.	
ϵ	Direct strain along generator.	
γ (Part 2)	Shear strain referred to axes along and normal to diaphragm section.	
E	Young's modulus of sheet-stringer combination.	
G	Shear modulus.	
σ	Direct stress along generator, positive for tension.	
τ	Shear stress in sheet, positive when arising from positive torque.	
$f=\sigma t_s'$	Direct load per unit length of periphery.	
$q=\tau t'$	Shear flux in sheet.	
P_m	Direct load in mth boom.	
P	Function proportional to boom loads in four-boom tube.	
T	Torque at any section, positive if clockwise on root to observer at the apex.	
T_k	Fourier coefficient for torque distribution.	

S_x, S_y	Shear resultants in X and Y directions, positive when acting in directions OX, OY on the root to observer at the apex.
M_x, M_y	Bending moments about X and Y axes, positive when resulting from positive shear.
F	Stress function.
g	Function of r.
h	Function of s. $\}$ $F=gh$ is a special stress function.
λ_i	ith root of secular (characteristic) equation for self-equilibrating end load system.
$\mu_i=\lambda_i\left(\frac{G}{E}\right)^{\frac{1}{2}}$	
v_i	Defined by equation (86a).
g_i	Function g associated with h_i.
h_i	Function h corresponding to eigenvalue λ_i.
g_B	Function g associated with generalised Bredt-Batho stress system.
h_B	Function h associated with generalised Bredt-Batho stress system
h_T	Function h defined in equation (122).
G_σ, G_τ	Functions of r associated with σ and τ respectively.
H_σ, H_τ	Functions of s associated with σ and τ respectively.
H	Defined by equation (231).
Ψ	Function of r.
Φ	Function of s.
α, β, γ (Part 4)	Arbitrary exponents.
a_i	Defined by equation (121).
δ_i	Defined by equation (162).
$\omega^2=\frac{(1+\alpha)}{r_0^2}\frac{E}{G}$	
k	Odd integer.
C_1, C_2, D_1, D_2	Constant coefficients.
$\oint$, Σ	Integral and sum over all walls of tube.
$\oint_s$, Σ_s	Integral and sum over outer walls only.
$\oint_N$	Integral over walls of Nth cell.
Suffix i	Refers to values associated with eigenvalue λ_i.
Suffix o	Refers to values at root.
Suffix c	Refers to values at free end of tube.
Suffix 1	Refers to values at section with concentrated load.
Indices i and o	Refer to values associated with inner and outer walls of multicell tubes.
Index oi	Refers to difference of values associated with outer and inner walls of multicell tube.

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2. STRAIN-DISPLACEMENT RELATIONS; WARPING OF CROSS-SECTION; SHEAR STRESS AT A BUILT-IN END OF TUBE.

2.1. Expressions for direct strain ϵ and shear strain γ.

Figure 1 represents a thin-walled conical tube which may be reinforced by longitudinal members. The fundamental assumption of this paper is that the cross-section of the tube is maintained by an indefinitely closely spaced system of diaphragms rigid in their own planes, but offering no resistance to deflections out of their planes. It follows from this assumption that the deformation of the tube is completely defined by the three displacements u, v, w, and the rotation θ (see Fig. 1 and the notation).

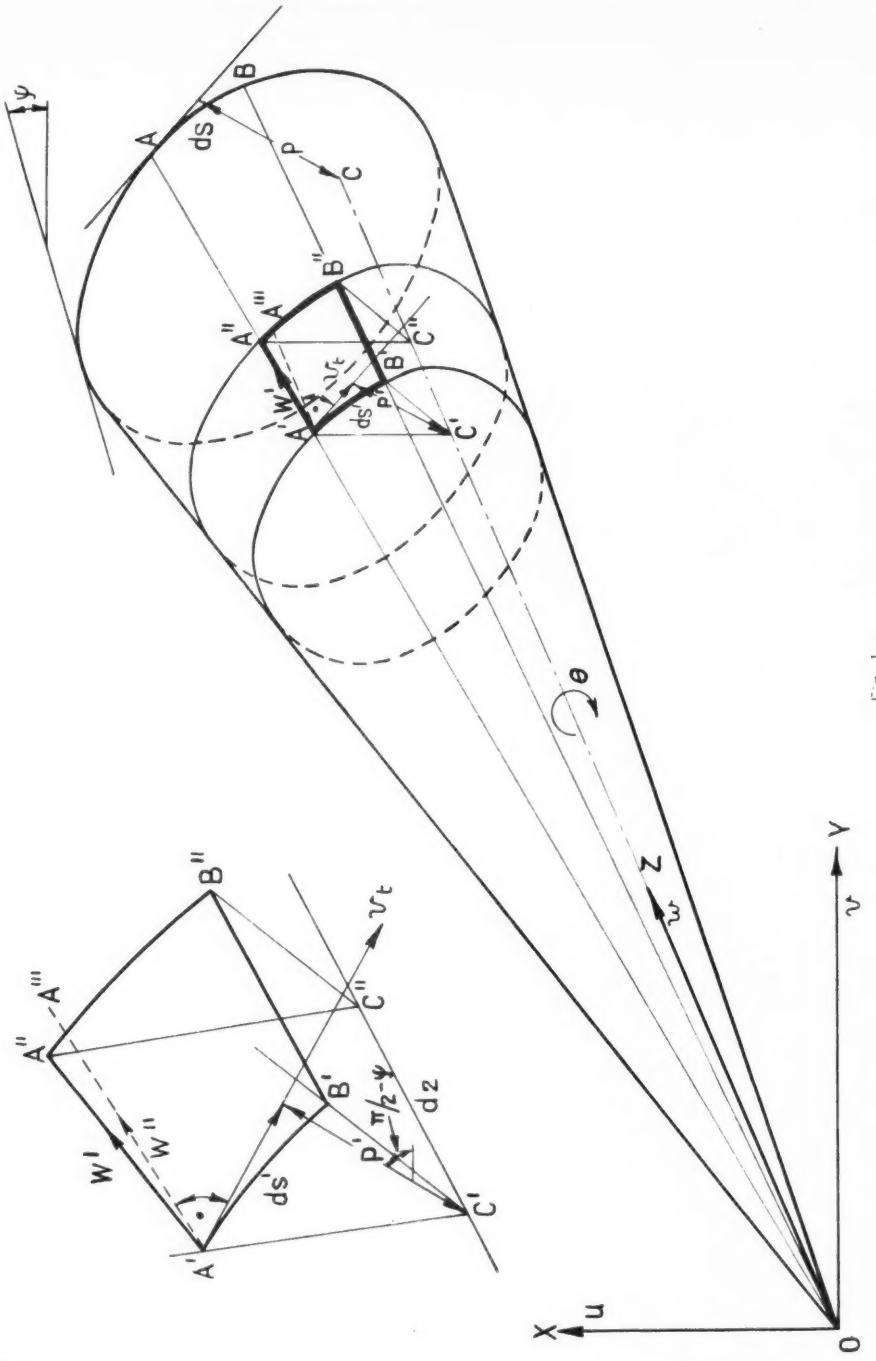

Fig. 1
Conical tube.

A further important assumption is that the angle between any generator and the axis OZ is so small as to make its square negligible compared with unity, so that one may write,

$$\left.\begin{aligned} l^2 = lm = m^2 = 0 \\ \text{and } n = 1 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The displacement w' along the generator through a point A' is given by,

$$w' = lu + mv + w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where l, m, 1 are the direction cosines of the generator through A'. The absence of a term involving θ is due to the fact that the tube is conical.

The direct strain ϵ along the generator is,

$$\begin{aligned} \epsilon = \frac{\partial w'}{\partial r} &= l\frac{\partial u}{\partial r} + m\frac{\partial v}{\partial r} + \frac{\partial w}{\partial r} \\ &= l\frac{du}{dz'} + m\frac{dv}{dz'} + \frac{\partial w}{\partial r} \quad . \quad . \quad . \quad . \quad . \quad (3) \end{aligned}$$

To find the shear strain γ along the diaphragm section at A', it is necessary to express the deformation of the tube in terms of the two orthogonal directions in the tangent plane at A', defined by the tangent to the diaphragm and the line $A'A'''$ normal to the tangent. The displacements in these two directions are denoted by v_t and w'' respectively, and the co-ordinate in the direction w'' by z''. The direction cosines of v_t are $\sin\psi$, $\cos\psi$,0. Then if l', m', n' are the direction cosines of $A'A'''$, the orthogonality of v_t and $A'A'''$, together with the co-planarity of both with the generator $A'A''$ yields,

$$l' \sin\psi + m' \cos\psi = 0$$

and,

$$\begin{vmatrix} l' & m' & n' \\ l & m & n \\ \sin\psi & \cos\psi & 0 \end{vmatrix} = 0$$

from which, with equation (1),

$$\left.\begin{aligned} l' &= \cos\psi\,(l\cos\psi - m\sin\psi) \\ m' &= -\sin\psi\,(l\cos\psi - m\sin\psi) \\ n' &= 1 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The displacements v_t and w'' may now be written in terms of u, v, w, and θ. Thus,

$$v_t = (u - y'\theta)\sin\psi + (v + x'\theta)\cos\psi \quad . \quad . \quad . \quad . \quad (5)$$

$$w'' = (u - y'\theta)l' + (v + x'\theta)\,m' + w \quad . \quad . \quad . \quad . \quad (6)$$

The shear strain γ is given by,

$$\gamma = \frac{\partial v_t}{\partial z''} + \frac{\partial w''}{\partial s'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Now,

$$\frac{\partial v_t}{\partial z''} = \frac{\partial v_t}{\partial z'} + l'\frac{\partial v_t}{\partial x'} + m'\frac{\partial v_t}{\partial y'} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and from equation (5),

$$\left.\begin{aligned} \frac{\partial v_t}{\partial z'} &= \sin\psi\frac{du}{dz'} + \cos\psi\frac{dv}{dz'} + \rho p\frac{d\theta}{dz'} \\ \frac{\partial v_t}{\partial x'} &= \theta\cos\psi \\ \frac{\partial v_t}{\partial y'} &= -\theta\sin\psi \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (8a)$$

Also from equation (6),

$$\frac{\partial w''}{\partial s'} = \frac{\partial w}{\partial s'} - \theta(l' \cos\psi - m' \sin\psi) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Hence using equations (8), (8a) and (9) in equation (7) one obtains,

$$\gamma = \frac{1}{\rho}\frac{\partial w}{\partial s} + \sin\psi \frac{du}{dz'} + \cos\psi \frac{dv}{dz'} + \rho p \frac{d\theta}{dz'} \quad . \quad . \quad . \quad . \quad (10)$$

The direct stress σ along the generator and the shear stress τ along the diaphragm section are obtained from equations (3) and (10). Thus,

$$\sigma = E\epsilon = E\left(\frac{\partial w}{\partial r} + \frac{x}{z}\frac{du}{dz'} + \frac{y}{z}\frac{dv}{dz'}\right) \quad . \quad . \quad . \quad . \quad (11)^*$$

and,

$$\tau = G\gamma = G\left(\frac{1}{\rho}\frac{\partial w}{\partial s} + \sin\psi \frac{du}{dz'} + \cos\psi \frac{dv}{dz'} + \rho p \frac{d\theta}{dz'}\right) \quad . \quad . \quad . \quad (12)$$

The shear stress along the generators will be assumed to have the same value as that given by equation (12) which is the shear stress along and normal to the diaphragm section. This assumption is correct for a cylindrical tube and is justified for a conical tube by the limitations imposed on the taper by the conditions expressed in equations (1). Many previous writers have used the same assumption. The exact specification of the stresses in a conical tube with rigid plane diaphragms is an extremely complex problem. Although the assumption of rigid diaphragms simplifies the mathematical description of the tube deformations and strains, the opposite is true as regards the stresses since the tube walls can no longer be considered isotropic or even, in general, to have a simple form of aelotropy. However, it seems most reasonable to associate the shear modulus G with the strain γ parallel to the direction in which the sheet is forced by the diaphragms to be effectively rigid. It is worth noting that if the tangential displacement v_t is taken normal to the generators, and is assumed to have the same value as expressed in equation (5), the shear stress along the generators may be written,

$$\tau = G\left\{\frac{\partial v_t}{\partial r} - \frac{v_t}{r} + \frac{\partial w'}{\partial s'}\right\}$$

which reduces to the same value as (12). (See Southwell, 1937 [18].)

The analysis of this paper would be exact, within the limits imposed by equation (1), for a conical tube structure consisting of a system of direct stress carrying rods lying along the generators and connected by purely shear resisting sheet. In this model the direct stresses perpendicular to the generators are not present. Their effect would be in any case small by the limitations imposed on the taper (see Fine, 1943 [6]).

This idealised model is used throughout this paper and is an adequate representation of an actual wing structure consisting of booms, stringers, sheet and ribs.

Another fact which vitiates any "exact" analysis based on an aelotropic material is the possible buckling of the sheet. The simplified model readily allows an approximate analysis of the effects of buckling, by a reduction of the sheet-thicknesses in accordance with the secant shear modulus of the buckled sheet.

(*) In formula (11) it has been assumed that the Poisson's ratio in the tube perpendicular to the generators is zero as along the diaphragms.

2.2. EXPRESSIONS FOR WARPING AND TWIST.

Integrating equation (12) with respect to s' one obtains,

$$\int_0^{s'} \tau ds' = G\left[(w-w_o)+\frac{du}{dz'}(x'-x_o')+\frac{dv}{dz'}(y'-y_o')+2A'_{os}\frac{d\theta}{dz'}\right] \quad . \quad . \quad (13)$$

The suffix "o" refers to the value of a variable at $s'=0$. Taking the integral over any closed cell "N,"

$$\oint_N \tau ds' = 2GA_N'\frac{d\theta}{dz'}, \text{ and hence,}$$

$$\frac{d\theta}{dz'} = \frac{1}{2A_N'G}\oint_N \tau ds' = \frac{1}{2A'G}\oint_S \tau ds' \quad . \quad . \quad . \quad . \quad (14)$$

Equation (14) is the well-known result for the rate of twist of a cylindrical or slightly conical tube under torque when the shear is distributed according to the Bredt-Batho formula. *The present derivation places no restrictions on the nature of the loading on the tube.* Now from equations (13) and (14),

$$w = w_o + \frac{1}{G}\int_0^{s'} \tau ds' - \frac{A_{os}}{GA_N}\oint_N \tau ds' - \frac{du}{dz'}(x'-x_o') - \frac{dv}{dz'}(y'-y_o') \quad . \quad . \quad (15)$$

The first, fourth, and fifth terms on the R.H.S. of equation (15) represent rigid body movements of the diaphragm sections. The remaining terms, which will be denoted by w^*, give the warping of the diaphragm sections out of their own planes.
Thus,

$$w^* = \frac{1}{G}\left(\int_0^{s'} \tau ds' - \frac{A_{os}}{A_N}\oint_N \tau ds'\right) \quad . \quad . \quad . \quad . \quad . \quad (16)$$

It is worth while noting that the expression for warping depends only on the shear stress distribution around the periphery. This simple result is due to the rigid diaphragm assumption.

When the stresses in a tube are found by the methods of the ordinary engineers' theory of bending and torsion the warping of sections as found from equation (16) will be in general incompatible with the direct stresses. Hence self-equilibrating systems of stresses will be set up which will have particular importance at sections where discontinuities of loading or structure occur, and also at axially constrained (*i.e.*, built-in) sections. In Part 3 the general theory of the self-equilibrating load systems for a single cell or many cell tube of arbitrary cross-section is developed.

If in equation (16) τ is made proportional to p' then $w^*=0$. In particular, for a single-cell tube under pure torque there will be no warping if,

$$t'p' = \text{function of } r \text{ only,}$$

and in such tubes the shear stress will follow the Bredt-Batho formula for any distribution of torque. Previous writers have noted this result only in the particular case of the rectangular tube.

When bending loads predominate it will be in general not possible to design an efficient structure with no warping. Special cases of doubly symmetrical tubes under bending have been discussed by Williams and Fine, 1940.[11]

2.3. SHEAR STRESS DISTRIBUTION AT BUILT-IN END OF TUBE.

A very interesting consequence of the rigid diaphragm assumption is that the shear stress distribution at a fully axially constrained section is statically determinate for any system of loading whatever. For when $w=0$ equation (12) becomes,

$$\tau=G\left(\sin\psi\ \frac{du}{dz'}+\cos\psi\ \frac{dv}{dz'}+p'\frac{d\theta}{dz'}\right) \quad . \quad . \quad . \quad . \quad (17)$$

The three unknowns $\frac{du}{dz'}$, $\frac{dv}{dz'}$, and $\frac{d\theta}{dz'}$ may be determined from the three statical equations at the section. The shear force components in the X and Y directions due to the direct stress flow f are given by,

$$\oint flds' \text{ and } \oint fmds' \text{ respectively.}$$

These expressions may be written,

$$\left.\begin{aligned}\oint flds'&=\frac{1}{r}\oint fx'ds'=\frac{M_y}{r}\\ \oint fmds'&=\frac{1}{r}\oint fy'ds'=\frac{M_x}{r}\end{aligned}\right| \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Hence the shear force components due to shear flow q only, are,

$$S_x-\frac{M_y}{r} \text{ and } S_y-\frac{M_x}{r} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

For equilibrium at the fully constrained section,

$$\left.\begin{aligned}\oint \tau t'\sin\psi ds'&=S_x-\frac{M_y}{r}\\ \oint \tau t'\cos\psi ds'&=S_y-\frac{M_x}{r}\\ \oint \tau t'p'ds'&=T\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The line integrals in equations (18) and (20) must be taken over every wall of a multi-cell tube with due attention to the sign of ψ, p' and τ. For example, in Fig. 2 the positive directions of s and τ are the same and are assigned arbitrarily in each wall. Then p_1' is positive since movement of the foot of the perpendicular along the positive direction of the tangent T_1 leads to a clockwise rotation of p_1' about C'. Similarly by this convention p_2' and p_3' are negative. ψ is the clockwise rotation of the tangent vector necessary to bring it into coincidence with the positive direction of the Y-axis.

Now substituting for τ from equation (17) in equation (20) one obtains the three following simultaneous equations for $\frac{du}{dz'}$, $\frac{dv}{dz'}$, and $\frac{d\theta}{dz'}$:—

$$\left.\begin{aligned}\frac{du}{dz'}\oint t'\sin^2\psi ds'+\frac{dv}{dz'}\oint t'\sin\psi\cos\psi ds'+\frac{d\theta}{dz'}\oint t'p'\sin\psi ds'&=\frac{1}{G}\left(S_x-\frac{M_y}{r}\right)\\ \frac{du}{dz'}\oint t'\sin\psi\cos\psi ds'+\frac{dv}{dz'}\oint t'\cos^2\psi ds'+\frac{d\theta}{dz'}\oint t'p'\cos\psi ds'&=\frac{1}{G}\left(S_y-\frac{M_x}{r}\right)\\ \frac{du}{dz'}\oint t'p'\sin\psi ds'+\frac{dv}{dz'}\oint t'p'\cos\psi ds'+\frac{d\theta}{dz'}\oint t'p'^2ds'&=\frac{T}{G}\end{aligned}\right| \quad . \quad (21)$$

Having $\frac{du}{dz'}$, $\frac{dv}{dz'}$, and $\frac{d\theta}{dz'}$ substitution (17) gives the shear stress distribution at a fully constrained section.

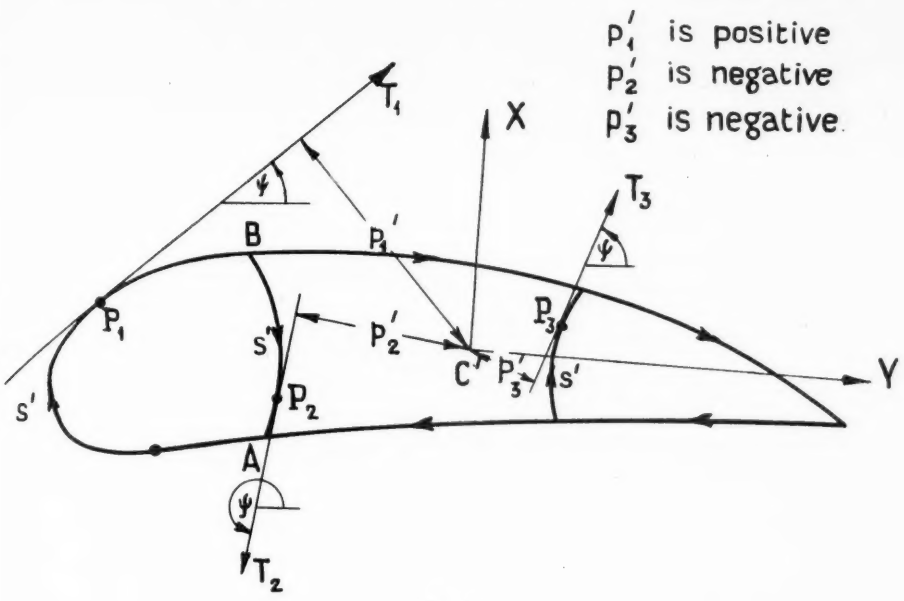

Fig. 2.
Sign convention for multi-cell tube.

3. THE ORTHOGONAL SELF-EQUILIBRATING END LOAD SYSTEMS; THE CONDITIONS FOR THEIR EXISTENCE.

In this part a complete theory of the conditions for the existence, and the form of the self-equilibrating orthogonal end load systems (eigenloads), which vary similarly along all generators, is given. It is shown that eigenloads of this type exist for arbitrary cross-section if the skin thicknesses and boom areas vary similarly along all generators.

3.1. THE DIFFERENTIAL EQUATION FOR THE STRESS FUNCTION F.

Consider a small element on the cone bounded by two generators AA' and BB' and two diaphragms AB and $A'B'$ (see Fig. 3).

By virtue of the assumption of 2.1 the equilibrium condition of this element is:

$$(f+df)(\rho+d\rho)ds - f\rho ds + (q+dq)\,dr - qdr = 0$$

Dividing by $drds$,

$$\frac{\partial}{\partial r}(\rho f) + \frac{\partial q}{\partial s} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where f and q are given by,

$$f = \sigma t_s' \text{ and } q = \tau t' \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Equation (22) is satisfied by a stress function F defined by

$$\rho f = \frac{\partial F}{\partial s} \text{ and } q = -\frac{\partial F}{\partial r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Solving (11) and (12) for $\frac{\partial w}{\partial z'}$ and $\frac{\partial w}{\partial s}$ respectively and substituting relations (24) one obtains,

$$\frac{\partial w}{\partial r} = \frac{1}{Et_s'\rho}\frac{\partial F}{\partial s} - \frac{x}{r_0}\frac{du}{dr} - \frac{y}{r_0}\frac{dv}{dr} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$\frac{\partial w}{\partial s} = -\frac{1}{Gt'}\rho\frac{\partial F}{\partial r} - p\rho^2\frac{d\theta}{dr} - \rho\sin\psi\frac{du}{dr} - \rho\cos\psi\frac{dv}{dr} \quad . \quad . \quad . \quad (26)$$

Differentiating equation (25) with respect to s and equation (26) with respect to r, subtracting and multiplying by $Gt'\rho$ one obtains,

$$\frac{Gt'}{E}\frac{\partial}{\partial s}\left(\frac{1}{t_s'}\frac{\partial F}{\partial s}\right) + \rho^2\frac{\partial^2 F}{\partial r^2} + t'\rho\frac{\partial}{\partial r}\left(\frac{\rho}{t'}\right)\frac{\partial F}{\partial r} = -Gt'\rho\Big[p\frac{d}{dr}\left(\rho^2\frac{d\theta}{dr}\right) + \rho\sin\psi\frac{d^2u}{dr^2} + \rho\cos\psi\frac{d^2v}{dr^2}\Big] \quad . \quad . \quad . \quad (27)$$

This is the fundamental differential equation for the stress function F.

For a cylindrical tube equation (27) reduces to,

$$\frac{Gt'}{E}\frac{\partial}{\partial s}\left(\frac{1}{t_s'}\frac{\partial F}{\partial s}\right) + \frac{\partial^2 F}{\partial z'^2} + t'\frac{\partial}{\partial z'}\left(\frac{1}{t'}\right)\frac{\partial F}{\partial z'} = -Gt'\left[p\frac{d^2\theta}{dz'^2} + \sin\psi\frac{d^2u}{dz'^2} + \cos\psi\frac{d^2v}{dz'^2}\right] \quad (27a)$$

3.2. The stress functions for self-equilibrating end load systems.

It is natural to assume the existence of end load systems in which the distributions of direct and shear flows are similar at all sections of the cone.

It follows from equations (24) that both $\frac{\partial F}{\partial s}$ and $\frac{\partial F}{\partial r}$ must be of the form

$$\text{Function } (s) \times \text{Function } (r)$$

Hence F can be expressed as,

$$F = g(r)h(s)$$

so that,

$$\sigma = \frac{1}{t_s'\rho}g\frac{dh}{ds} \text{ and } \tau = -\frac{dg}{dr}\frac{h}{t'} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

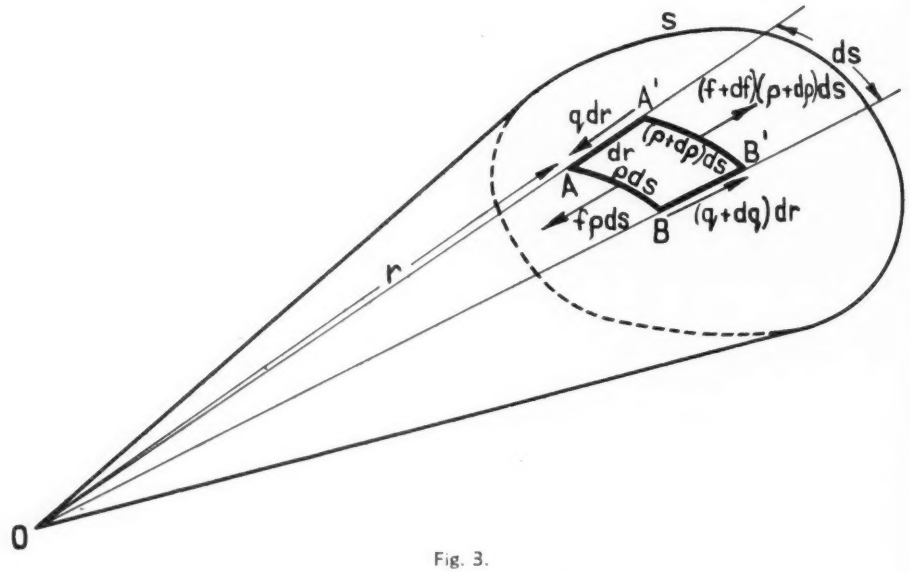

Fig. 3.
Stress resultants on element of tube

It is assumed in (28) that $h(s)$ is periodic over any closed path of the tube cross-section and that the values of h are independent of the circuit chosen. This ensures that there is no resultant end load on the tube.

Substituting (28) into (27),

$$\rho^2 h\frac{d^2g}{dr^2}+t'\rho\frac{\partial}{\partial r}\left(\frac{\rho}{t'}\right)h\frac{dg}{dr}+\frac{Gt'}{E}\frac{\partial}{\partial s}\left(\frac{1}{t_s'}\frac{dh}{ds}\right)g=-Gt'\rho\left[p\frac{d}{dr}\left(\rho^2\frac{d\theta}{dr}\right)+\rho\sin\psi\frac{d^2u}{dr^2}+\rho\cos\psi\frac{d^2v}{dr^2}\right]. \quad (29)$$

Assuming for the moment that u and v derive from a rotation of the diaphragms in their planes about points on a straight line through the apex of the cone, it is possible to write,

$$u=+\rho\eta\theta \text{ and } v=-\rho\xi\theta \quad (30)$$

where ξ and η are the co-ordinates of the point of intersection of the assumed axis of rotation and the root section.

From (30),

$$\left.\begin{aligned}\frac{d^2u}{dr^2}&=\rho\eta\frac{d^2\theta}{dr^2}+2\frac{\eta}{r_o}\frac{d\theta}{dr}\\ \frac{d^2v}{dr^2}&=-\rho\xi\frac{d^2\theta}{dr^2}-2\frac{\xi}{r_o}\frac{d\theta}{dr}\end{aligned}\right\} \quad (31)$$

Hence,

$$\rho\sin\psi\frac{d^2u}{dr^2}+\rho\cos\psi\frac{d^2v}{dr^2}=(\eta\sin\psi-\xi\cos\psi)\frac{d}{dr}\left(\rho^2\frac{d\theta}{dr}\right) \quad (32)$$

By substitution of equation (32) into (29),

$$\rho^2h\frac{d^2g}{dr^2}+t'\rho\frac{\partial}{\partial r}\left(\frac{\rho}{t'}\right)h\frac{dg}{dr}+\frac{Gt'}{E}\frac{\partial}{\partial s}\left(\frac{1}{t_s'}\frac{dh}{ds}\right)g=-Gt'\rho\hat{p}\frac{d}{dr}\left(\rho^2\frac{d\theta}{dr}\right). \quad (33)$$

where,

$$\hat{p}=p-\xi\cos\psi+\eta\sin\psi \quad (34)$$

It will be noted that $\hat{p}$ is the perpendicular from the point (ξ, η) to the tangent at the root cross-section.

From equations (14), (24) and (28),

$$\frac{d\theta}{dr}=-\frac{1}{2A_N G\rho}\oint_N\frac{1}{t'}\frac{\partial F}{\partial r}ds=-\frac{1}{2A_N G\rho}\frac{dg}{dr}\oint_N\frac{h}{t'}ds=-\frac{1}{2AG\rho}\frac{dg}{dr}\oint_S\frac{h}{t'}ds \quad (35)$$

In general the last expression in equation (35) will be used.

From equation (35),

$$\frac{d}{dr}\left(\rho^2\frac{d\theta}{dr}\right)=-\frac{1}{2AG}\oint_S\frac{h}{t'}ds\left\{\frac{1}{r_o}\frac{dg}{dr}+\rho\frac{d^2g}{dr^2}\right\}+\frac{1}{2AG}\oint_S\frac{h}{t'^2}\frac{\partial t'}{\partial r}ds\,.\,\rho\frac{dg}{dr} \quad (36)$$

The second term on the L.H.S. of (29) may be written,

$$\rho\left\{\frac{1}{r_o}-\rho\frac{1}{t'}\frac{\partial t'}{dr}\right\}h\frac{dg}{dr}$$

By substitution of the last expression and (36) into (29) one obtains,

$$\left\{h-\frac{\hat{p}t'}{2A}\oint_S\frac{h}{t'}ds\right\}\rho^2\frac{d^2g}{dr^2}+\left[\frac{1}{r_o}\left\{h-\frac{\hat{p}t'}{2A}\oint_S\frac{h}{t'}ds\right\}-\rho\left\{\frac{h}{t'}\frac{\partial t'}{\partial r}-\frac{\hat{p}t'}{2A}\oint_S\frac{h}{t'^2}\frac{\partial t'}{\partial r}ds\right\}\right]\rho\frac{dg}{dr}+\frac{Gt'}{E}\frac{\partial}{\partial s}\left(\frac{1}{t_s'}\frac{dh}{ds}\right)g=0 \quad (37)$$

Since *a priori* g is independent of s the equation obtained by dividing equation (37) by,

$$\left\{h - \frac{\hat{p}t'}{2A} \oint_S \frac{h}{t'}\, ds\right\}$$

must not involve s.

The term under $\frac{dg}{dr}$ will be independent of s if,

$$t' = tt^* \qquad (38)$$

where t is a function of s and t^* a function solely of r. In fact this term becomes,

$$\left[\frac{1}{r_o} - \frac{\rho}{t^*}\frac{dt^*}{dr}\right]\rho\,\frac{dg}{dr} \qquad (39)$$

One condition that the term under g is independent of s is that

$$t_s' = t_s t_s^* \qquad (40)$$

where t_s is a function of s and t_s^* a function solely of r.

Using (40) this term becomes,

$$\frac{\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh}{ds}\right)}{\left\{\frac{h}{t} - \frac{\hat{p}}{2A}\oint_S \frac{h}{t}\, ds\right\}}\,\frac{Gt^*}{Et_s^*}\,g \qquad (41)$$

Since expression (41) must be independentof s,

$$\frac{\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh}{ds}\right)}{\left\{\frac{h}{t} - \frac{\hat{p}}{2A}\oint_S \frac{h}{t}\, ds\right\}} = -\lambda^2 \qquad (42)$$

where λ is a constant.

Using (39), (41) and (42) in equation (37) one obtains the differential equation for g,

$$r^2\frac{d^2g}{dr^2} + r\left(1 - \frac{r}{t^*}\frac{dt^*}{dr}\right)\frac{dg}{dr} - \mu^2 r_o^2\,\frac{t^*}{t_s^*}\,g = 0 \qquad (43)$$

where,

$$\mu^2 = \frac{G}{E}\,\lambda^2 \qquad (44)$$

Equation (42) may be written,

$$\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh}{ds}\right) + \lambda^2\left\{\frac{h}{t} - \frac{(p - \xi\cos\psi + \eta\sin\psi)}{2A}\oint_S \frac{h}{t}\, ds\right\} = 0 \qquad (45)$$

3.2.1. *Discussion of the differential equation in h ; the eigenvalues* λ.

It will be shown that when the stress system represented by $F = gh$ is self-equilibrating the boundary conditions for the function h are homogeneous, and that for this reason solutions of equation (45) exist only for a discrete set, in general an infinite set of λ values. These values are the eigenvalues

$$\lambda_1, \lambda_2, \ldots, \lambda_i, \ldots$$

and the corresponding solutions of equation (45) are the eigenfunctions,

$$h_1, h_2, \ldots, h_i, \ldots$$

Furthermore, since

$$\hat{p} = p - \xi\cos\psi + \eta\sin\psi$$

is the perpendicular from an undetermined point it can be seen from equation (45) that its solutions are invariant with both the position of the origin C and the direction of the axes X, Y with respect to the root cross-section. Hence the functions h are independent of any sweepback or sweep-up of the cone as long as

$$n \approx 1$$

For further consideration of equation (45) it is necessary to express the stress resultants T, S_x and S_y in terms of the functions g, h.

Expressing the direct and shear flows in terms of g and h, from (24),

$$f = \frac{g}{\rho}\frac{dh}{ds}, \quad q = -h\frac{dg}{dr} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Using equation (46),

$$T = \oint qp'ds' = -\rho^2 \frac{dg}{dr}\oint hpds \qquad . \quad . \quad . \quad . \quad (47)$$

In deriving the expressions for the shear resultants the contribution of the direct stress must be considered.

Thus,

$$\left.\begin{aligned} S_x &= \oint q \sin\psi ds' + \oint fld s' \\ S_y &= \oint q \cos\psi ds' + \oint fmds' \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad . \quad (48)$$

Again using equations (46),

$$\left.\begin{aligned} S_x &= -\rho\frac{dg}{dr}\oint h\sin\psi ds + \frac{g}{r_o}\oint \frac{dh}{ds}xds \\ S_y &= -\rho\frac{dg}{dr}\oint h\cos\psi ds + \frac{g}{r_o}\oint \frac{dh}{ds}yds \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad (49)$$

By partial integration of the second integrals on the R.H.S. of equation (49) over each wall,

$$\oint \frac{dh}{ds}xds = \sum_{m=1}^{m=n}\left(\frac{1}{t_s}\frac{dh}{ds}\right)_m B_m x_m + \left[\oint hx\right] - \oint h\sin\psi ds \qquad . \quad . \quad (50)$$

where the notation $\left[\oint hx\right]$ denotes the sum of the differences of hx at the two ends of each continuous wall, and the second term has to be taken over all booms. The value $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_m$ is by virtue of equations (46) proportional to the stress in the mth boom. For a single-cell tube with no booms the first two terms on the R.H.S. of (50) are obviously zero. The same result is true for any single- or multi-cell tube with booms as will appear later when the boundary conditions of the h functions are discussed (see equation 63).

Hence,

$$\oint \frac{dh}{ds}xds = -\oint h\sin\psi ds \qquad . \quad . \quad . \quad . \quad . \quad (50a)$$

and similarly,

$$\oint \frac{dh}{ds}yds = -\oint h\cos\psi ds \qquad . \quad . \quad . \quad . \quad . \quad (50b)$$

Using equations (50a) and (50b) in equation (49),

$$\left.\begin{aligned} S_x &= -\left(\rho\frac{dg}{dr} + \frac{g}{r_o}\right)\oint h\sin\psi ds \\ S_y &= -\left(\rho\frac{dg}{dr} + \frac{g}{r_o}\right)\oint h\cos\psi ds \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad (51)$$

The moments M_x and M_y will be given by,

$$\left.\begin{aligned} M_x &= + \oint fy'ds' = +\rho g \oint \frac{dh}{ds}\, yds = -\rho g \oint h\cos\psi ds \\ M_y &= + \oint fx'ds' = +\rho g \oint \frac{dh}{ds}\, xds = -\rho g \oint h\sin\psi ds \end{aligned}\right\} \quad . \quad . \quad (52)$$

It may be noted that an alternative derivation of equations (51) is by differentiation of equations (52) with respect to r.

The limiting forms of equations (47), (51) and (52) for a cylindrical tube are,

$$T = -\frac{dg}{dz'}\oint hpds \quad . \quad . \quad . \quad . \quad . \quad . \quad (47a)$$

$$\left.\begin{aligned} S_x &= -\frac{dg}{dz'}\oint h\sin\psi ds \\ S_y &= -\frac{dg}{dz'}\oint h\cos\psi ds \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (51a)$$

$$\left.\begin{aligned} M_x &= -g\oint h\cos\psi ds \\ M_y &= -g\oint h\sin\psi ds \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (52a)$$

For a self-equilibrating end load system,

$$S_x = S_y = T = 0 \quad . \quad . \quad . \quad . \quad . \quad (53)$$

Hence from equations (47) and (51) one obtains three homogeneous equations in h, namely,

$$\left.\begin{aligned} \oint hpds &= 0 \\ \oint h\sin\psi ds = \oint h\cos\psi ds &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (54)$$

For each wall in which t, t_s and ψ are continuous functions there will be a separate solution of the differential equation (45) each containing in general two arbitrary constants. Note that the at present undetermined constants λ^2, ξ, η and the magnitude $\oint_S \frac{h}{t}ds$ are common to all the separate solutions.

The boundary conditions for h will now be considered. In a single-cell tube in which t, t_s and ψ are continuous between $s=0$ and $s=S$, but not necessarily at the point ($s=0$ or $s=S$) itself, there is only one solution of equation (45) with two arbitrary constants C_1 and C_2. The conditions of continuity in q and σ at the beginning and end of the closed circuit yield,

$$\left.\begin{aligned} h(0) &= h(S) \\ \text{and} \quad \left(\frac{1}{t_s}\frac{dh}{ds}\right)_{s=0} &= \left(\frac{1}{t_s}\frac{dh}{ds}\right)_{s=S} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (55)$$

where S is the periphery of the root cross-section.

The general form of the solution of equation (45) is,

$$h = C_1\Phi_1 + C_2\Phi_2 + \oint_S \frac{h}{t}ds\,.\,\Phi_3 + \xi\oint_S \frac{h}{t}\,ds\,.\,\Phi_4 + \eta\oint_S \frac{h}{t}\,ds\,.\,\Phi_5 \quad . \quad (56)$$

where Φ_1 to Φ_5 are functions in s. The first two terms in equation (56) represent the complementary function while the last three terms are particular integrals of the non-homogeneous differential equation (45).

By substitution of (56) into (54) and (55) five homogeneous linear simultaneous equations in the five unknowns C_1, C_2, $\oint_S \frac{h}{t}\,ds$, $\xi \oint_S \frac{h}{t}\,ds$, $\eta \oint_S \frac{h}{t}\,ds$ are found. Non-zero solutions of these equations exist only if the determinant formed by the coefficients of the five unknowns is identically zero.

The roots of this determinantal equation in λ are the eigenvalues

$$\lambda_1, \lambda_2, \ldots, \lambda_i \ldots$$

of the problem. This equation is usually called a secular equation and in general will be a transcendental one having an infinite set of roots, which are independent of the position of the origin C.

For any particular value λ_i the ratios

$$C_{1i} : C_{2i} : \oint_S \frac{h_i}{t}\,ds : \xi_i \oint_S \frac{h_i}{t}\,ds : \eta_i \oint_S \frac{h_i}{t}\,ds$$

can be determined, thus yielding unique values in ξ_i and η_i, the magnitude of the remaining unknowns being indefinite.

Introducing the condition,

$$\oint \frac{h_i^2}{t}\,ds = 1 \qquad \ldots\ldots \quad (57)$$

the significance of which will appear later, the values of C_{1i}, C_{2i} and $\oint_S \frac{h_i}{t}ds$ are fixed. Thus the eigenfunction h_i corresponding to the eigenvalue λ_i becomes a definite function of the geometry of the root cross-section, but is independent of the position of the apex of the cone. The simplest illustration of the complementary function of the differential equation (45) is the case where t and t_s are constant. The homogeneous part of equation (45) then becomes,

$$\frac{d^2 h_i}{ds^2} + \lambda_i^2 \frac{t_s}{t} h_i = 0 \qquad \ldots\ldots \quad (58)$$

and its solution,

$$h_i = C_{1i} \sin\left\{\sqrt{\frac{t_s}{t}}\lambda_i s\right\} + C_{2i} \cos\left\{\sqrt{\frac{t_s}{t}}\lambda_i s\right\} \qquad \ldots\ldots \quad (59)$$

The transcendental character of the secular equation in λ is now apparent.

The effect on the analysis of additional discontinuities in t, t_s and/or ψ, or the presence of concentrated members (booms, flanges, stringers) will next be discussed. Such members are assumed to be concentrated along generators.

The variation with r of the areas of the booms is conditioned by equation (40) which yields,

$$B' = \rho t_s^* B \qquad \ldots\ldots \quad (60)$$

For each additional discontinuity there will be a separate solution of equation (45) with two additional unknowns of the type C_1 and C_2. But each discontinuity provides two extra boundary conditions. Thus from the equilibrium condition at a boom,

$$q(s)_- - q(s)_+ = \frac{d}{dr}(B'\sigma(s)) \qquad \ldots\ldots \quad (61)$$

The suffices + and − indicate the s- increasing and s- decreasing sides at the point s. With equations (23) and (46) equation (61) becomes,

$$h(s)_+ - h(s)_- = B\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{s+} = B\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{s-} \quad . \quad . \quad . \quad . \quad (62)$$

Equations (62) reduce to equations (55) when $B=0$.

It is evident that in any single-cell tube with $n>1$ discontinuities the number of unknown coefficients of the type C_1 and C_2 is $2n$, so that in all there will be $2n+3$ unknowns. But there are $2n$ homogeneous equations of the form (62) and three of the form (54), that is $2n+3$ equations. Thus the arguments leading to the secular equation in λ_i and the existence of the eigenfunctions h_i apply exactly as before.

To complete the present section it is necessary only to discuss the boundary conditions at the junction of three walls as occurs in a multi-cell tube.

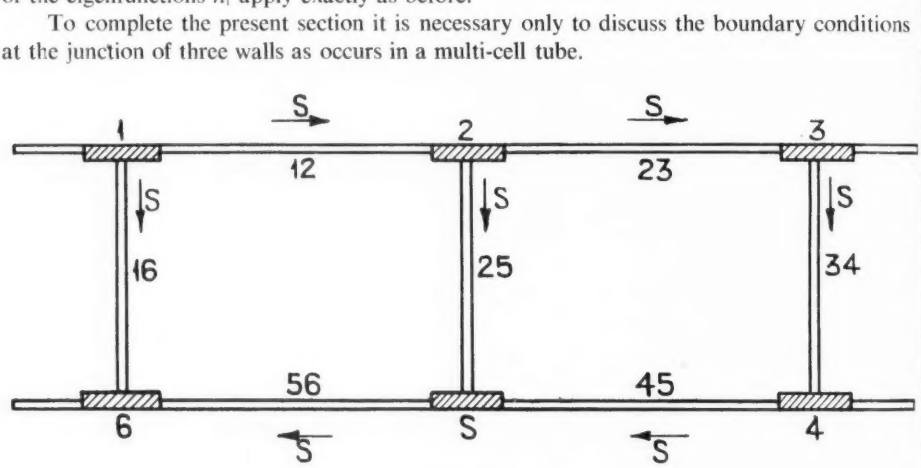

Fig. 4
Multi-cell tube with booms.

In Fig. 4 two adjoining cells are shown. The walls (12), (23) and (25) meet in a boom 2 with an area B_2 and the directions in which s is considered positive are indicated by arrows adjacent to each wall. The equilibrium conditions at 2 yield the three equations,

$$h_{2+} + h_{2d} - h_{2-} = B_2\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2+} = B_2\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2d} = B_2\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2-} \quad . \quad . \quad (63)$$

where h_{2d} and $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2d}$ are obtained by applying the solution of equation (45) over the wall (25) at the point 2. The index d indicates the downward direction at 2, *i.e.*, the wall (25).

If there is no boom at 2 equations (63) reduce to,

$$\left.\begin{aligned} h_{2+} + h_{2d} - h_{2-} &= 0 \\ \left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2+} &= \left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2d} = \left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2-} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (63a)$$

Equations (62), (63) and (63a) prove that the sum of the first two terms on the R.H.S. of equation (50) is in fact zero.

In multi-cell tubes of the type illustrated in Fig. 2 the number W of walls is connected with the number J of junctions by the relation,

$$3J = 2W$$

Since to each junction there corresponds three equations of the type (63) and for each wall,

assumed continuous, there are two unknowns of the type C_1 and C_2, it follows that the number of equations is,

$$3J+3$$

and the number of unknowns is

$$2W+3$$

Thus, again, there are as many homogeneous linear equations as unknowns. A similar argument will be found to apply in the case of any multi-cell tube whatever, with any number of additional discontinuities. The derivation of the secular equation follows similarly in all cases.

The solution of equation (45) takes a particularly simple form in the practically important case where the walls of the tube are straight and of constant thicknesses as in box beams and polygonal tubes. Adding the particular integral for $\hat{p}$ constant to the complementary function given in equation (59) one obtains,

$$h_1=C_1\sin\left(\sqrt{\frac{t}{t_s}}\lambda_1 s\right)+C_2\cos\left(\sqrt{\frac{t}{t_s}}\lambda_1 s\right)+\frac{\hat{p}_1 t}{2A}\oint_S\frac{h_1}{t}\,ds \qquad . \quad . \quad (59a)$$

3.2.2. *The special case where one or more walls are effective only in resisting shear loads.*

The special case to be considered in this section is an idealisation which is of great practical use. The device of treating certain or all of the walls of a tube as effective only in shear, *i.e.*, $t_s=0$, has been much applied by previous authors. A very great simplification arises when t_s is zero throughout, since in this case the secular equation in λ^2 is algebraic with a finite number of roots.

When t_s is zero over a wall the shear flow and hence h must be constant and the differential equation (45) becomes meaningless. However, integrating equation (45) over the wall (12) (Fig. 5),

$$\left(\frac{1}{t_s}\frac{dh}{ds}\right)_2-\left(\frac{1}{t_s}\frac{dh}{ds}\right)_1+\lambda^2\int_1^2\frac{h}{t}\,ds=\frac{\lambda^2}{2A}\left\{2A_{12}-\xi(y_2-y_1)+\eta(x_2-x_1)\right\}\oint\frac{h}{t}\,ds \qquad (64)$$

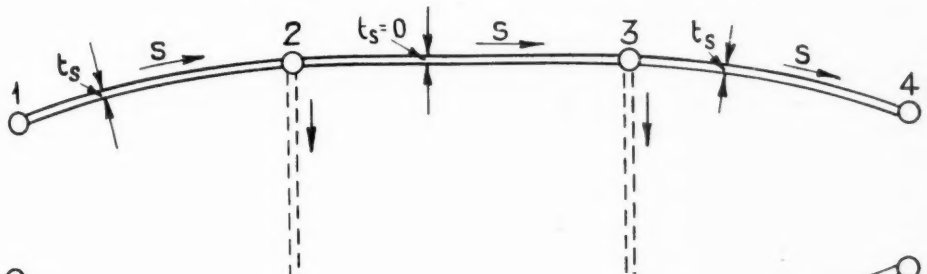

Fig. 5.
Tube with walls effective only in shear.

The expressions $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{1,2}$ are proportional to the direct stresses at points 1 and 2 and have definite values even if t_s vanishes. Assuming in Fig. 5 that t_s is zero over the wall (2, 3)

and that t is constant over this wall, the corresponding equation (64) becomes,

$$\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{3+} - \left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2-} + \lambda^2 \frac{s_{23}}{t_{23}} h_{23} = \frac{\lambda^2}{2A} \{ 2A_{23} - \xi (y_3 - y_2) + \eta (x_3 - x_2) \} \oint_S \frac{h}{t} ds \quad (65)$$

The value of $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{3+}$ is obtained from the solution of the differential equation (45) over the wall (34) where t_s is assumed finite; similarly for $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2-}$.

If there are booms at the points 2 and 3 equations (62) may be used to obtain the following modified form of equation (65),

$$\frac{1}{B_3}(h_{3+} - h_{23}) - \frac{1}{B_2}(h_{23} - h_{2-}) + \lambda^2 \frac{s_{23}}{t_{23}} h_{23} = \frac{\lambda^2}{2A} \left\{ 2A_{23} - \xi (y_3 - y_2) + \eta (x_3 - x_2) \right\} \oint_S \frac{h}{t} ds \quad (65a)$$

If t_s in the walls (12) and (34) is also zero h_{3+} and h_{2-} may be replaced in equation (65a) by the constant values h_{34} and h_{12} respectively. Thus one obtains,

$$\frac{1}{B_3}(h_{34} - h_{12}) - \frac{1}{B_2}(h_{23} - h_{12}) + \lambda^2 \frac{s_{23}}{t_{23}} h_{23} = \frac{\lambda^2}{2A} \{ 2A_{23} - \xi (y_3 - y_2) + \eta (x_3 - x_2) \} \oint_S \frac{h}{t} ds \quad (66)$$

In a single-cell tube with n booms and walls effective only in shear, there are n equations of the type (66) and the three equilibrium conditions of equations (54) which may now be written,

$$\left.\begin{aligned} \sum_1^n A_{m,\,m+1}\ h_{m,\,m+1} &= 0 \\ \sum_1^n (x_{m+1} - x_m)\, h_{m,\,m+1} &= \sum_1^n (y_{m+1} - y_m)\, h_{m,\,m+1} = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (67)$$

Whereas in the case where $t_s \neq 0$ there were two unknowns C_1 and C_2 for each wall, in the present case there is only one unknown, namely, the value $h_{m,\,m+1}$ itself. Thus the number of unknowns is $n+3$ which is, as before, equal to the number of equations. The argument when some of the walls have finite values of t_s is similar.

Rewriting equation (66) in its general form for an n-boom single-cell tube with $t_s = 0$ throughout,

$$\frac{1}{B_m} h_{m-1,\,m} + \left(\lambda^2 \frac{s_{m,\,m+1}}{t_{m,\,m+1}} - \frac{1}{B_m} - \frac{1}{B_{m+1}} \right) h_{m,\,m+1} + \frac{1}{B_{m+1}} h_{m+1,\,m+2}$$
$$= \frac{\lambda^2}{2A} \left\{ 2A_{m,\,m+1} - \xi (y_{m+1} - y_m) + \eta (x_{m+1} - x_m) \right\} \sum_S \frac{s_{m,\,m+1}}{t_{m,\,m+1}} h_{m,\,m+1} \quad . \quad . \quad (66a)$$

The corresponding secular equation follows at once from equations (66a) and (67) and takes the form (see facing page):—

It will be seen that the position of the origin C and the directions of the axes X, Y occur only in the last three rows and columns. For any other origin C and directions of axes a new set of values $A_{i,i+1}$, $x_{i+1} - x_i$, $y_{i+1} - y_i$ will be obtained. But these latter values may be obtained by linear combinations of the original values. Hence the determinant in equation (68) is invariant with the origin C as was found in the general case also.

Since the determinant in equation (68) is symmetrical all the roots λ^2 of the secular equation are real and positive. This equation is of the $(n-3)$th degree in λ^2 so that there are $(n-3)$ positive eigenvalues λ. Thus in the four-boom tube there is only one eigenvalue. Physical reasoning confirms this conclusion, as only one self-equilibrating end load system and one degree of freedom in warping can exist in a four-boom tube. This important case is considered in detail in Part 4.3.

$$\Delta = \begin{vmatrix}
\left[\lambda^2 \frac{s_{12}}{t_{12}} - \frac{1}{B_1} - \frac{1}{B_2}\right] & \frac{1}{B_2} & 0 & 0 & 0 & \frac{1}{B_1} & A_{12} & (x_2 - x_1) & (y_2 - y_1) \\
\frac{1}{B_2} & \left[\lambda^2 \frac{s_{23}}{t_{23}} - \frac{1}{B_2} - \frac{1}{B_3}\right] & \frac{1}{B_3} & 0 & 0 & 0 & A_{23} & (x_3 - x_2) & (y_3 - y_2) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \frac{1}{B_m} & \left[\lambda^2 \frac{s_{m,\,m+1}}{t_{m,\,m+1}} - \frac{1}{B_m} - \frac{1}{B_{m+1}}\right] & \frac{1}{B_{m+1}} & 0 & A_{m,\,m+1} & (x_{m+1} - x_m) & (y_{m+1} - y_m) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \frac{1}{B_{n-1}} & \left[\lambda^2 \frac{s_{n-1,\,n}}{t_{n-1,\,n}} - \frac{1}{B_{n-1}} - \frac{1}{B_n}\right] & \frac{1}{B_n} & A_{n-1,\,n} & (x_n - x_{n-1}) & (y_n - y_{n-1}) \\
\frac{1}{B_1} & 0 & 0 & 0 & \frac{1}{B_n} & \left[\lambda^2 \frac{s_{n,\,1}}{t_{n,\,1}} - \frac{1}{B_n} - \frac{1}{B_1}\right] & A_{n,\,1} & (x_1 - x_n) & (y_1 - y_n) \\
A_{12} & A_{23} - - - - - - - A_{m,\,m+1} & & A_{n-1,\,n} & A_{n,\,1} & 0 & 0 & 0 & \\
(x_2 - x_1) & (x_3 - x_2) - - - - - - - (x_{m+1} - x_m) & & (x_n - x_{n-1}) & (x_1 - x_n) & 0 & 0 & 0 & \\
(y_2 - y_1) & (y_3 - y_2) - - - - - - - (y_{m+1} - y_m) & & (y_n - y_{n-1}) & (y_1 - y_n) & 0 & 0 & 0 &
\end{vmatrix} = 0 \quad . \quad (68)$$

From the determinant of equation (68) the ratios,

$$h_{12} : h_{23} : \ldots : h_{n,1} : \sum_S \frac{s_{m,m+1}}{t_{m,m+1}} h_{m,m+1} : \xi \sum_S \frac{s_{m,m+1}}{t_{m,m+1}} h_{m,m+1} : \eta \sum_S \frac{s_{m,m+1}}{t_{m,m+1}} h_{m,m+1} \quad (69)$$

can be determined for each eigenvalue λ.

Introducing again the relation (57), which now takes the form,

$$\sum \frac{s_{m,m+1}}{t_{m,m+1}} h^2_{m,m+1} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

the h-functions take definite values.

The case of a multi-cell tube with one or more walls where $t_s = 0$ will now be investigated. It is assumed in Fig. 5 that t_s is zero over the wall (2, 3) and that two additional walls (2, 6) and (3, 5) with finite values of t_s are provided. It can immediately be seen that the integrated equations (64) and (65) apply also in the present case. By virtue of equations (63a) it is possible to substitute in equation (65) the values $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{3d}$ and $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2d}$ for the values $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{3+}$ and $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2-}$ respectively. The term $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{3d}$ is obtained by applying the solution of the differential equation (45) over the wall (3, 5) at the point 3. The index d indicates the downward direction at the point 3, *i.e.*, the wall (3, 5). A similar argument applies for the term $\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{2d}$.

If there are booms B_2 and B_3 at the points 2 and 3 equations (63) may be applied in equation (65) to obtain,

$$\frac{1}{B_3}(h_{3+} + h_{3d} - h_{23}) - \frac{1}{B_2}(h_{23} + h_{2d} - h_{2-}) + \lambda^2 \frac{s_{23}}{t_{23}} h_{23}$$
$$= \frac{\lambda^2}{2A}\left\{2A_{23} - \xi(y_3 - y_2) + \eta(x_3 - x_2)\right\} \oint_S \frac{h}{t}\, ds \quad . \quad . \quad . \quad (71)$$

If t_s is zero also in the walls (1, 2), (3, 4), (3,5), and (2,6) the values h_{3+}, h_{3d}, h_{2d}, and h_{2-} may be replaced by the constant values h_{12}, h_{34}, h_{35} and h_{26} respectively. Hence one obtains in this case,

$$\frac{1}{B_3}(h_{34} + h_{35} - h_{23}) - \frac{1}{B_2}(h_{23} + h_{26} - h_{12}) + \lambda^2 \frac{s_{23}}{t_{23}} h_{23}$$
$$= \frac{\lambda^2}{2A}\left\{2A_{23} - \xi(y_3 - y_2) + \eta(x_3 - x_2)\right\} \oint_S \frac{h}{t}\, ds \quad . \quad . \quad . \quad (72)$$

The equilibrium conditions (67) have to be taken over all the walls.

From equations (67) and (72) the secular equation for a multi-cell tube with n booms and $t_s = 0$ throughout can easily be derived. The determinant is again symmetrical and the resulting algebraic equation is of the $(n-3)$th degree in λ^2 —as in the n-boom single-cell tube. No further discussion of the multi-cell tube is necessary here as the analysis proceeds exactly as in the single-cell case. In certain cases the multi-cell tube may be reduced to an equivalent single-cell tube (see 4.3.2.).

3.2.3. *The orthogonal properties of the h-functions, the normalised h-functions.*

The importance of the h-functions lies in the fact that they have orthogonal properties, which allow their use in a "generalised" Fourier series representing any self-equilibrating end load system. This series will in general contain a term corresponding to each eigenvalue.

The differential equations (45) corresponding to two distinct eigenvalues λ_i and λ_j are,

$$\left.\begin{aligned}\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)+\lambda_i^2\left\{\frac{h_i}{t}-\frac{\hat{p}_i}{2A}\oint_s\frac{h_i}{t}ds\right\}=0\\ \frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_j}{ds}\right)+\lambda_j^2\left\{\frac{h_j}{t}-\frac{\hat{p}_j}{2A}\oint_s\frac{h_j}{t}ds\right\}=0\end{aligned}\right| \quad . \quad . \quad . \quad . \quad . \quad (73)$$

Multiplying the first equation by h_j and the second by h_i, subtracting and integrating over all walls one obtains,

$$\oint h_j\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)ds-\oint h_i\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_j}{ds}\right)ds+(\lambda_i^2-\lambda_j^2)\oint\frac{h_ih_j}{t}ds$$
$$-\lambda_i^2\frac{1}{2A}\oint\hat{p}_ih_jds\oint_s\frac{h_i}{t}ds+\lambda_j^2\frac{1}{2A}\oint\hat{p}_jh_ids\oint_s\frac{h_j}{t}ds=0. \quad . \quad . \quad . \quad (74)$$

Now,

$$\oint p_ih_jds=\oint\hat{p}_jh_ids=0$$

since the h-functions are for self-equilibrating systems (see equation (54)).

The first integral in equation (74) becomes by partial integration and arguments similar to those used to derive equation (50a),

$$\oint h_j\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)ds=-\oint\frac{1}{t_s}\frac{dh_i}{ds}\frac{dh_j}{ds}ds \quad . \quad . \quad . \quad . \quad (75a)$$

and similarly for the second integral,

$$-\oint h_i\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_j}{ds}\right)ds=+\oint\frac{1}{t_s}\frac{dh_i}{ds}\frac{dh_j}{ds}ds \quad . \quad . \quad . \quad . \quad (75b)$$

Hence the sum of the first two terms in equation (74) is zero and one obtains,

$$(\lambda_j^2-\lambda_i^2)\oint\frac{h_ih_j}{t}ds=0$$

and since $\lambda_j\neq\lambda_i$,

$$\oint\frac{h_ih_j}{t}ds=0, \text{ when } i\neq j \quad . \quad . \quad . \quad . \quad . \quad (76)$$

Equation (76) shows that the eigenfunctions h_i and h_j are orthogonal with the weight function $\frac{1}{t}$.

Multiplying the first of equations (73) by h_j and integrating over all walls, one obtains by virtue of relations (54) and (76),

$$\oint h_j\frac{a}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)ds=0$$

Hence from equation (75a),

$$\oint\frac{1}{t_s}\frac{dh_i}{ds}\frac{dh_j}{ds}ds=0, \text{ when } i\neq j \quad . \quad . \quad . \quad . \quad . \quad (77)$$

Thus the derivatives of the eigenfunctions h_i and h_j are also orthogonal with the weight function $\frac{1}{t_s}$.

For the n-boom single-cell tube with walls effective only in shear, the orthogonality relations (76) and (77) become,

$$\sum_{m=1}^{n}\frac{s_{m,\,m+1}}{t_{m,\,m+1}}(h_i)_{m,\,m+1}(h_j)_{m,\,m+1}=0 \quad . \quad . \quad . \quad . \quad (76a)$$

and,

$$\sum_{m=1}^{n} \frac{1}{B_m} [(h_i)_{m,\,m+1} - (h_i)_{m-1,\,m}] [(h_j)_{m,\,m+1} - (h_j)_{m-1,\,m}] = 0 \quad . \quad . \quad (77a)$$

Relations corresponding to these equations may be written down for the *n*-boom multi-cell tube.

It is convenient to normalise the *h*-functions by means of the relation,

$$\oint \frac{h_i^2}{t} ds = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

or for the *n*-boom tube.

$$\sum \frac{s_{m,\,m+1}}{t_{m,\,m+1}} (h_i)^2_{m,\,m+1} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

Multiplying the first of equations (73) by h_i, integrating over all walls and applying equation (57),

$$\oint \frac{1}{t_s} \left(\frac{dh_i}{ds}\right)^2 ds = \lambda_i^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

Again, for the *n*-boom single-cell tube,

$$\sum_{m=1}^{n} \frac{1}{B_m} [(h_i)_{m,\,m+1} - (h_i)_{m-1,\,m}]^2 = \lambda_i^2 \quad . \quad . \quad . \quad . \quad . \quad (78a)$$

In all that follows, unless otherwise stated, the *h*-functions are to be assumed normalised by the conditions (57) or (70).

3.2.4. *Discussion of the differential equation in g.*

The differential equation in g as derived in 3.2 is

$$r^2 \frac{d^2g}{dr^2} + r\left(1 - \frac{r}{t^*}\frac{dt^*}{dr}\right)\frac{dg}{dr} - \mu^2 r_0^2 \frac{t^*}{t_s^*} g = 0 \quad . \quad . \quad . \quad . \quad (43)$$

The general solution of this equation may be written,

$$g = D_1 \Psi_1(r) + D_2 \Psi_2(r) \quad . \quad . \quad . \quad . \quad . \quad (79)$$

where D_1, D_2 are constants and Ψ_1, Ψ_2 are functions of r.

To each eigenvalue λ_i there will correspond a solution g_i. The constants D_{1i} and D_{2i} of the g_i function will be determined by the boundary conditions for stress and/or displacement at the two ends of the tube. From equation (46) it will be seen that a specified direct stress at an end gives the value of g at that end. Similarly it will be seen in 3.2.5. that specified warping displacements at an end give the value of $\frac{dg}{dr}$ at that end.

In particular, for a completely free end,

$$g = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

and for a fully built-in end,

$$\frac{dg}{dr} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

When t^* and t_s^* are arbitrary functions equations (43) cannot be solved in terms of known functions but it will always be possible to obtain approximate solutions in series or numerical solutions.

The present analysis will be restricted to forms of the functions t^*, t_s^* which permit solutions in terms of polynomial, hyperbolic, or Bessel functions.

A fairly general mode of variation in skin thickness is represented by the equations,

$$\left.\begin{aligned} t^* &= \rho^\alpha \\ t_s^* &= \rho^\beta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

where α and β may have any positive or negative values.

By equation (60) the boom areas will be,

$$B' = \rho^{1+\beta} B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82a)$$

Substituting (82) in equation (43).

$$r^2 \frac{d^2g}{dr^2} + (1-\alpha) r \frac{dg}{dr} - \mu^2 r_o^2 \rho^{\alpha-\beta} g = 0 \quad . \quad . \quad . \quad . \quad . \quad (83)$$

The general solution of this equation may be written,

$$g = r^{\alpha/2} \left\{ D_1 I_{\frac{\alpha}{\alpha-\beta}} \left[\frac{2\mu r_o}{\alpha-\beta} \rho^{(\alpha-\beta)/2} \right] + D_2 K_{\frac{\alpha}{\alpha-\beta}} \left[\frac{2\mu r_o}{\alpha-\beta} \rho^{(\alpha-\beta)/2} \right] \right\} \quad . \quad . \quad (84)$$

where I and K are the modified Bessel functions of the first and second kinds and order $\alpha/(\alpha-\beta)$.

When $\alpha=\beta$ both the order and the argument of the Bessel functions become infinite and solution (84) cannot be used. However, in this case equation (83) reduces to,

$$r^2 \frac{d^2g}{dr^2} + (1-\alpha) r \frac{dg}{dr} - \mu^2 r_o^2 g = 0 \quad . \quad . \quad . \quad . \quad . \quad (85)$$

This homogeneous linear differential equation has a simple polynomial solution,

$$g = r^{\alpha/2} [D_1 r^\nu + D_2 r^{-\nu}] \quad . \quad . \quad . \quad . \quad . \quad (86)$$

where

$$\nu = \frac{1}{2} [\alpha^2 + 4\mu^2 r_o^2]^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (86a)$$

When the skin thicknesses are constant along the length, *i.e.*, when $\alpha=\beta=0$, solution (86) reduces to,

$$g = D_1 r^{\mu r_o} + D_2 r^{-\mu r_o} \quad . \quad . \quad . \quad . \quad . \quad (87)$$

For linear taper of the skin thicknesses, *i.e.*, when $\alpha=\beta=1$, the second term on the L.H.S. of equation (85) vanishes, and ν becomes,

$$\nu = \frac{1}{2} [1 + 4\mu^2 r_o^2]^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (86b)$$

Special consideration of equation (43) is necessary when the tube is cylindrical, *i.e.*, when $r_o \to \infty$. In this case equation (43) becomes,

$$\frac{d^2g}{dz'^2} - \frac{1}{t^*} \frac{dt^*}{dz'} \frac{dg}{dz'} - \mu^2 \frac{t^*}{t_s^*} g = 0 \quad . \quad . \quad . \quad . \quad (43a)$$

where t^* and t_s^* are now functions of z'/c.

When the skin thicknesses are constant, *i.e.*, when $t^* = t_s^* = 1$ equation (43a) reduces to,

$$\frac{d^2g}{dz'^2} - \mu^2 g = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43b)$$

Note that the same equation may be obtained by taking $r_o \to \infty$ in equations (82) and (83). The solution of equation (43b) is,

$$g = D_1 \cosh \mu z' + D_2 \sinh \mu z' \quad . \quad . \quad . \quad . \quad (88)$$

When t^* and t_s^* are arbitrary functions equation (43a) can always be solved numerically or approximately in series. A fairly general thickness variation is obtained by taking,

$$\left.\begin{aligned} t^* &= \left(\frac{z'+d}{c}\right)^\alpha \\ t_s^* &= \left(\frac{z'+d}{c}\right)^\beta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

where c is the tube length and d is a constant.

Substituting equations (89) in (43a),

$$\frac{d^2g}{dz'^2} - \frac{\alpha}{z'+d}\frac{dg}{dz'} - \mu^2\left(\frac{z'+d}{c}\right)^{\alpha-\beta} g=0 \quad . \quad . \quad . \quad . \quad (90)$$

The general solution of equation (90) may be written,

$$g=(z'+d)^{(1+\alpha)/2}\left\{ D_1 I_\kappa\left[\frac{2\mu(z'+d)}{2+\alpha-\beta}\left(\frac{z'+d}{c}\right)^{(\alpha-\beta)/2}\right] + D_2 K_\kappa\left[\frac{2\mu(z'+d)}{2+\alpha-\beta}\left(\frac{z'+d}{c}\right)^{(\alpha-\beta)/2}\right]\right\} \quad . \quad . \quad . \quad (91)$$

where $\kappa=(1+\alpha)/(2+\alpha-\beta)$ and I_κ and K_κ are the modified Bessel functions.

Solution (91) takes a simpler form for the particular case $\alpha=\beta$. In this case,

$$g=(z'+d)^{(1+\alpha)/2}\{ D_1 I_{(1+\alpha)/2}[\mu(z'+d)]+D_2 K_{(1+\alpha)/2}[\mu(z'+d)]\} \quad . \quad (92)$$

For linear taper of t' and t_s', $\alpha=\beta=1$ and equation (92) takes the particularly simple form,

$$g=(z'+d)\{ D_1 I_1[\mu(z'+d)]+D_2 K_1[\mu(z'+d)]\} \quad . \quad . \quad (93)$$

3.2.5. *Expression for warping due to a self-equilibrating load system.*

Using equations (28) and (30) in equation (26) one obtains for the displacement w due to a stress system specified by $F=gh$.

$$\frac{\partial w}{\partial s} = -\frac{1}{Gt'}\rho\frac{dg}{dr}h-(p-\xi\cos\psi+\eta\sin\psi)\rho^2\frac{d\theta}{dr}+(\xi\cos\psi-\eta\sin\psi)\rho\frac{\theta}{r_o}$$

With equations (34), (36) and (38) the last expression becomes,

$$\frac{\partial w}{\partial s} = -\frac{1}{Gt^*}\rho\frac{dg}{dr}\left\{\frac{h}{t}-\frac{\dot{p}}{2A}\oint_S\frac{h}{t}ds\right\}+(\xi\cos\psi-\eta\sin\psi)\rho\frac{\theta}{r_o}$$

For a self-equilibrating eigenload system, defined by $F=gh$, the last relation, by virtue of the differential equation (45), reduces to,

$$w=\frac{1}{Gt^*}\rho\frac{1}{\lambda^2}\frac{dg}{dr}\left(\frac{1}{t_s}\frac{dh}{ds}\right)+(\xi y-\eta x)\rho\frac{\theta}{r_o}+\text{function of } r \quad . \quad . \quad (94)$$

The last two terms represent rigid body displacements and hence the warping w^* is given by the first term of equation (94). Thus the warping w_i^* due to a self-equilibrating end load system, specified by $F=g_ih_i$, becomes,

$$w_i^*=\frac{1}{Gt^*}\rho\frac{1}{\lambda_i^2}\frac{dg_i}{dr}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right) \quad . \quad . \quad . \quad . \quad . \quad (95)$$

The same expression can be derived, using the warping relation (16). It follows from equation (95) that the warping w_i^* is proportional to $\frac{dg_i}{dr}$; this relation is used when the boundary conditions are given in terms of specified warping discussed in 3.2.4.

In a single-cell tube with booms, and walls effective only in shear, the warping at the boom B_m may be written down from equation (95) as,

$$(w_i^*)_m=\frac{1}{Gt^*}\rho\frac{1}{\lambda_i^2}\frac{dg_i}{dr}\frac{1}{B_m}[(h_i)_{m,\,m+1}-(h_i)_{m-1,\,m}] \quad . \quad . \quad (96)$$

The appropriate formulæ for multi-cell tubes can easily be derived.

It will be noticed that since $\frac{1}{t_s}\frac{dh_i}{ds}$ is proportional to the direct stress so also is the warping w_i^*. In fact this proportionality of stress and warping is a fundamental and unique property of the eigenloads.

Williams and Fine, 1945[8], investigated the stress distribution at the built-in end of a cylindrical tube under torque on the assumption that the direct stresses at the root were proportional to the warping that would occur if there were no axial constraint. It can be seen from the above analysis that this assumption would be justified only if the free warping could be approximately nullified by just one eigenload system. This will be in general untrue, but for a four-boom tube, which has only one eigenload, the method is exact.

When $t^* = \rho^n$ equation (95) becomes,

$$w_i^* = \frac{1}{G} \rho^{1-n} \frac{1}{\lambda_i^2} \frac{dg_i}{dr} \left(\frac{1}{t_s} \frac{dh_i}{ds} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (95a)$$

For a cylindrical tube equation (95) reduces to,

$$w_i^* = \frac{1}{Gt^*} \frac{1}{\lambda_i^2} \frac{dg_i}{dz'} \left(\frac{1}{t_s} \frac{dh_i}{ds} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (95b)$$

It is sometimes useful to have an expression for the derivative of the warping with respect to s. From the above analysis,

$$\frac{\partial w_i^*}{\partial s} = - \frac{1}{Gt^*} \rho \frac{dg_i}{dr} \left\{ \frac{h_i}{t} - \frac{\hat{p}_i}{2A} \oint_S \frac{h_i}{t} ds \right\} \quad . \quad . \quad . \quad . \quad . \quad (95c)$$

Note that equation (95c) is applicable for non-self-equilibrating load systems, defined by a stress function $F = gh$.

4. GENERAL ANALYSIS FOR A CONICAL OR CYLINDRICAL TUBE UNDER ARBITRARY TORQUE DISTRIBUTION.

4.1. Concentrated torque applied at any section of a built-in tube.

In this section the case of a concentrated torque applied at any section of a built-in tube is considered. Any case of distributed torque can be obtained by integration of the single torque solution. The analysis falls naturally into two parts.

(a) the deduction of a stress-system for a constant torque, irrespective of the end conditions and the position of the applied torque.

(b) the deduction of the self-equilibrating systems necessary to satisfy the boundary conditions at the ends of the tube and at the section where the torque is applied (see Fig. 6). The latter load systems are combinations of the eigenloads investigated in 3.2.

4.1.1. *A particular stress-function for constant torque.*

If a solution in the form $F = gh$ is feasible for a constant torque T, it follows from equation (47),

$$- \rho^2 \frac{dg}{dr} \oint hpds = T \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

that $\rho^2 \frac{dg}{dr}$ must be constant, so a possible solution for g is of the form,

$$g \propto \frac{1}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

It is convenient to assume that,

$$g = \frac{r_o^2}{r} \frac{T}{2A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

where A is the total area of the root cross-section.

With (98) equation (47) becomes,

$$\oint hpds = 2A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

The shear force resultants are given by equations (51), and substitution of equation (98) into (51) shows that the conditions of zero shear resultants are automatically satisfied. However, the conditions,

$$\oint h \sin \psi ds = \oint h \cos \psi ds = 0 \quad . \quad . \quad . \quad . \quad . \quad (100)$$

have still to be applied in order to satisfy the conditions of zero moments (see equations (52)).

The differential equation for a gh stress system is given by equation (37) of 3.2. Substituting into the latter for g, equation (98), and for t' and t'_s relations (38) and (40), and dividing by t/r one obtains,

$$\frac{Gt^*}{Et_s^*}\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh}{ds}\right)+\frac{1}{r_o^2}\left(1+\frac{r}{t^*}\frac{dt^*}{dr}\right)\left\{\frac{h}{t}-\frac{\hat{p}}{2A}\oint_S \frac{h}{t}\,ds\right\}=0 \quad . \quad . \quad (101)$$

Since equation (101) must be independent of r, it follows that a constant torque solution in the form gh is only possible if,

$$t^* = t_s^* = \rho^\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

With equation (102) relation (101) becomes,

$$\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh}{ds}\right)+\frac{1+\alpha}{r_o^2}\frac{E}{G}\left\{\frac{h}{t}-\frac{\hat{p}}{2A}\oint_S \frac{h}{t}\,ds\right\}=0 \quad . \quad . \quad . \quad (103)$$

Comparing (103) with (45) it can be seen that a solution of the form $g \propto \frac{1}{r}$ is only feasible if,

$$\frac{1+\alpha}{r_o^2}\frac{E}{G}$$

is not equal to an eigenvalue λ_i^2, since in this case the solution of (103) would be the self-equilibrating system h_i. The exceptional case when

$$\lambda_i^2 = \frac{1+\alpha}{r_o^2}\frac{E}{G} \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

will be excluded from the present analysis, as it has little physical importance. The interested reader may obtain the results for this case by taking the limit.

$$\lim \left\{\frac{1+\alpha}{r_o^2}\frac{E}{G}\right\} \to \lambda_i^2$$

in the final stress function formulæ, or *ab initio* by assuming $g \propto \frac{\ln r}{r}$ (see also 4.2.3.).

The special equation when one or more walls are effective only in shear may be obtained immediately from 3.2.2 by writing,

$$\omega^2 = \frac{1+\alpha}{r_o^2}\frac{E}{G} \quad . \quad . \quad . \quad . \quad . \quad . \quad (105)$$

in place of λ^2.

In particular rewriting equation (66a) with this substitution for an n-boom single-cell tube with $t_s = 0$ throughout, one obtains,

$$\frac{1}{B_m}h_{m-1,\,m}+\left(\omega^2\frac{s_{m,\,m+1}}{t_{m,\,m+1}}-\frac{1}{B_m}-\frac{1}{B_{m+1}}\right)h_{m,\,m+1}+\frac{1}{B_{m+1}}h_{m+1,\,m+2}$$

$$=\frac{\omega^2}{2A}\left\{2A_{m,\,m+1}-\xi(y_{m+1}-y_m)+\eta(x_{m+1}-x_m)\right\}\sum_S \frac{s_{m,\,m+1}}{t_{m,\,m+1}}h_{m,\,m+1} \quad . \quad (106)$$

The equilibrium conditions replacing equations (67) are,

$$\left.\begin{aligned} \sum_{1}^{n} A_{m,\,m+1} h_{m,\,m+1} &= A \\ \sum_{1}^{n} (x_{m+1} - x_m)\, h_{m,\,m+1} &= \sum_{1}^{n} (y_{m+1} - y_m)\, h_{m,\,m+1} = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (107)$$

A similar analysis may be used for the multi-cell tube.

The non-homogeneous character of the boundary condition (99) or (107) determines completely the constant torque h-function. When it is desirable to distinguish between the eigenvalue h-solutions and the constant torque h-solution the latter will be denoted by h_T.

Before considering the general solution of equations (103) or (106) it is convenient first to investigate the special cases when $r_o \to \infty$, *i.e.*, the cylindrical tube, or when $\alpha = -1$.

4.1.2. *The cylindrical tube of uniform cross-section and the conical tube with* $\alpha = -1$. *The generalised Bredt-Batho function* h_B.

In both these special cases the second term in the differential equation (103) vanishes. Thus equation (103) reduces to,

$$\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh}{ds}\right) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (103a)$$

Since $\frac{1}{t_s}\frac{dh}{ds}$ is proportional to the direct stress equation (103a) implies constant direct stress in all the walls which would be contrary to the condition of no resultant end load. Thus the direct stress would be zero throughout the tube so that

$$\frac{1}{t_s}\frac{dh}{ds} = 0, \text{ or } h = \text{constant}$$

in any one wall. For a single-cell tube the equilibrium condition (99) yields,

$$h = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

The complete stress function for the cylindrical tube is therefore (see equation (47a)),

$$F = -\frac{T}{2A} z' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

The stress system of (109) represents the ordinary Bredt-Batho distribution the shear flow being,

$$q = -\frac{\partial F}{\partial z'} = \frac{T}{2A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

The stress-function for the conical tube with $\alpha = -1$ becomes,

$$F = \frac{T}{2A}\frac{r_o^2}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

and the shear flow,

$$q = -\frac{\partial F}{\partial r} = -\frac{T}{2A'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

Thus the Bredt-Batho formula for tubes under constant torque is true only in the case of a cylindrical tube with constant thickness in the longitudinal direction, or in a conical tube in which the thickness varies inversely with the distance from the apex.

In an N-cell tube of the type shown in Fig.4 there are $3(N-1)$ separate walls and hence $3(N-1)$ unknown values of h.

The equations for h are considered under three headings.

(1) The equilibrium condition in torque,

$$\Sigma A_w h_w = A \qquad (113)$$

where the suffix w refers to all walls and A_w is the swept area corresponding to each wall.

(2) In taking $r_o \to \infty$ or $\alpha = -1$ in equation (103) the condition of identical rotation of all walls is lost and must be reintroduced. There are $N-1$ independent equations of the type of equation (35).

$$\frac{1}{2A_M} \sum_M \frac{s}{t} h = \frac{1}{2A} \sum_S \frac{s}{t} h \qquad (114)$$

where the suffix M refers to the Mth cell and $\sum_M \frac{s}{t} h$ is taken over the periphery of the Mth cell.

(3) There are $2N-3$ independent equations at the joints of the cells expressing the equilibrium of the shear flows. Thus in Fig. 5 the typical equation for joint 2 is,

$$h_{23} + h_{26} - h_{12} = 0 \qquad (115)$$

It will be found that the two equilibrium conditions for shear may be obtained from equations (115) and are therefore not additional conditions. They may of course be used in place of any two of equations (115). Thus in all there are always 3 $(N-1)$ independent equations for determining the 3 $(N-1)$ unknown values of h. These h's correspond to the generalised Bredt-Batho shear flows and will be denoted by h_B.

The warping w_B^* for a conical tube due to the Bredt-Batho stress system $g_B h_B = \frac{T}{2A} \frac{r_o^2}{r} h_B$ can be written by virtue of equation (95c),

$$\frac{\partial w_B^*}{\partial s} = -\frac{1}{Gt^*} \rho \frac{dg_B}{dr} \left\{ \frac{h_B}{t} - \frac{\hat{p}}{2A} \oint_S \frac{h_B}{t} ds \right\} \qquad (116)$$

or for $t^* = \rho^n$,

$$\frac{\partial w_B^*}{\partial s} = \frac{T}{2AG} \rho^{-(1+\alpha)} \left\{ \frac{h_B}{t} - \frac{\hat{p}}{2A} \oint_S \frac{h_B}{t} ds \right\} \qquad (116a)$$

and for $\alpha = -1$,

$$\frac{\partial w_B^*}{\partial s} = \frac{T}{2AG} \left\{ \frac{h_B}{t} - \frac{\hat{p}}{2A} \oint_S \frac{h_B}{t} ds \right\} \qquad (116b)$$

This latter formula is also valid for the cylindrical tube.

Integrating equation (116a) between 0 and s,

$$w_B^* = \frac{T}{2AG} \rho^{-(1+\alpha)} \left\{ \int_0^s \frac{h_B}{t} ds - \frac{A_{os}}{A} \oint_S \frac{h_B}{t} ds \right\} \qquad (117a)$$

Proceeding similarly in equation (116b),

$$w_B^* = \frac{T}{2AG} \left\{ \int_0^s \frac{h_B}{t} ds - \frac{A_{os}}{A} \oint_S \frac{h_B}{t} ds \right\} \qquad (117b)$$

The swept areas A_{os} can be taken from any origin.

An important relationship between the function h_B and the eigenfunction h_i will now be demonstrated. The function h_i satisfies the differential equation,

$$\frac{d}{ds}\left(\frac{1}{t_s} \frac{dh_i}{ds}\right) + \lambda_i^2 \left\{ \frac{h_i}{t} - \frac{\hat{p}_i}{2A} \oint_S \frac{h_i}{t} ds \right\} = 0 \qquad (45)$$

Multiplying equation (45) by h_B and integrating over all walls, one obtains, using (99) and (103a),

$$\oint \frac{h_i h_B}{t}\, ds = \oint_S \frac{h_i}{t}\, ds \qquad (118)$$

4.1.3. *Expression of the constant torque function h_T in terms of h_B and the eigenfunctions h_i.*

It will now be shown that the solution h_T of either equations (103) or (106) may be expressed in terms of the Bredt-Batho function h_B considered in 4.1.2 and the eigenfunctions h_i. By the definitions for the h_B and h_T functions the stress system $h_T - h_B$ will be self-equilibrating and hence one may write,

$$h_T = h_B + \sum_i a_i h_i \qquad (119)$$

where a_i are undetermined coefficients.

Substituting equation (119) into the differential equation (103) for h_T, one obtains,

$$\sum_i a_i \frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right) + \omega^2 \left\{ \frac{h_B}{t} - \frac{\hat{p}_T}{2A}\oint_S \frac{h_B}{t}\, ds \right\} + \omega^2 \sum_i a_i \left\{ \frac{h_i}{t} - \frac{\hat{p}_i}{2A}\oint_S \frac{h_T}{t} ds \right\} = 0 \qquad (120)$$

Multiplying equation (120) by h_i, integrating over all walls and using relations (54), (57), (76), (78) and (118),

$$-\lambda_i^2 a_i + \omega^2 \oint_S \frac{h_i}{t} ds + \omega^2 a_i = 0$$

or

$$a_i = \frac{\omega^2}{\lambda_i^2 - \omega^2}\oint_S \frac{h_i}{t} ds = \frac{(1+\alpha)}{\mu_i^2 r_0^2 - (1+\alpha)}\oint_S \frac{h_i}{t}\, ds \qquad (121)$$

With equation (121) formula (119) becomes,

$$h_T = h_B + \sum_i \frac{1+\alpha}{\mu_i^2 r_0^2 - (1+\alpha)}\, h_i \oint_S \frac{h_i}{t}\, ds \qquad (122)$$

From (119) it follows that the warping w_T^* associated with h_T can be expressed as follows,

$$w_T^* = w_B^* + \sum_i a_i w^*_{iB} \qquad (123)$$

where w^*_{iB} is the warping due to a stress system $g_B h_i$. Substituting into (123) equations (95) and (117) one obtains,

$$w_T^* = \frac{T}{2AG}\rho^{-(1+\alpha)} \left\{ \int_0^S \frac{h_B}{t} ds - \frac{A_{os}}{A}\oint_S \frac{h_B}{t}\, ds \right\}$$
$$- \frac{T}{2AG}\rho^{-(1+\alpha)} \sum_i \frac{\omega^2}{\lambda_i^2(\lambda_i^2 - \omega^2)}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)\oint_S \frac{h_i}{t} ds \qquad (124)$$

An alternative formula is,

$$\frac{\partial w_T^*}{\partial s} = \frac{T}{2AG}\rho^{-(1+\alpha)} \left\{ \frac{h_B}{t} - \frac{\hat{p}}{2A}\oint_S \frac{h_B}{t}\, ds \right\}$$
$$+ \frac{T}{2AG}\rho^{-(1+\alpha)} \sum_i \frac{\omega^2}{\lambda_i^2 - \omega^2} \left\{ \frac{h_i}{t} - \frac{\hat{p}_i}{2A}\oint_S \frac{h_i}{t} ds \right\} \oint_S \frac{h_i}{t}\, ds \qquad (124a)$$

4.1.4. *The complementary stress functions for the conical tube to satisfy the boundary conditions; the complete stress system.*

In this paragraph the second problem stated in 4.1 will be investigated.

Fig. 6 shows a built-in conical tube with a concentrated torque T applied at section $r=r_1$.

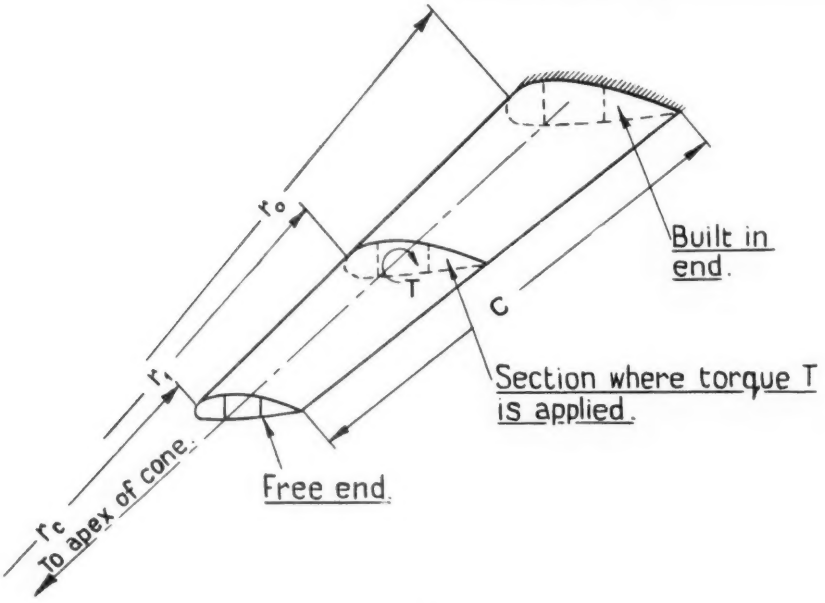

Fig. 6.
Built-in tube with concentrated torque at intermediate section.

Over the region between the tip and the loaded section, *i.e.*, for $r_1 \geqslant r \geqslant r_c$, the torque is zero and the stress function F' consists entirely of eigenloads.

Hence,

$$F' = \sum_i g_i' h_i \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (125a)$$

In the remaining part of the tube ($r_1 \leqslant r \leqslant r_o$) which is under constant torque T the stress function F may be written,

$$F = \sum_i g_i h_i + \frac{T}{2A} \frac{r_o^2}{r} h_T$$

or substituting equation (122) for h_T,

$$F = \sum \left(g_i + \frac{T}{2A} \frac{r_o^2}{r} \frac{(1+\alpha)}{\mu_1^2 r_o^2 - (1+\alpha)} \oint_S \frac{h_i}{t} ds \right) h_i + \frac{T}{2A} \frac{r_o^2}{r} h_B \qquad . \quad (125b)$$

The functions g_i and g_i' differ only in the constants D_1 and D_2 of equation (86).

The expressions for the stresses are,

for $r_c \leqslant r \leqslant r_1$,

$$\left.\begin{aligned} \sigma' &= \rho^{-(1+\alpha)} \sum_i g_i' \left(\frac{1}{t_s} \frac{dh_i}{ds} \right) \\ \tau' &= -\rho^{-\alpha} \sum_i \frac{dg_i'}{dr} \frac{h_i}{t} \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad . \quad (126a)$$

and for $r_1 \leqslant r \leqslant r_0$,

$$\left.\begin{aligned} \sigma &= \rho^{-(1+\alpha)} \sum_i \left(g_i + \frac{T r_0}{2A\rho} \frac{\omega^2}{\lambda_i^2 - \omega^2} \oint_S \frac{h_i}{t}\, ds \right) \left(\frac{1}{t_s} \frac{dh_i}{ds} \right) \\ \tau &= -\rho^{-\alpha} \sum_i \left(\frac{dg_i}{dr} - \frac{T}{2A\rho^2} \frac{\omega^2}{\lambda_i^2 - \omega^2} \oint_S \frac{h_i}{t}\, ds \right) \frac{h_i}{t} + \frac{T}{2A} \rho^{-(2+\alpha)} \frac{h_B}{t} \end{aligned}\right\} \quad . \quad . \quad (126b)$$

Using equations (95c) and (124a) the warping expression can be written,

for $r_0 \leqslant r \leqslant r_1$,

$$G \frac{\partial w^{*\prime}}{\partial s} = -\rho^{-(1+\alpha)} \sum_i \rho^2 \frac{dg_i'}{dr} \left\{ \frac{h_i}{t} - \frac{\hat{p}_i}{2A} \oint_S \frac{h_i}{t}\, ds \right\} \quad . \quad . \quad . \quad (127a)$$

and for $r_1 \leqslant r \leqslant r_0$,

$$\begin{aligned} G \frac{\partial w^*}{\partial s} = -\rho^{-(1+\alpha)} \sum_i & \left\{ \rho^2 \frac{dg_i}{dr} - \frac{T}{2A} \frac{\omega^2}{\lambda_i^2 - \omega^2} \oint_S \frac{h_i}{t}\, ds \right\} \left\{ \frac{h_i}{t} - \frac{\hat{p}_i}{2A} \oint_S \frac{h_i}{t}\, ds \right\} \\ & + \frac{T}{2A} \rho^{-(1+\alpha)} \left\{ \frac{h_B}{t} - \frac{\hat{p}}{2A} \oint_S \frac{h_B}{t}\, ds \right\} \quad . \quad . \quad . \quad . \quad (127b) \end{aligned}$$

The four boundary conditions are:—

$$\left.\begin{aligned} \sigma' &= 0 \qquad \text{for } r = r_0 \\ \sigma' &= \sigma \\ \frac{\partial w^{*\prime}}{\partial s} &= \frac{\partial w^*}{\partial s} \end{aligned}\right\} \text{ for } r = r_1$$

and

$$\frac{\partial w^*}{\partial s} = 0 \qquad \text{for } r = r_0$$

There are four constants D_1', D_2', D_1 and D_2 to be determined for each eigenvalue λ_i.

Expressing the first two boundary conditions in terms of the relations (126a) and (126b), multiplying both sides of the equation by $\frac{dh_i}{ds}$, integrating over all walls and using the orthogonal and normal properties given by equations (77) and (78) one obtains,

$$[g_i']_{r=r_0} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (129)$$

and

$$[g_i']_{r=r_1} - [g_i]_{r=r_1} = \frac{T r_0}{2A\rho_1} \frac{\omega^2}{\lambda_i^2 - \omega^2} \oint_S \frac{h_i}{t}\, ds \quad . \quad . \quad . \quad (130)$$

Expressing the last two boundary conditions in terms of the relations (127a) and (127b), multiplying both sides of the equations by h_i, integrating over all walls and using the orthogonal and normal properties given by equations (75) and (57) one obtains,

$$\left[\frac{dg_i'}{dr}\right]_{r=r_1} - \left[\frac{dg_i}{dr}\right]_{r=r_1} = -\frac{T}{2A\rho_1^2} \frac{\lambda_i^2}{\lambda_i^2 - \omega^2} \oint_S \frac{h_i}{t}\, ds \quad . \quad . \quad (131)$$

and

$$\left[\frac{dg_i}{dr}\right]_{r=r_0} = \frac{T}{2A} \frac{\lambda_i^2}{\lambda_i^2 - \omega^2} \oint_S \frac{h_i}{t}\, ds \quad . \quad . \quad . \quad (132)$$

Equations (129) to (132) are the necessary four relations for the calculation of the coefficients D_1', D_2', D_1 and D_2 appearing in the equations (86) and (86a) for the g-functions.

These equations are,

$$\left.\begin{aligned} g_1' &= r^{\alpha/2}\{D'_{1i}r^{\nu_i}+D'_{2i}r^{-\nu_i}\} \\ g_i &= r^{\alpha/2}\{D_{1i}r^{\nu_i}+D_{2i}r^{-\nu_i}\} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (133)$$

where $\nu_i = \frac{1}{2}[\alpha^2+4\mu_i{}^2r_o{}^2]^{1/2}$

Substituting relations (133) into the boundary conditions four simultaneous linear equations for the D-coefficients are obtained. The stress-functions may then be written,

$$F' = \frac{Tr_o}{2A}\rho^{\alpha/2}\sum_i(\bar{D}'_{1i}\rho^{\nu_i}+\bar{D}'_{2i}\rho^{-\nu_i})h_i\oint_S\frac{h_i}{t}\,ds \quad . \quad . \quad . \quad (134a)$$

and

$$F = \frac{Tr_o}{2A}\rho^{\alpha/2}\sum_i\left(\bar{D}_{1i}\rho^{\nu_i}+\bar{D}_{2i}\rho^{-\nu_i}+\frac{(1+\alpha)}{\mu_i{}^2r_o{}^2-(1+\alpha)}\rho^{-(\alpha+2)/2}\right)h_i\oint_S\frac{h_i}{t}\,ds+\frac{Tr_o}{2A\rho}h_B \quad . \quad (134b)$$

where,

$$\left.\begin{aligned} \bar{D}'_{1i} &= \frac{\mu^2r_o{}^2}{2\{\mu^2r_o{}^2-(1+\alpha)\}}\,\frac{\left(1+\frac{\alpha}{2}-\nu\right)\rho_1{}^{-\nu}-\left(1+\frac{\alpha}{2}+\nu\right)\rho_1{}^{\nu}+2\nu\rho_1{}^{(\alpha+2)/2}}{\nu\rho_1{}^{(\alpha+2)/2}\rho_c{}^{\nu}\left[\left(\frac{\alpha}{2}+\nu\right)\rho_c{}^{-\nu}-\left(\frac{\alpha}{2}-\nu\right)\rho_c{}^{\nu}\right]} \\ \bar{D}'_{2i} &= -\bar{D}'_{1i}\rho_c{}^{2\nu} \end{aligned}\right\} \quad . \quad . \quad (135a)$$

$$\bar{D}_{1i} = \frac{\mu^2r_o{}^2}{2\{\mu^2r_o{}^2-(1+\alpha)\}} \times \frac{\left(\frac{\alpha}{2}-\nu\right)\left(1+\frac{\alpha}{2}-\nu\right)(\rho_c/\rho_1)^{\nu}-\left(\frac{\alpha}{2}+\nu\right)\left(1+\frac{\alpha}{2}+\nu\right)(\rho_1/\rho_c)^{\nu}+2\nu\left(\frac{\alpha}{2}+\nu\right)\rho_1{}^{(\alpha+2)/2}\rho_c{}^{-\nu}}{\nu\left(\frac{\alpha}{2}+\nu\right)\rho_1{}^{(\alpha+2)/2}\left[\left(\frac{\alpha}{2}+\nu\right)\rho_c{}^{-\nu}-\left(\frac{\alpha}{2}-\nu\right)\rho_c{}^{\nu}\right]}$$

$$\bar{D}_{2i} = -\frac{\mu^2r_o{}^2}{2\{\mu^2r_o{}^2-(1+\alpha)\}} \times \frac{\left(\frac{\alpha}{2}-\nu\right)\left(1+\frac{\alpha}{2}-\nu\right)(\rho_c/\rho_1)^{\nu}-\left(\frac{\alpha}{2}+\nu\right)\left(1+\frac{\alpha}{2}+\nu\right)(\rho_1/\rho_c)^{\nu}+2\nu\left(\frac{\alpha}{2}-\nu\right)\rho_1{}^{(\alpha+2)/2}\rho_c{}^{\nu}}{\nu\left(\frac{\alpha}{2}-\nu\right)\rho_1{}^{(\alpha+2)/2}\left[\left(\frac{\alpha}{2}+\nu\right)\rho_c{}^{-\nu}-\left(\frac{\alpha}{2}-\nu\right)\rho_c{}^{\nu}\right]} \quad . \quad . \quad (135b)$$

The suffix i has been omitted from the symbols μ and ν for clarity in printing.

Relations (126a) and (126b) may now be written,

for $r_c \leqslant r \leqslant r_1$,

$$\left.\begin{aligned} \sigma' &= \frac{T}{2A}\sum_i G'_{\sigma i}H_{\sigma i} \\ \tau' &= \frac{T}{2A}\sum_i G'_{\tau i}H_{\tau i} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (136a)$$

and for $r_1 \leqslant r \leqslant r_o$,

$$\left.\begin{aligned} \sigma &= \frac{T}{2A}\sum_i G_{\sigma i}H_{\sigma i} \\ \tau &= \frac{T}{2A}\left[\rho^{-(\alpha+2)}\frac{h_B}{t}+\sum_i G_{\tau i}H_{\tau i}\right] \end{aligned}\right\} \quad . \quad . \quad . \quad (136b)$$

The G-functions are,

$$\left.\begin{aligned} G'_{\sigma i} &= \rho^{-(\alpha+2)/2}\,[\bar{D}'_{1i}\rho^{\nu_i} + \bar{D}'_{2i}\rho^{-\nu_i}]\mu_i r_o \\ G'_{\tau i} &= -\rho^{-(\alpha+2)/2}\left[\bar{D}'_{1i}\left(\frac{\alpha}{2}+\nu_i\right)\rho^{\nu_i} + \bar{D}'_{2i}\left(\frac{\alpha}{2}-\nu_i\right)\rho^{-\nu_i}\right] \end{aligned}\right\} \quad . \quad . \quad (137a)$$

$$G_{\sigma i} = \rho^{-(\alpha+2)/2}\left[\bar{D}_{1i}\rho^{\nu_i} + \bar{D}_{2i}\rho^{-\nu_i} + \frac{1+\alpha}{\mu_i^2 r_o^2 - (1+\alpha)}\rho^{-(\alpha+2)/2}\right]\mu_i r_o \quad . \quad . \quad (137b)$$

$$G_{\tau i} = -\rho^{-(\alpha+2)/2}\left[\bar{D}_{1i}\left(\frac{\alpha}{2}+\nu_i\right)\rho^{\nu_i} + \bar{D}_{2i}\left(\frac{\alpha}{2}-\nu_i\right)\rho^{-\nu_i} - \frac{1+\alpha}{\mu_i^2 r_o^2 - (1+\alpha)}\rho^{-(\alpha+2)/2}\right]$$

The H-functions are,

$$\left.\begin{aligned} H_{\sigma i} &= \mu_i^{-1}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)\oint_S \frac{h_i}{t}\,ds \\ H_{\tau i} &= \frac{h_i}{t}\oint_S \frac{h_i}{t}\,ds \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

It will be observed that the G- and H-functions are analogous to the g- and h-functions; in fact G_σ is proportional to g, while G_τ is proportional to $\frac{dg}{dr}$. Similarly H_σ is proportional to $\frac{dh}{ds}$, while H_τ is proportional to h. It should be observed that the G-functions have the same analytical form for all values of i. This is a great advantage for numerical work. The H-functions are independent of the loading and are in fact functions of the geometric and elastic properties of the root cross-section.

An interesting check on the formula for the shear stress is obtained by putting $\rho = 1$ in the second formula (136b) and the second formula (137b).

Then,

$$[G_{\tau i}]_{\rho=1} = -1$$

and the shear stress at the root is,

$$\tau = \frac{T}{2A}\left[\frac{h_B}{t} - \Sigma_i H_{\tau i}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

or

$$\tau = \frac{T}{2At}\left[h_B - \Sigma_i h_i \oint_S \frac{h_i}{t}\,ds\right] \quad . \quad . \quad . \quad . \quad . \quad (139a)$$

This result agrees with the more general theory of 2.3 and shows that the shear stress at the root depends only on the torque and geometric and elastic properties at the root.

Typical plots of the function G_σ and G_τ for the case $\alpha = 0$ are given in Figs. 7, 8, 9 and 10 for a built-in tube with taper ratio 2:1. The curves are given for concentrated torque at 0.4 and 0.8 span from the root and for several values of the parameter μc. The abscissa ζ is given by,

$$\zeta = \frac{\text{Distance from root}}{\text{Span}}$$

Note that,

$$\zeta = (1-\rho)/(1-\rho_c)$$

and

$$c = r_o(1-\rho_c)$$

$$\text{Taper ratio} = 1/\rho_c$$

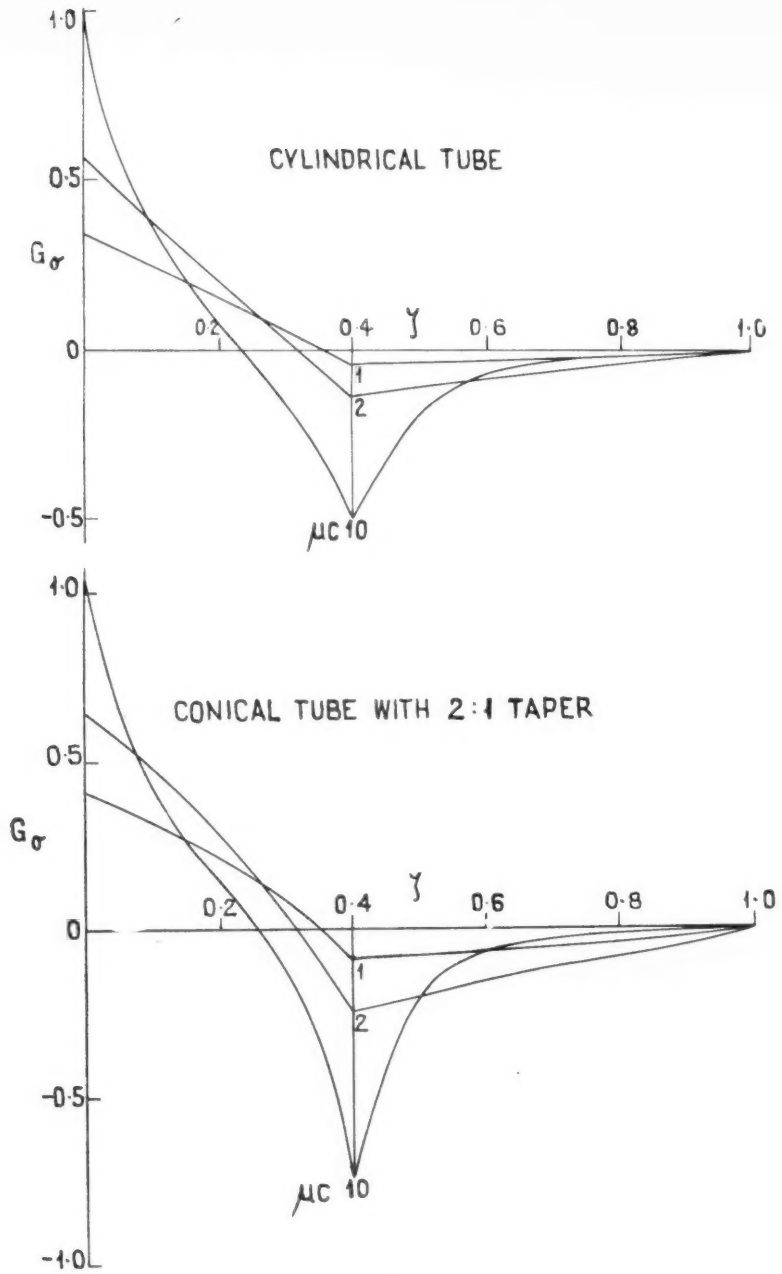

Fig. 7.
G_σ functions for concentrated torque at 0.4 length from built-in root.

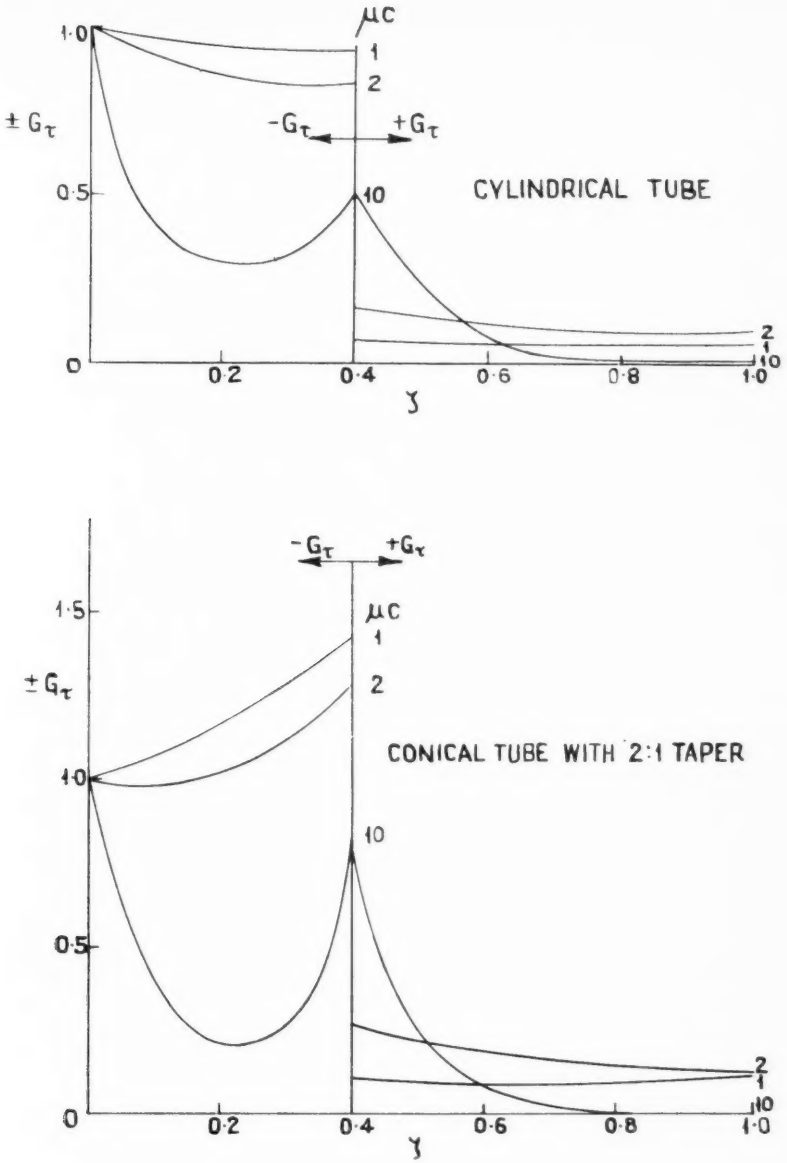

Fig. 8.

G_τ functions for concentrated torque at 0.4 length from built-in root.

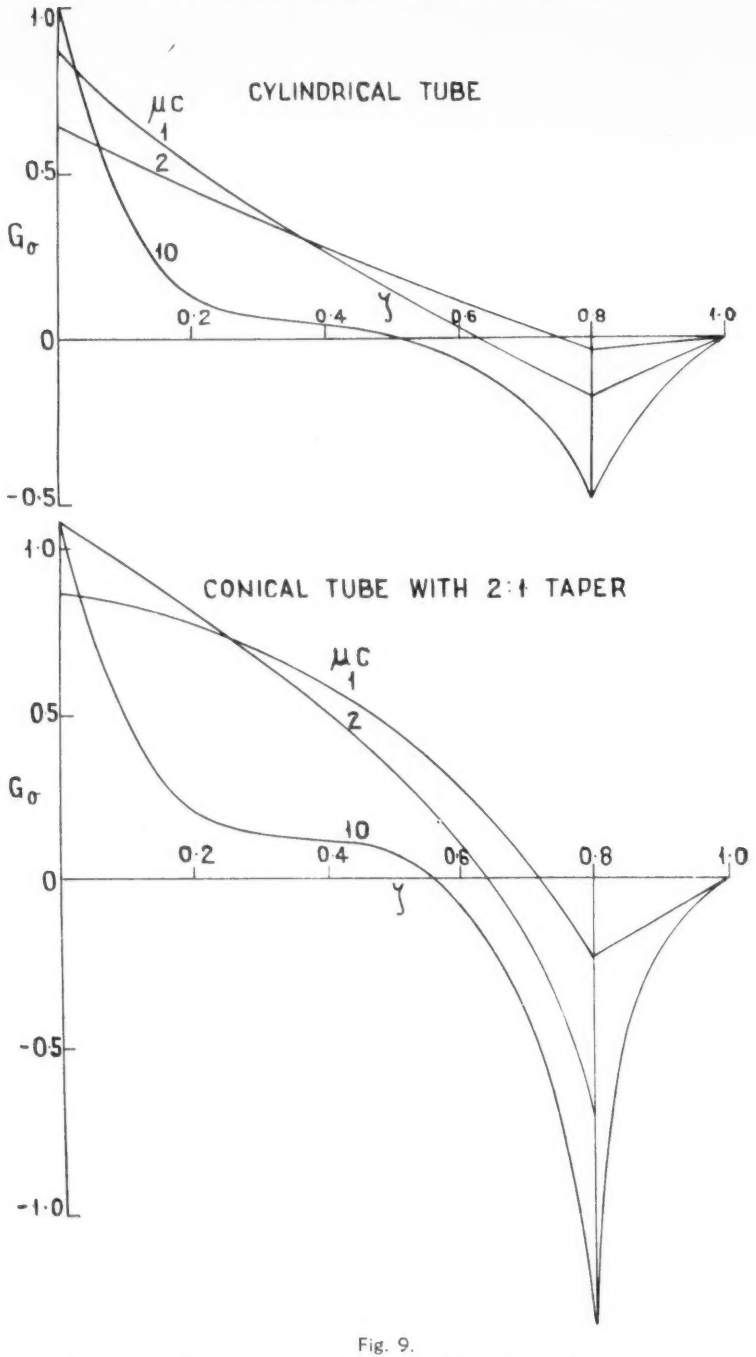

Fig. 9.
G_r functions for concentrated torque at 0.8 length from built-in root.

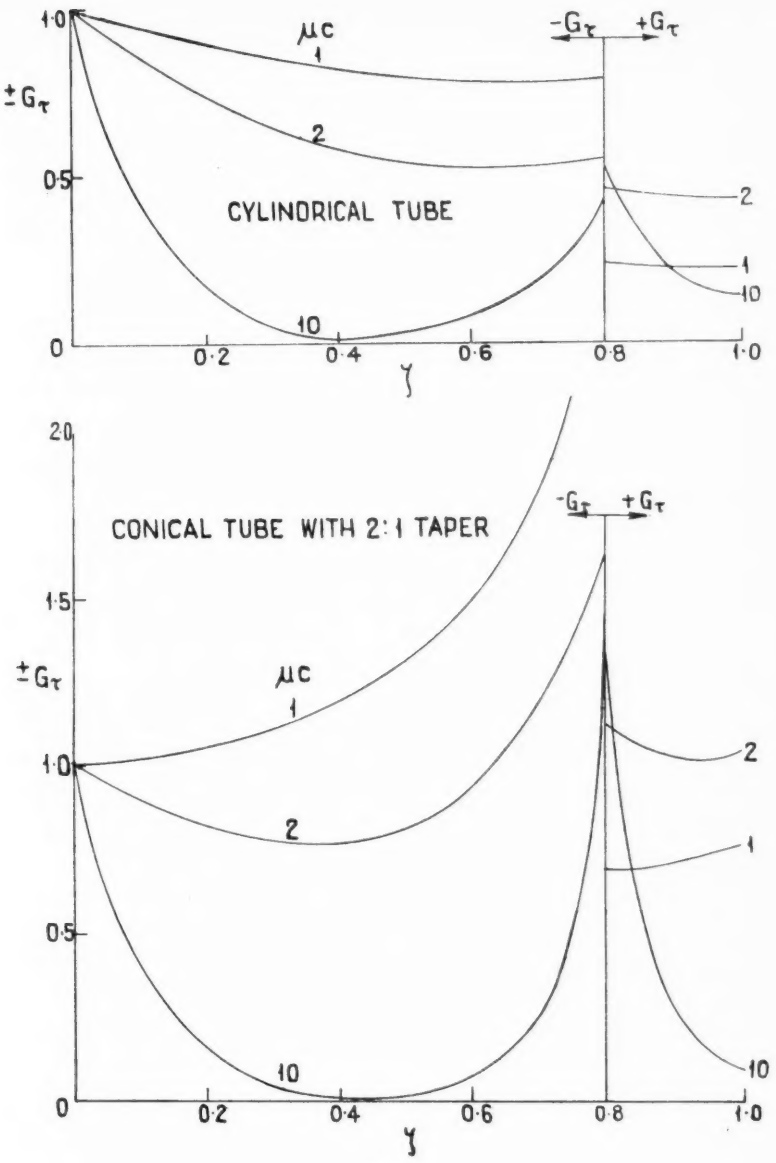

Fig. 10.
G_τ functions for concentrated torque at 0.8 length from built-in root.

4.1.5. *The stress function and stresses in the cylindrical tube.*

Consider now the case of the cylindrical tube loaded with a concentrated torque T at the section defined by $z'=z_1$.

The boundary conditions analogous to those of equations (129) to (132) are,

$$[g_i']_{z'=0}=0 \qquad (129a)$$

$$[g_i']_{z'=z_1}-[g_i]_{z'=z_1}=0 \qquad (130a)$$

$$\left[\frac{dg_i'}{dz'}\right]_{z'=z_1}-\left[\frac{dg_i}{dz'}\right]_{z'=z_1}=-\frac{T}{2A}\oint_S \frac{h_i}{t}\,ds \qquad (131a)$$

$$\left[\frac{dg_i}{dz'}\right]_{z'=c}=\frac{T}{2A}\oint_S \frac{h_i}{t}\,ds \qquad (132a)$$

The g_i and g_i' functions take the form given in equation (88),

$$\left.\begin{aligned} g_i'&=D'_{1i}\cosh\mu_i z'+D'_{2i}\sinh\mu_i z' \\ g_i&=D_{1i}\cosh\mu_i z'+D_{2i}\sinh\mu_i z' \end{aligned}\right\} \qquad (140)$$

Proceeding as in 4.1.4 the stress-functions are found to be:—

For $0\leqslant z'\leqslant z_1$,

$$F'=\frac{Tc}{2A}\sum_i \frac{[1-\cosh\mu_i(c-z_1)]\sinh\mu_i z'}{\mu_i c\cosh\mu_i c}\,h_i\oint_S \frac{h_i}{t}\,ds \qquad (141a)$$

For $z_1\leqslant z'\leqslant c$,

$$F=\frac{Tc}{2A}\sum_i \frac{\sinh\mu_i z'-\sinh\mu_i z_1\cosh\mu_i z'}{\mu_i c\cosh\mu_i c}\,h_i\oint_S \frac{h_i}{t}\,ds-\frac{Tz'}{2A}h_B \qquad (141b)$$

The formulæ for the stresses are the same as (136a) and (136b) with $\rho=1$. In the modified formulæ the H-functions remain as before (equations (138)), but the G-functions now become:—

For $0\leqslant z'\leqslant z_1$,

$$\left.\begin{aligned} G'_{\sigma i}&=\frac{1-\cosh\mu_i(c-z_1)}{\cosh\mu_i c}\sinh\mu_i z' \\ G'_{\tau i}&=-\frac{1-\cosh\mu_i(c-z_1)}{\cosh\mu_i c}\cosh\mu_i z' \end{aligned}\right\} \qquad (142a)$$

For $z_1\leqslant z'\leqslant c$,

$$\left.\begin{aligned} G_{\sigma i}&=\frac{\sinh\mu_i z'-\sinh\mu_i z_1\cosh\mu_i(c-z')}{\cosh\mu_i c} \\ G_{\tau i}&=-\frac{\cosh\mu_i z'+\sinh\mu_i z_1\sinh\mu_i(c-z')}{\cosh\mu_i c} \end{aligned}\right\} \qquad (142b)$$

Note that the shear stress distribution at the root is again given by formula (139). Typical plots of G_σ and G_τ for the cylindrical tube are given in Figs. 7, 8, 9 and 10 for comparison with those for the 2 : 1 taper conical tube. Note that in this case $\zeta=(c-z')/c$.

4.2. GENERAL THEORY FOR AN ARBITRARY DISTRIBUTION OF TORQUE.

In 4.1 it has been shown how a solution may be built up for the case of a single concentrated torque, and that the final result may be regarded as the addition of two stress systems. The first system consists of a number, generally infinite, of self-equilibrating systems in which the h-functions are identical with the eigenfunctions h_i discussed in 3.2.1. The second system is simply the shear stress distribution according to Bredt-Batho's formula or its generalisation for a multi-cell tube.

This particular result suggests that the method may be generalised for the case of arbitrary distribution of torque.

4.2.1. *The differential equation in the* g*-function for arbitrary torque distribution.*

Following the preamble of the previous paragraph the stress function F is assumed to take the form,

$$F = \sum_i g_i h_i + g_B h_B \qquad . \quad . \quad . \quad . \quad . \quad (143)$$

In this equation g_i and g_B are the unknown functions of r. It should be noted that the g_i in equation (143) includes the effect of the torque distribution as well as the end conditions. Thus in the special case of the concentrated torque (equation 125b) the present g_i is identical with the expression,

$$\left(g_i + \frac{T}{2A} \frac{r_o^2}{r} \frac{(1+\alpha)}{\mu_i^2 r_o^2 - (1+\alpha)} \oint_S \frac{h_i}{t} ds \right)$$

The $g_B h_B$ term in equation (143) corresponds to the Bredt-Batho distribution which is statically equivalent to the applied load.

For the solution of the present problem it is necessary only to know the derivatives $\frac{dg_B}{dr}$ and $\frac{d^2 g_B}{dr^2}$ of the function g_B. These derivatives may be obtained at once from the equilibrium condition in torque equation (47). Thus, using equation (99),

$$T = -\rho^2 \frac{dg_B}{dr} \oint h_B p ds = -2A\rho^2 \frac{dg_B}{dr}$$

or

$$\left.\begin{aligned} \frac{dg_B}{dr} &= -\frac{T}{2A\rho^2} \\ \text{and} \qquad \frac{d^2 g_B}{dr^2} &= \frac{1}{2A\rho^2}\left[\frac{2T}{r} - \frac{dT}{dr}\right] \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad (144)$$

The differential equation (27) must now be written in the terms of the stress function F given in equation (143) with the relations (144). Assuming that the sheet thicknesses vary in accordance with relations (38) and (40) and noting that $\frac{dh_B}{ds} = 0$ the differential equation becomes,

$$\begin{aligned} \sum_i \left\{ \rho^2 \frac{h_i}{t} \frac{d^2 g_i}{dr^2} + \rho^2 \frac{d}{dr}\left[\ln(\rho/t^*)\right] \frac{h_i}{t} \frac{dg_i}{dr} + \frac{Gt^*}{Et_s^*} \frac{d}{ds}\left(\frac{1}{t_s} \frac{dh_i}{ds}\right) g_i \right\} \\ + \frac{h_B}{2At} \left\{ T\left[\frac{2}{r} - \frac{d}{dr}\ln(\rho/t^*)\right] - \frac{dT}{dr} \right\} \\ = -Gt^* \rho \left[p \frac{d}{dr}\left(\rho^2 \frac{d\theta}{dr}\right) + \rho \sin\psi \frac{d^2 u}{dr^2} + \rho \cos\psi \frac{d^2 v}{dr^2} \right] \qquad . \quad . \quad (145) \end{aligned}$$

Multiplying this equation by h_i, integrating over all walls using the orthogonality and normality relations (76), (77), (57) and (78), and the conditions expressed by equations (54) and (118) one obtains,

$$r^2 \frac{d^2 g_i}{dr^2} + r\left(1 - \frac{r}{t^*}\frac{dt^*}{dr}\right)\frac{dg_i}{dr} - \mu_i^2 r_o^2 \frac{t^*}{t_s^*} g_i = -\frac{r_o^2}{2A} \oint_S \frac{h_i}{t} ds \left\{ \frac{T}{r}\left[1 + \frac{r}{t^*}\frac{dt^*}{dr}\right] - \frac{dT}{dr} \right\} \qquad . \quad (146)$$

For the cylindrical tube equation (146) reduces to,

$$\frac{d^2 g_i}{dz'^2} - \frac{1}{t^*}\frac{dt^*}{dz'} - \mu_i^2 \frac{t^*}{t_s^*} g_i = -\frac{1}{2A} \oint_S \frac{h_i}{t} ds \left\{ \frac{1}{t^*}\frac{dt^*}{dz'} T - \frac{dT}{dz'} \right\} \qquad . \quad (146a)$$

The L.H.S. of equations (146) are identical with the L.H.S. of equations (43). For an arbitrary distribution of T, t^* and/or t_s^* the solution of this equation may be found by numerical methods. When t^*, t_s^*, and T and their derivatives are continuous over a section of the tube the solution of equation (146) over this section is the sum of the complementary function (79), involving the two undetermined constants D_{1i} and D_{2i}, and a particular solution. An interesting special case occurs when,

$$\left.\begin{aligned} &T=T_0 t^* \quad \text{for cylindrical tubes} \\ \text{or}\quad &T=T_0\rho t^* \quad \text{for conical tubes} \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad . \quad (147)$$

Then the R.H.S. of equation (146) is zero and so also is the particular solution. Hence for the torque distribution of (147) the stresses may be distributed according to the Bredt-Batho formula provided the ends of the tube are free. The physical interpretation is given by the fact that when $T/\rho t^*$ is constant the warping is independent of r.

The expressions for the stresses are,

$$\left.\begin{aligned} \sigma &= \frac{1}{\rho t_s^*}\sum_i g_i\left(\frac{1}{t_s}\frac{dh_i}{ds}\right) \\ \tau &= -\frac{1}{t^*}\left[\sum_i \frac{dg_i}{dr}\frac{h_i}{t}+\frac{dg_B}{dr}\frac{h_B}{t}\right] \end{aligned}\right| \qquad . \quad . \quad . \quad . \quad . \quad (148)$$

The warping is given by,

$$\frac{\partial w^*}{\partial s}=\sum_i \frac{\partial w_i^*}{\partial s}+\frac{\partial w_B^*}{\partial s} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (149)$$

Substituting equation (95c) for $\frac{\partial w_i^*}{\partial s}$ and (116) for $\frac{\partial w_B^*}{\partial s}$ one obtains, using equation (144),

$$G\frac{\partial w^*}{\partial s}=-\frac{\rho}{t^*}\sum_i \frac{dg_i}{dr}\left\{\frac{h_i}{t}-\frac{\hat{p}_i}{2A}\oint_S \frac{h_i}{t}\,ds\right\}+\frac{T}{2A\rho t^*}\left\{\frac{h_B}{t}-\frac{\hat{p}}{2A}\oint_S \frac{h_B}{t}\,ds\right\}. \quad (150)$$

The undetermined constants D_{1i} and D_{2i} are obtained from the boundary conditions which will now be considered.

At a free end of the tube $r=r_e$ the stress σ is zero and condition (129) derived in 4.1.4 may be applied. Thus,

$$[\bar{g}_i]_{r=r_e}=0 \qquad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

At a built-in end $r=r_o$,

$$\frac{\partial w^*}{\partial s}=0$$

so that the R.H.S. of equation (150) is zero. Multiplying this equation by h_i and proceeding as in 4.1.4 one derives,

$$\left[\frac{dg_i}{dr}\right]_{r=r_o}=\frac{T_o}{2A}\oint_S \frac{h_i}{t}\,ds \qquad . \quad . \quad . \quad . \quad (152)$$

where T_o is the torque at $r=r_o$.

If there are no discontinuities in load or structure between $r=r_e$ and $r=r_o$ the two conditions (151) and (152) are sufficient to determine the coefficients D_{1i} and D_{2i} in the complementary function which holds over the whole region.

Each discontinuity in T, t^*, and/or t_s^* requires two additional constants D which can be found from the two additional boundary conditions at the discontinuity. The particular case of a concentrated torque has been treated in 4.1.4. The general case of a discontinuity

in T, t^*, and/or t_s^* will now be considered. Denoting the two sides of the discontinuity at $r=r_1$ by the suffices R and L, one obtains from the equality of direct stress flow,

$$\sum_i \frac{1}{t_s}\frac{dh_i}{ds}[g_{iR}]_{r=r_1} = \sum_i \frac{1}{t_s}\frac{dh_i}{ds}[g_{iL}]_{r=r_1}$$

and hence by the procedure used to establish equations (129) and (151),

$$[g_{iR}]_{r=r_1} - [g_{iL}]_{r=r_1} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (153)$$

From the equality of the warping on each side of the discontinuity, application of equation (150) and a procedure similar to that used to establish equations (131) and (132), yields,

$$\frac{1}{t_R^*}\left[\frac{dg_{iR}}{dr}\right]_{r=r_1} - \frac{1}{t_L^*}\left[\frac{dg_{iL}}{dr}\right]_{r=r_1} = \frac{1}{2A\rho_1^2}\left\{\frac{T_{1R}}{t_R^*} - \frac{T_{1L}}{t_L^*}\right\}\oint_S \frac{h_i}{t}\,ds \quad . \quad (154)$$

where T_{1R} and T_{1L} are the values of the torque on each side of $r=r_1$.

4.2.2. *The conical tube with $t^*=\rho^\alpha$ and $t_s^*=\rho^\beta$.*

In this case equation (146) reduces to,

$$r^2\frac{d^2g_i}{dr^2} + (1-\alpha)\,r\frac{dg_i}{dr} - \mu_i^2 r_o^2 \rho^{\alpha-\beta} g_i = -\frac{r_o^2}{2A}\oint_S \frac{h_i}{t}\,ds\left\{\frac{T(1+\alpha)}{r} - \frac{dT}{dr}\right\} \quad . \quad (155)$$

The complementary functions Ψ_1 and Ψ_2 of this equation are given by equation (84). Denoting by V the R.H.S. of equation (155) the complete solution of this equation may be written,

$$g = \Psi_1\int \frac{V\Psi_2}{r^2(\Psi_1'\Psi_2 - \Psi_2'\Psi_1)}\,dr - \Psi_2\int\frac{V\Psi_1}{r^2(\Psi_1'\Psi_2 - \Psi_2'\Psi_1)}\,dr \quad . \quad . \quad (156)$$

where the dashes denote differentiation with respect to r. In general the solution (156) cannot be integrated in a closed form.

4.2.3. *The particular case when $\alpha=\beta$.*

When $t^*=t_s^*=\rho^\alpha$ equation (155) becomes,

$$r^2\frac{d^2g_i}{dr^2} + (1-\alpha)\,r\frac{dg_i}{dr} - \mu_i^2 r_o^2 g_i = -\frac{r_o^2}{2A}\oint_S \frac{h_i}{t}\,ds\left\{\frac{T(1+\alpha)}{r} - \frac{dT}{dr}\right\} \quad . \quad . \quad (157)$$

The complementary function of equation (157) is given by equation (86), namely,

$$g_i = r^{\alpha/2}\{D_{1i}r^{\nu_i} + D_{2i}r^{-\nu_i}\} \quad . \quad . \quad . \quad . \quad . \quad (86)$$

where

$$\nu_i = \tfrac{1}{2}[\alpha^2 + 4\mu_i^2 r_o^2]^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (86a)$$

The case of the concentrated torque investigated in 4.1 could also be analysed by means of equation (157) noting that for constant torque T the particular solution is,

$$g = \frac{T}{2A}\frac{r_o^2}{r}\frac{(1+\alpha)}{\mu_i^2 r_o^2 - (1+\alpha)}\oint_S \frac{h_i}{t}\,ds \quad . \quad . \quad . \quad . \quad (158)$$

This agrees with the second term in parenthesis on the R.H.S. of equation (125b)

In the exceptional case when $\mu_i^2 r_o^2 = (1+\alpha)$ the particular solution becomes,

$$g = \frac{T}{2A}\frac{r_o^2}{r}\ln r\,.\,\frac{1+\alpha}{2+\alpha}\oint_S \frac{h_i}{t}\,ds \quad . \quad . \quad . \quad . \quad (159)$$

4.2.4. *The built-in tube with $t^* = t_s^* = \rho^\alpha$ under a distributed torque $T = T_o\rho^\gamma$.*

Inspection of equation (157) shows that it is homogeneous in r if T is of the form,

$$T = T_o\rho^\gamma \qquad (160)$$

where γ is any real number. For such a torque distribution the particular solution is,

$$g = \frac{T_o r_o}{2A}\rho^{\gamma-1}\delta_i \oint_S \frac{h_i}{t}\,ds \qquad (161)$$

where,

$$\delta_i = -\frac{\gamma-(1+\alpha)}{\mu_i^2 r_o^2 - [\gamma-(1+\alpha)](\gamma-1)} \qquad (162)$$

This solution applies only when

$$\mu_i^2 r_o^2 \neq [\gamma-(1+\alpha)](\gamma-1)$$

Adjusting to the boundary conditions (152) and (153) and using (144) the stress function F is obtained as,

$$F = \frac{T_o r_o}{2A}\sum_i \{\rho^{\alpha/2}[\bar{D}_{1i}\rho^{\nu_i} + \bar{D}_{2i}\rho^{-\nu_i}] + \delta_i\rho^{\gamma-1}\} h_i \oint_S \frac{h_i}{t}\,ds - \frac{T_o r_o}{2A}\frac{\rho^{\gamma-1}}{\gamma-1}h_B \qquad (163)$$

where,

$$\bar{D}_{1i} = -\frac{\frac{\alpha}{2}-\nu_i}{\mu_i^2 r_o^2 - [\gamma-(1+\alpha)](\gamma-1)} \cdot \frac{\left\{[\gamma-(1+\alpha)]\rho_c^{\gamma-(2+\alpha)/2} + \left(\frac{\alpha}{2}+\nu_i\right)\rho_c^{-\nu_i}\right\}}{\left\{\left(\frac{\alpha}{2}+\nu_i\right)\rho_c^{-\nu_i} - \left(\frac{\alpha}{2}-\nu_i\right)\rho_c^{\nu_i}\right\}}$$

$$\bar{D}_{2i} = +\frac{\frac{\alpha}{2}+\nu_i}{\mu_i^2 r_o^2 - [\gamma-(1+\alpha)](\gamma-1)} \cdot \frac{\left\{[\gamma-(1+\alpha)]\rho_c^{\gamma-(2+\alpha)/2} + \left(\frac{\alpha}{2}-\nu_i\right)\rho_c^{\nu_i}\right\}}{\left\{\left(\frac{\alpha}{2}+\nu_i\right)\rho_c^{-\nu_i} - \left(\frac{\alpha}{2}-\nu_i\right)\rho_c^{\nu_i}\right\}} \qquad (164)$$

The stresses are given by,

$$\left.\begin{aligned}\sigma &= \frac{T_o}{2A}\sum_i G_{\sigma i}H_{\sigma i}\\ \tau &= \frac{T_o}{2A}\left[\rho^{\gamma-(\alpha+2)}\frac{h_B}{t} + \sum_i G_{\tau i}H_{\tau i}\right]\end{aligned}\right\} \qquad (165)$$

where H_σ and H_τ are the values given by equations (138), and the G-functions are as follows:—

$$\left.\begin{aligned}G_{\sigma i} &= \rho^{-(\alpha+2)/2}[\bar{D}_{1i}\rho^{\nu_i} + \bar{D}_{2i}\rho^{-\nu_i} + \delta_i\rho^{\gamma-(\alpha+2)/2}]\,\mu_i r_o\\ G_{\tau i} &= -\rho^{-(\alpha+2)/2}\left[\bar{D}_{1i}\left(\frac{\alpha}{2}+\nu_i\right)\rho^{\nu_i} + \bar{D}_{2i}\left(\frac{\alpha}{2}-\nu_i\right)\rho^{-\nu_i} + \delta_i(\gamma-1)\rho^{\gamma-(\alpha+2)/2}\right]\end{aligned}\right\} \qquad (166)$$

The coefficients δ_i, D_{1i} and D_{2i} are as in equations (162) and (164). The shear stresses at the root are again given by equation (139) with T_o in place of T.

In the exceptional case when,

$$\mu_i^2 r_o^2 = [\gamma-(1+\alpha)](\gamma-1)$$

the appropriate particular solution of equation (157) is,

$$g = \frac{T_o r_o}{2A}\rho^{\gamma-1}\ln r \cdot \frac{1+\alpha-\gamma}{2+\alpha-2\gamma}\oint_S \frac{h_i}{t}ds \qquad (167)$$

The solution may be completed as in the general case after equation (161).

4.2.5. *The built-in tube under uniformly distributed torque.*

A uniformly distributed torque may be represented by,

$$T = T_o \frac{\rho - \rho_c}{1 - \rho_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (168)$$

where T_o is the torque at the root.

This torque may be separated into a constant part $-\frac{T_o \rho_c}{1-\rho_c}$ for which the solution is given in 4.1 with $\rho_1 = \rho_c$, and a linearly varying part $\frac{T_o}{1-\rho_c}\rho$ which may be obtained from 4.2.4 by putting $\gamma = 1$.

The stresses are found to be,

$$\left.\begin{aligned} \sigma &= \frac{T_o}{2A} \sum_i G_{\sigma i} H_{\sigma i} \\ \tau &= \frac{T_o}{2A}\left[\frac{\rho - \rho_c}{1-\rho_c}\rho^{-(\alpha+2)}\frac{h_B}{t} + \sum_i G_{\tau i} H_{\tau i}\right] \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (169)$$

where the H-functions are given in equations (138) and the G-functions are as follows:—

$$\left.\begin{aligned} G_{\sigma i} &= \rho^{-(\alpha+2)/2}\left\{\bar{D}_{1i}\rho^{\nu_i} + \bar{D}_{2i}\rho^{-\nu_i} - \frac{1}{1-\rho_c}\left[\frac{1+\alpha}{\mu_i^2 r_o^2 - (1+\alpha)}\rho_c - \frac{\alpha}{\mu_i^2 r_o^2}\rho\right]\rho^{-(\alpha+2)/2}\right\}\mu_i r_o \\ G_i &= -\rho^{-(\alpha+2)/2}\left\{\bar{D}_{1i}\left(\frac{\alpha}{2}+\nu_i\right)\rho^{\nu_i} + \bar{D}_{2i}\left(\frac{\alpha}{2}-\nu_i\right)\rho^{-\nu_i} + \frac{\rho_c}{1-\rho_c}\cdot\frac{1+\alpha}{\mu_i^2 r_o^2 - (1+\alpha)}\rho^{-(\alpha+2)/2}\right\} \end{aligned}\right\} \quad . \quad (170)$$

where,

$$\left.\begin{aligned} \bar{D}_{1i} &= \frac{\left(\frac{\alpha}{2}+\nu_i\right)\rho_c^{-\nu_i}[(1-\rho_c)\mu_i^2 r_o^2 - (1+\alpha)] + \rho_c^{-\alpha/2}[\mu_i^2 r_o^2 + \alpha(1+\alpha)]}{[\mu_i^2 r_o^2 - (1+\alpha)]\left(\frac{\alpha}{2}+\nu_i\right)(1-\rho_c)\left[\left(\frac{\alpha}{2}+\nu_i\right)\rho_c^{-\nu_i} - \left(\frac{\alpha}{2}-\nu_i\right)\rho_c^{\nu_i}\right]} \\ \bar{D}_{2i} &= -\frac{\left(\frac{\alpha}{2}-\nu_i\right)\rho_c^{\nu_i}[(1-\rho_c)\mu_i^2 r_o^2 - (1+\alpha)] + \rho_c^{-\alpha/2}[\mu_i^2 r_o^2 + \alpha(1+\alpha)]}{[\mu_i^2 r_o^2 - (1+\alpha)]\left(\frac{\alpha}{2}-\nu_i\right)(1-\rho_c)\left[\left(\frac{\alpha}{2}+\nu_i\right)\rho_c^{-\nu_i} - \left(\frac{\alpha}{2}-\nu_i\right)\rho_c^{\nu_i}\right]} \end{aligned}\right\} \quad . \quad (171)$$

Typical plots of the functions G_σ and G_τ for the case $\alpha = 0$ are given in Figs. 11 and 12 for a built-in tube with taper ratio 2:1 and for various values of the parameter μc.

4.2.6. *The cylindrical tube with linearly varying thicknesses.*

The general differential equation (146a) for the cylindrical tube under arbitrary torque may be solved numerically for any variation of t^*, t_s^* and/or T. In this section the discussion will be restricted to the case where the thicknesses vary linearly so that in equations (89) $\alpha = \beta = 1$. The differential equation becomes,

$$\frac{d^2 g_i}{dz'^2} - \frac{1}{z'+d}\frac{dg_i}{dz'} - \mu_i^2 g_i = -\frac{1}{2A}\oint_S \frac{h_i}{t}\,ds\left\{\frac{T}{z'+d} - \frac{dT}{dz'}\right\} \quad . \quad . \quad . \quad (172)$$

The complementary function is given in equation (93) and using equation (156) with r^2 replaced by unity the general solution may be written,

$$\left.\begin{aligned} g_i &= \bar{z} I_1(\mu_i \bar{z}) \int \frac{V K_1(\mu_i \bar{z})\,d\bar{z}}{\mu_i \bar{z}\{I_o(\mu_i \bar{z}) K_1(\mu_i \bar{z}) + K_o(\mu_i \bar{z}) I_1(\mu_i \bar{z})\}} \\ &\quad - \bar{z} K_1(\mu_i \bar{z}) \int \frac{V I_1(\mu_i \bar{z})\,d\bar{z}}{\mu_i \bar{z}\{I_o(\mu_i \bar{z}) K_1(\mu_i \bar{z}) + K_o(\mu_i \bar{z}) I_1(\mu_i \bar{z})\}} \end{aligned}\right\} \quad . \quad . \quad (173)$$

where $\bar{z}=z'+d$, and V is the R.H.S. of equation (172). I_0 and K_1 are the modified Bessel functions of the first and second kinds and of zero and first orders. In general integrations of equation (173) cannot be obtained in closed form.

The arbitrary constants in equation (173) can be obtained from the boundary conditions (151) to (154) with $\rho=1$, and t_s^* and t^* as in equations (89) with $\alpha=\beta=1$.

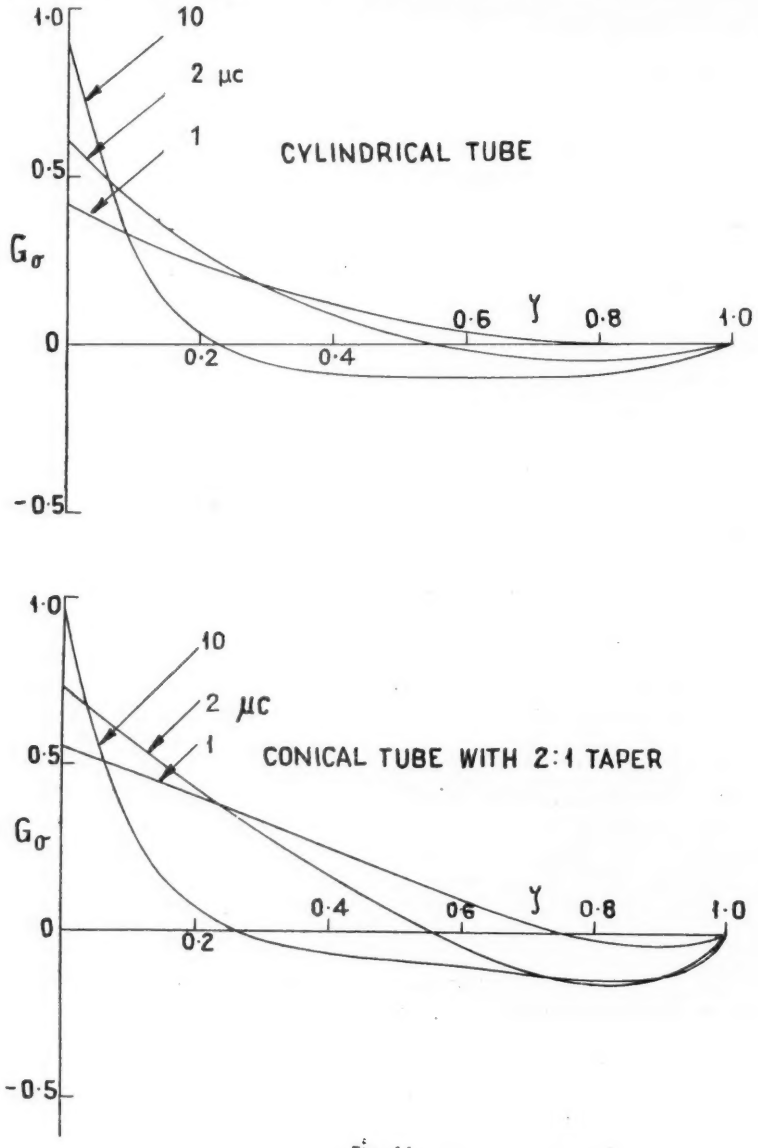

Fig. 11.
G_σ functions for uniformly distributed torque with built-in root.

4.2.7. *The cylindrical tube of constant thickness longitudinally and linearly varying torque.*

When t_s^* and t^* are constant equation (146a) reduces to,

$$\frac{d^2 g_i}{dz'^2} - \mu_i^2 g_i = \frac{1}{2A} \oint_S \frac{h_i}{t}\, ds \cdot \frac{dT}{dz'} \quad . \quad . \quad . \quad . \quad . \quad (174)$$

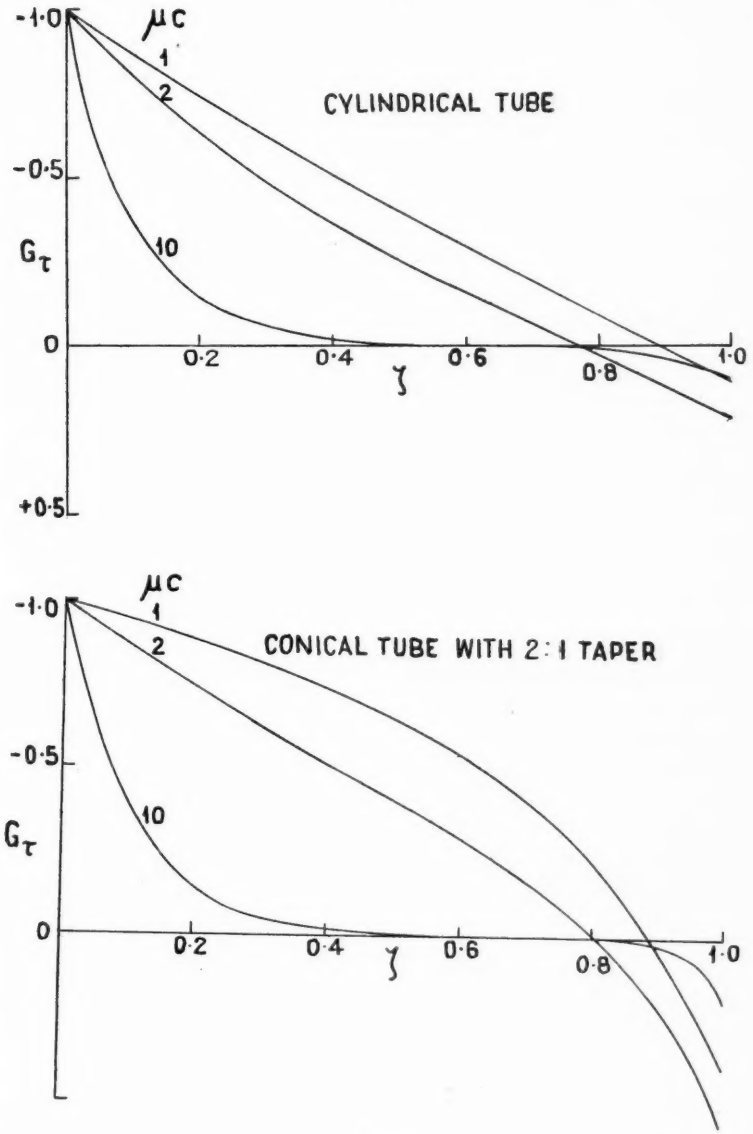

Fig. 12.
G_τ functions for uniformly distributed torque with built-in root

The complementary function is,

$$g_1 = D_{1i} \cosh \mu_i z' + D_{2i} \sinh \mu_i z' \quad . \quad . \quad . \quad . \quad . \quad (88)$$

Equation (174) will now be applied to the solution of a built-in tube under a torque increasing linearly from zero at $z' = z_1$ to T_o at $z' = c$.

Such a torque distribution may be represented as follows:—

$$\left.\begin{aligned} &\text{For } 0 \leqslant z' \leqslant z_1, \quad T = 0 \\ &\text{For } z_1 \leqslant z' \leqslant c, \quad T = T_o \frac{z' - z_1}{c - z_1} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (175)$$

The complete solution of equation (174) may be written,

For $0 \leqslant z' \leqslant z_1$,

$$g_1' = D'_{1i} \cosh \mu_i z' + D'_{2i} \sinh \mu_i z'$$

and for $z_1 \leqslant z' < c$

$$g_1 = D_{1i} \cosh \mu_i z' + D_{2i} \sinh \mu_i z' - \frac{T_o}{2A} \frac{1}{(c - z_1)\mu_i^2} \oint_S \frac{h_i}{t}\, ds \quad . \quad . \quad (176)$$

The D coefficients are determined by the boundary conditions (see equations (151) to (154)),

$$\left.\begin{aligned} [g_1']_{z'=0} &= 0 \\ \left[\frac{dg_1}{dz'}\right]_{z'=c} &= \frac{T_o}{2A} \oint_S \frac{h_i}{t}\, ds \\ [g_1']_{z'=z_1} - [g_1]_{z'=z_1} &= 0 \\ \left[\frac{dg_1'}{dz'}\right]_{z'=z_1} - \left[\frac{dg_1}{dz'}\right]_{z'=z_1} &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (177)$$

Substituting equations (176) into (177) and using (144) one obtains the stress functions,

For $0 \leqslant z' \leqslant z_1$

$$F' = \frac{T_o c}{2A} \sum_i \frac{[\mu_i (c - z_1) - \sinh \mu_i (c - z_1)] \sinh \mu_i z'}{\mu_i c \mu_i (c - z_1) \cosh \mu_i c} h_i \oint_S \frac{h_i}{t}\, ds \quad . \quad . \quad . \quad (178a)$$

and for $z_1 \leqslant z' \leqslant c$

$$F = \frac{T_o c}{2A} \sum_i \frac{\cosh \mu_i z_1 \cosh \mu_i (c - z') + \mu_i (c - z_1) \sinh \mu_i z' - \cosh \mu_i c}{\mu_i c \mu_i (c - z_1) \cosh \mu_i c} h_i \oint_S \frac{h_i}{t}\, ds - \frac{T_o}{4A} \frac{(z' - z_1)^2}{(c - z_1)} h_8$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (178b)$$

Note that the appropriate form of equation (144) for the cylindrical tube is

$$\frac{dg_B}{dz'} = -\frac{T}{2A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (144a)$$

from which the last expression on the R.H.S. of equation (178b) may be derived.

The expressions for the stresses are:—

For $0 \leqslant z' \leqslant z_1$

$$\left.\begin{aligned} \sigma' &= \frac{T_o}{2A} \sum_i G'_{\sigma i} H_{\sigma i} \\ \tau' &= \frac{T_o}{2A} \sum_i G'_{\tau i} H_{\tau i} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (179a)$$

For $z_1 \leqslant z' \leqslant c$,

$$\left.\begin{aligned} \sigma &= \frac{T_o}{2A} \sum_i G_\sigma H_{\sigma i} \\ \tau &= \frac{T_o}{2A}\left[\frac{z'-z_1}{c-z_1}\,\frac{h_B}{t} + \sum_i G_{\tau i} H_{\tau i}\right] \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (179b)$$

where H_σ and H_τ are the values given by equations (138), and the G-functions are as follows:—

$$\left.\begin{aligned} G'_{\sigma i} &= \frac{[\mu_i (c-z_1) - \sinh \mu_i (c-z_1)] \sinh \mu_i z'}{\mu_i (c-z_1) \cosh \mu_i c} \\ G'_{\tau i} &= -\frac{[\mu_i (c-z_1) - \sinh \mu_i (c-z_1)] \cosh \mu_i z'}{\mu_i (c-z_1) \cosh \mu_i c} \end{aligned}\right\} \quad . \quad . \quad (180a)$$

$$\left.\begin{aligned} G_{\sigma i} &= \frac{\cosh \mu_i z_1 \cosh \mu_i (c-z') + \mu_i (c-z_1) \sinh \mu_i z' - \cosh \mu_i c}{\mu_i (c-z_1) \cosh \mu_i c} \\ G_{\tau i} &= -\frac{\mu_i (c-z_1) \cosh \mu_i z' - \cosh \mu_i z_1 \sinh \mu_i (c-z')}{\mu_i (c-z_1) \cosh \mu_i c} \end{aligned}\right\} \quad . \quad . \quad (180b)$$

The shear stresses at the root are again given by equation (139) with T_o in place of T.

The case of a cylindrical tube with uniformly distributed torque over the whole length can be obtained from the above equations by the substitution $z_1 = 0$. Typical plots of G_σ and G for a cylindrical tube under uniformly distributed torque are given in Figs. 11 and 12 for comparison with those for the 2 : 1 taper conical tube.

4.2.8. *The cylindrical tube of constant thickness longitudinally and arbitrary torque distribution.*

An arbitrary torque distribution may be represented by the Fourier series,

$$T = \sum_{k\ \mathrm{odd}}^{\infty} T_k \cos \frac{k\pi z'}{2c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (181)$$

By taking a sufficient number of terms this series will represent any torque distribution. The theoretical zero at $z' = c$ necessitates, however, a large number of terms to represent adequately the torque near the root. To avoid this it is best to work with the Fourier series for $T - T_o$, where T_o is the value of the torque at the root, and to combine the distribution $T - T_o$ with the constant torque case included in the analysis of 4.1.5.

It is only necessary to solve for the typical term

$$T_k \cos \frac{k\pi z'}{2c}$$

of the distribution $T - T_o$.

For such a torque distribution the general solution of equation (174) is,

$$g_i = D_{1i} \cosh \mu_i z' + D_{2i} \sinh \mu_i z' - \frac{T_k}{2A}\,\frac{(k\pi/2c)}{\mu_i^2 + (k\pi/2c)^2} \sin \frac{k\pi z'}{2c} \oint_S \frac{h_i}{t}\, ds \quad . \quad (182)$$

Adjusting to the two first boundary conditions of equations (177) it can be seen that

$$D_{1i} = D_{2i} = 0.$$

Thus the stress function F_k becomes,

$$F_k = -\frac{T_k}{2A}\left(\frac{2c}{k\pi}\right) \sin\left(\frac{k\pi z'}{2c}\right) \left\{ \sum_i \frac{k^2\pi^2}{4\mu_i^2 c^2 + k^2\pi^2} h_i \oint_S \frac{h_i}{t}\, ds + h_B \right\} \quad . \quad . \quad (183)$$

The stresses are,

$$\sigma_k = -\frac{T_k}{2A}\left(\frac{2c}{k\pi}\right)\sin\left(\frac{k\pi z'}{2c}\right)\sum_i \frac{k^2\pi^2}{4\mu_i{}^2c^2+k^2\pi^2}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)\oint_S \frac{h_i}{t}\,ds$$

$$\tau_k = \frac{T_k}{2A}\cos\left(\frac{k\pi z'}{2c}\right)\left\{\sum_i \frac{k^2\pi^2}{4\mu_i{}^2c^2+k^2\pi^2}\left(\frac{h_i}{t}\right)\oint_S \frac{h_i}{t}\,ds + \frac{h_B}{t}\right\} \quad . \quad . \quad (184)$$

The final total stresses will be,

$$\left.\begin{aligned}\sigma &= \sigma_c + \sum_{k\ \text{odd}}^{\infty}\sigma_k \\ \tau &= \tau_c + \sum_{k\ \text{odd}}^{\infty}\tau_k\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (185)$$

where σ_c and τ_c are the stresses due to the constant torque T_o (see 4.1.5.).

The interested reader may note that the results obtained by Hemp, 1943[12] for a doubly symmetrical rectangular tube with booms under torque can be easily derived from equations (184). This will be shown in detail in Part 6 of this paper.

4.3. The four-boom tube in general.

The four-boom tube with walls effective only in shear is often a very practical idealisation of actual wing-structures. In fact much of the previous literature has been restricted to very special cases such as rectangular and trapezoidal box beams. The analysis in the case of the four-boom tube is much simplified for the reasons given in 3.2.2., namely that there is only one eigenvalue.

No theoretical limitation need be placed on the number of shear carrying walls connecting the four booms. However, all cases may be made to depend on the solution for the single-cell tube of the type illustrated in Fig. 13. For this basic case the complete analysis will be given. Explicit formulæ for the reduction of the multi-cell tube of the type shown in Fig. 14 to the single-cell case are also presented. The important practical problem of a wing structure shown in Fig. 15 will appear as a particular example.

$$A_{12} = A_{12}^{*} + N_{12}$$

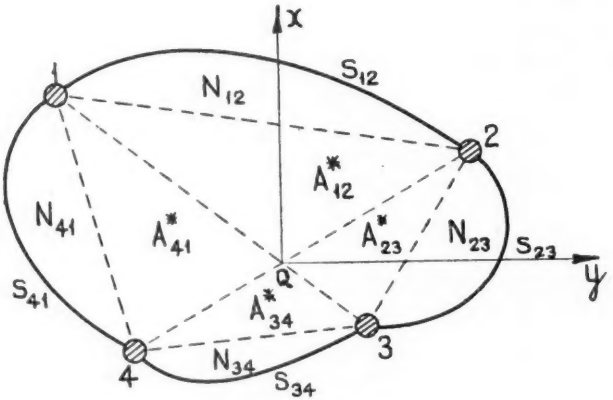

Fig. 13.
The four-boom single-cell tube.

4.3.1. *The eigenvalue λ and the h-function for the single-cell four-boom tube.*

The single-cell four-boom tube is a special case of the n-boom single-cell tube discussed in 3.2.2. For $n=4$ the secular equation (68) becomes:—

$$\Delta = \begin{vmatrix} \left[\lambda^2 \frac{s_{12}}{t_{12}} - \frac{1}{B_1} - \frac{1}{B_2}\right] & \frac{1}{B_2} & 0 & \frac{1}{B_1} & A_{12} & (x_2-x_1) & (y_2-y_1) \\ \frac{1}{B_2} & \left[\lambda^2 \frac{s_{23}}{t_{23}} - \frac{1}{B_2} - \frac{1}{B_3}\right] & \frac{1}{B_3} & 0 & A_{23} & (x_3-x_2) & (y_3-y_2) \\ 0 & \frac{1}{B_3} & \left[\lambda^2 \frac{s_{34}}{t_{34}} - \frac{1}{B_3} - \frac{1}{B_4}\right] & \frac{1}{B_4} & A_{34} & (x_4-x_3) & (y_4-y_3) \\ \frac{1}{B_1} & 0 & \frac{1}{B_4} & \left[\lambda^2 \frac{s_{41}}{t_{41}} - \frac{1}{B_4} - \frac{1}{B_1}\right] & A_{41} & (x_1-x_4) & (y_1-y_4) \\ A_{12} & A_{23} & A_{34} & A_{41} & 0 & 0 & 0 \\ (x_2-x_1) & (x_3-x_2) & (x_4-x_3) & (x_1-x_4) & 0 & 0 & 0 \\ (y_2-y_1) & (y_3-y_2) & (y_4-y_3) & (y_1-y_4) & 0 & 0 & 0 \end{vmatrix} = 0 \quad . \quad (186)$$

Expanding this determinant by the Laplace method using the minors of the three last columns it can be seen that the secular equation is linear in λ^2. The numerical part of this equation is obtained by putting $\lambda^2 = 0$ in the determinant of equation (186). Adding then in the new determinant the second, third and fourth rows and columns to the first row and column respectively the numerical part becomes:

$$\begin{vmatrix}
0 & 0 & 0 & 0 & A & 0 & 0 \\
0 & -\left(\frac{1}{B_2}+\frac{1}{B_3}\right) & \frac{1}{B_3} & 0 & A_{23} & (x_3-x_2) & (y_3-y_2) \\
0 & \frac{1}{B_3} & -\left(\frac{1}{B_3}+\frac{1}{B_4}\right) & \frac{1}{B_4} & A_{34} & (x_4-x_3) & (y_4-y_3) \\
0 & 0 & \frac{1}{B_4} & -\left(\frac{1}{B_4}+\frac{1}{B_1}\right) & A_{41} & (x_1-x_4) & (y_1-y_4) \\
A & A_{23} & A_{34} & A_{41} & 0 & 0 & 0 \\
0 & (x_3-x_2) & (x_4-x_3) & (x_1-x_4) & 0 & 0 & 0 \\
0 & (y_3-y_2) & (y_4-y_3) & (y_1-y_4) & 0 & 0 & 0
\end{vmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (187)$$

$= -A^2 \times$ [Determinant obtained from the above by omitting the fifth row and column].

In the latter determinant the coefficient of $\frac{1}{B_1}$ is by the Laplace expansion,

$$-\begin{vmatrix} (x_3-x_2) & (y_3-y_2) \\ (x_4-x_3) & (y_4-y_3) \end{vmatrix}^2 = -4\,(A_{234})^2$$

where A_{234} is the area of the triangle (2, 3, 4). Similarly the coefficient of $\frac{1}{B_2}$ will be $-4\,(A_{341})^2$ etc.

Thus the numerical part of the secular equation becomes,

$$4A^2 \sum_1^4 \frac{(A_{234})^2}{B_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (188)$$

Considering now the term involving λ^2 one sees that the multiplier of the typical term $\lambda^2 \frac{s_{12}}{t_{12}}$ is the first minor of the determinant (187). This minor is by Laplace's expansion,

$$-\begin{vmatrix} A_{23} & (x_3-x_2) & (y_3-y_2) \\ A_{34} & (x_4-x_3) & (y_4-y_3) \\ A_{41} & (x_1-x_4) & (y_1-y_4) \end{vmatrix}^2$$
$$= -4\,[A_{23}A_{341} + A_{34}\,(A_{234} - A_{123}) + A_{41}A_{234}]^2 \quad . \quad . \quad . \quad (189)$$

As stated in 3.2.1 and 3.2.2 the origin C and the direction of the axes X, Y can be chosen arbitrarily. The formulæ are simplified by taking the origin at Q, the point of intersection of the diagonals (1, 3) and (2, 4). In this case expression (189) becomes,

$$= -4\,[A^*_{34}A + (A^*_{12}A_{34} + A^*_{34}A_{12}) - (A^*_{41}A_{23} + A^*_{23}A_{41})]^2$$
$$= -4\,[A^*_{34}A + (A^*_{41}N_{23} + A^*_{23}N_{41}) - (A^*_{12}N_{34} + A^*_{34}N_{12})]^2$$

where A^*_{12} etc. are the areas of the triangles $(Q,1,2)$ etc. and

$$N_{12}=A_{12}-A^*_{12} \text{ etc.} \quad . \quad . \quad . \quad . \quad . \quad (190)$$

see also Fig. 13.

Thus the coefficient of the typical term $\lambda^2\frac{s_{12}}{t_{12}}$ becomes,

$$-4A^2(\bar{A}_{34})^2$$

where,

$$\bar{A}_{34}=A^*_{34}+\bar{N} \quad . \quad . \quad . \quad . \quad . \quad (191)$$

and

$$\bar{N}=\frac{1}{A}[(A^*_{41}N_{23}+A^*_{23}N_{41})-(A^*_{12}N_{34}+A^*_{34}N_{12})] \quad . \quad . \quad . \quad (192)$$

Proceeding similarly for the other terms in λ^2 one finds,

$$\lambda^2=\frac{\sum_1^4\frac{(A_{234})^2}{B_1}}{\sum_1^4\frac{s_{12}}{t_{12}}(\bar{A}_{34})^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (193)$$

where,

$$\bar{A}_{12}=A^*_{12}+\bar{N},\ \bar{A}_{23}=A^*_{23}-\bar{N},\ \bar{A}_{41}=A^*_{41}-\bar{N} \quad . \quad . \quad . \quad (191a)$$

For a quadrilateral tube $N=0$ and hence

$$\bar{A}_{34}=A^*_{34}=A_{34} \text{ and etc.} \quad . \quad . \quad . \quad . \quad . \quad (194)$$

Explicit expressions for $\mu=\lambda\sqrt{\frac{G}{E}}$ have been given previously for doubly and singly symmetrical rectangular tubes (Reissner, 1926[1], Ebner, 1933[2], Williams, 1936[4], Kuhn, 1942[7] Cox, 1942[5], Fine, 1943[6]) and for singly symmetrical trapezoidal tubes (Dunne, 1945[13]), but only in relation to uniform parallel tubes. The significance of λ as an eigenvalue of the cross-section geometry was not apparent in these investigations.

The next step is to determine the h-functions.

Expanding the minors of the elements in the first row of determinant (186) by the Laplace method or directly from the three equilibrium equations one obtains,

$$h_{12}\begin{vmatrix} A_{23} & (x_3-x_2) & (y_3-y_2)\\ A_{34} & (x_4-x_3) & (y_4-y_3)\\ A_{41} & (x_1-x_4) & (y_1-y_4)\end{vmatrix} = -h_{23}\begin{vmatrix} A_{34} & (x_4-x_3) & (y_4-y_3)\\ A_{41} & (x_1-x_4) & (y_1-y_4)\\ A_{12} & (x_2-x_1) & (y_2-y_1)\end{vmatrix}$$

$$= h_{34}\begin{vmatrix} A_{41} & (x_1-x_4) & (y_1-y_4)\\ A_{12} & (x_2-x_1) & (y_2-y_1)\\ A_{23} & (x_3-x_2) & (y_3-y_2)\end{vmatrix} = -h_{41}\begin{vmatrix} A_{12} & (x_2-x_1) & (y_2-y_1)\\ A_{23} & (x_3-x_2) & (y_3-y_2)\\ A_{34} & (x_4-x_3) & (y_4-y_3)\end{vmatrix} \quad (195)$$

By virtue of equations (189),

$$\begin{vmatrix} A_{23} & (x_3-x_2) & (y_3-y_2)\\ A_{34} & (x_4-x_3) & (y_4-y_3)\\ A_{41} & (x_1-x_4) & (y_1-y_4)\end{vmatrix} = 2\bar{A}_{34}A \quad . \quad . \quad (189a)$$

Hence from equation (195) and the four equations of the form of (189a),

$$\frac{h_{12}}{\bar{A}_{34}} = \frac{-h_{23}}{\bar{A}_{41}} = \frac{h_{34}}{\bar{A}_{12}} = \frac{-h_{41}}{\bar{A}_{23}} = \text{const.} \quad . \quad . \quad . \quad . \quad (196)$$

For a quadrilateral tube,

$$\frac{h_{12}}{A_{34}} = \frac{-h_{23}}{A_{41}} = \frac{h_{34}}{A_{12}} = \frac{-h_{41}}{A_{23}} = \text{const.} \quad . \quad . \quad . \quad . \quad (196a)$$

Using the normalising relationship of equation (70) one obtains,

$$h_{12} = \frac{\bar{A}_{34}}{\left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2\right]^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (197)$$

and similar expressions for h_{23} etc.

The boundary integral $\oint_s \frac{h}{t}\, ds$ is,

$$\sum_1^4 \frac{s_{12}}{t_{12}} h_{12} = \frac{1}{\left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2\right]^{1/2}} \left\{ \frac{s_{12}}{t_{12}} \bar{A}_{34} - \frac{s_{23}}{t_{23}} \bar{A}_{41} + \frac{s_{34}}{t_{34}} \bar{A}_{12} - \frac{s_{41}}{t_{41}} \bar{A}_{23} \right\} . \quad (198)$$

The differences of the h values in adjacent walls take the form,

$$\left.\begin{aligned} h_{12} - h_{41} &= \frac{\bar{A}_{34} + \bar{A}_{23}}{\left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2\right]^{1/2}} = \frac{A_{234}}{\left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2\right]^{1/2}} \\ h_{23} - h_{12} &= -\frac{\bar{A}_{41} + \bar{A}_{34}}{\left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2\right]^{1/2}} = -\frac{A_{341}}{\left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2\right]^{1/2}} \\ h_{34} - h_{23} &= \frac{A_{412}}{\left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2\right]^{1/2}}, \quad h_{41} - h_{34} = -\frac{A_{123}}{\left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2\right]^{1/2}} \end{aligned}\right\} \quad . \quad (199)$$

4.3.2. *The eigenvalue λ and the h-function for the multi-cell four-boom tube; reduction to an equivalent single-cell tube.*

The type of multi-cell tube to be considered is shown in Fig. 14. There need be no restriction on the number of walls connecting the booms but for the present analysis it is sufficient to examine the typical case of two walls connecting each pair of consecutive booms.

Some explanation of the notation used is necessary. The outer walls are denoted by the index o and the inner walls by the index i, thus s^i_{12} is the length of the inner wall joining the booms 1 and 2 and A^o_{12} is the swept area corresponding to the outer wall (1, 2).

The problem of the multi-cell tube under end loads will be made to depend on the simpler problem of the single-cell tube treated in 4.3.1. It will be shown that it is possible to replace each pair of walls by a single wall in such a way that the resulting single-cell tube has the same total deformations (u, v, θ) and boom displacements (w) as the original multi-cell tube. This single-cell tube which will be called the equivalent tube will have the same eigenvalue λ. All quantities related to the equivalent tube will be without indices.

The assumption of equal twist in both the original and equivalent tube means that the expression,

$$\frac{1}{A} \oint_s \frac{h}{t}\, ds$$

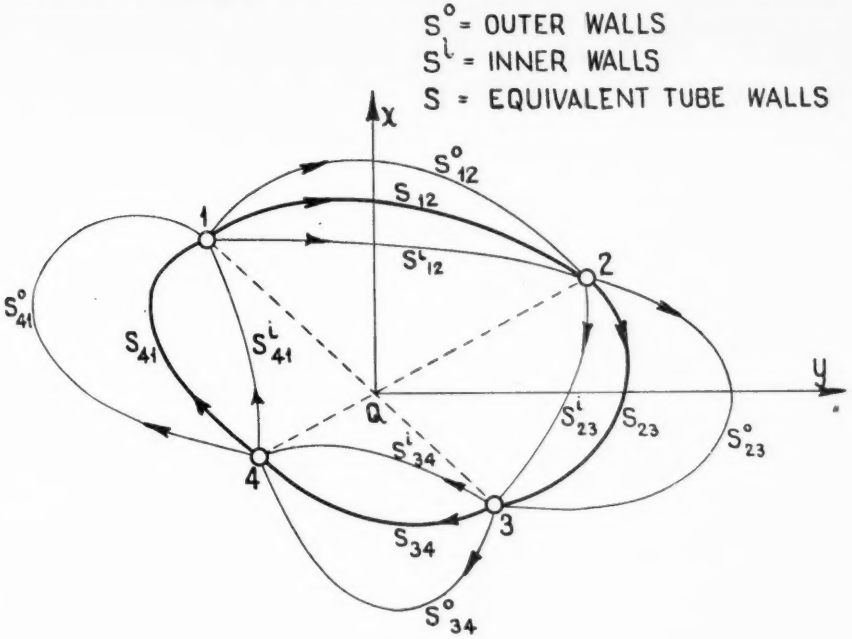

Fig. 14.
The four-boom multi-cell tube and equivalent single-cell tube.

will have the same value for any closed circuit of either tube. In what follows it is to be understood to apply to the periphery of the single-cell equivalent tube.

The typical equation of the form (72) for the inner wall (1, 2) is,

$$\frac{1}{B_2}[(h^i_{23}+h^o_{23})-(h^i_{12}+h^o_{12})]-\frac{1}{B_1}[(h^i_{12}+h^o_{12})-(h^i_{41}+h^o_{41})]+\lambda^2\frac{s^i_{12}}{t^i_{12}}h^i_{12}$$
$$=\frac{\lambda^2}{2A}[2A^i_{12}-\xi(y_2-y_1)+\eta(x_2-x_1)]\oint_S\frac{h}{t}\,ds \quad . \quad . \quad . \quad (200)$$

The corresponding equation for the outer wall (1, 2) is obtained by writing i in place of o and vice versa.

From the equality of twist in all cells (equation (14)) one obtains,

$$\frac{1}{A^{oi}_{12}}\left(\frac{s^o_{12}}{t^o_{12}}h^o_{12}-\frac{s^i_{12}}{t^i_{12}}h^i_{12}\right)=\frac{1}{A}\oint_S\frac{h}{t}\,ds \quad . \quad . \quad . \quad (201)$$

where A^{oi}_{12} denotes the area between the outer and inner walls so that,

$$A^{oi}_{12}=A^o_{12}-A^i_{12} \quad . \quad . \quad . \quad . \quad . \quad . \quad (202)$$

Assuming

$$h'_{12}=h^o_{12}+h^i_{12} \text{ etc.} \quad . \quad . \quad . \quad . \quad . \quad (203)$$

and solving equations (201) and (203) for h^o_{12} and h^i_{12} one obtains,

$$\left.\begin{aligned}h^o_{12}&=\frac{(s^i_{12}/t^i_{12})}{(s^o_{12}/t^o_{12})+(s^i_{12}/t^i_{12})}h'_{12}+\frac{(A^{oi}_{12}/A)}{(s^o_{12}/t^o_{12})+(s^i_{12}/t^i_{12})}\oint_S\frac{h}{t}\,ds\\ h^i_{12}&=\frac{(s^o_{12}/t^o_{12})}{(s^o_{12}/t^o_{12})+(s^i_{12}/t^i_{12})}h'_{12}-\frac{(A^{oi}_{12}/A)}{(s^o_{12}/t^o_{12})+(s^i_{12}/t^i_{12})}\oint_S\frac{h}{t}\,ds\end{aligned}\right\} \quad . \quad . \quad (204)$$

and similar equations for the other walls.

Substituting from equations (203) and (204) in equation (200),

$$\frac{1}{B_2}(h'_{23}-h'_{12})-\frac{1}{B_1}(h'_{12}-h'_{41})+\lambda^2\frac{s_{12}}{t_{12}}h'_{12}$$
$$=\frac{\lambda^2}{2A}[2A^{\circ}_{12}-\xi(y_2-y_1)+\eta(x_2-x_1)]\oint_S\frac{h}{t}\,ds \quad . \quad . \quad . \quad (205)$$

where

$$\frac{s_{12}}{t_{12}}=\frac{(s^{\circ}_{12}/t^{\circ}_{12})(s^{i}_{12}/t^{i}_{12})}{(s^{\circ}_{12}/t^{\circ}_{12})+(s^{i}_{12}/t^{i}_{12})} \quad . \quad . \quad . \quad . \quad . \quad (206)$$

and,

$$A^{e}_{12}=A^{i}_{12}+\frac{(s^{i}_{12}/t^{i}_{12})}{(s^{\circ}_{12}/t^{\circ}_{12})+(s^{i}_{12}/t^{i}_{12})}A^{oi}_{12}=\frac{(s^{\circ}_{12}/t^{\circ}_{12})A^{i}_{12}+(s^{i}_{12}/t^{i}_{12})A^{\circ}_{12}}{(s^{\circ}_{12}/t^{\circ}_{12})+(s^{i}_{12}/t^{i}_{12})} \quad . \quad (207)$$

There are three equations of the type (205) for the remaining equivalent walls (2, 3), (3, 4) and (4, 1).

The zero shear conditions are,

$$\left.\begin{aligned}\sum_1^4(x_2-x_1)(h^{\circ}_{12}+h^{i}_{12})&=\sum_1^4(x_2-x_1)h'_{12}=0\\ \sum_1^4(y_2-y_1)(h^{\circ}_{12}+h^{i}_{12})&=\sum_1^4(y_2-y_1)h'_{12}=0\end{aligned}\right\} \quad . \quad . \quad . \quad (208)$$

The zero torque condition is,

$$\sum_1^4(A^{\circ}_{12}h^{\circ}_{12}+A^{i}_{12}h^{i}_{12})=0$$

so that from equations (204) and (207),

$$\sum_1^4 A^{e}_{12}h'_{12}+\frac{(A^{m})^2}{A}\oint_S\frac{h}{t}\,ds=0 \quad . \quad . \quad . \quad . \quad . \quad (209)$$

where,

$$(A^{m})^2=\sum_1^4\frac{(A^{oi}_{12})^2}{(s^{\circ}_{12}/t^{\circ}_{12})+(s^{i}_{12}/t^{i}_{12})} \quad . \quad . \quad . \quad . \quad . \quad (210)$$

Comparing equations (205), (208) and (209) with those for a single-cell tube it is evident that the only difference is the additional term,

$$\frac{(A^{m})^2}{A}\oint_S\frac{h}{t}\,ds$$

in equation (209).

Transforming equations (205), (208) and (209) by means of the substitution,

$$h'_{12}=h_{12}+\frac{K}{A}\oint_S\frac{h}{t}\,ds \quad . \quad . \quad . \quad . \quad . \quad (211)$$

where K is at present an undetermined constant one obtains,

$$\frac{1}{B_2}(h_{23}-h_{12})-\frac{1}{B_1}(h_{12}-h_{41})+\lambda^2\frac{s_{12}}{t_{12}}h_{12}$$
$$=\frac{\lambda^2}{2A}\left[2\left(A^{\circ}_{12}-K\frac{s_{12}}{t_{12}}\right)-\xi(y_2-y_1)+\eta(x_2-x_1)\right]\oint_S\frac{h}{t}\,ds \quad . \quad . \quad (205a)$$

$$\sum_1^4(x_2-x_1)h_{12}=\sum_1^4(y_2-y_1)h_{12}=0 \quad . \quad . \quad . \quad . \quad (208a)$$

and

$$\sum_1^4 A^{e}_{12}h_{12}+\frac{[KA^{e}+(A^{m})^2]}{A}\oint_S\frac{h}{t}\,ds=0$$

where

$$A^e = \sum_1^4 A^e{}_{12},$$

so that

$$\sum_1^4 \left\{ A^e{}_{12} + \frac{[KA^e + (A^m)^2]}{A} \frac{s_{12}}{t_{12}} \right\} h_{12} = 0 \quad . \quad . \quad . \quad . \quad (209a)$$

Comparing equations (205a), (208a) and (209a) with the equations of the single-cell tube with four booms, it is evident that the swept area of the equivalent wall (1, 2) must be expressible as,

$$A_{12} = A^e{}_{12} - K\frac{s_{12}}{t_{12}} = A^e{}_{12} + \frac{KA^e + (A^m)^2}{A}\frac{s_{12}}{t_{12}} \quad . \quad . \quad . \quad . \quad (212)$$

From the first two equations in (212),

$$A = \sum_1^4 A_{12} = A^e - K\sum_1^4 \frac{s_{12}}{t_{12}} \quad . \quad . \quad . \quad . \quad . \quad (212a)$$

Substituting for A into the last equation of (212),

$$A^e{}_{12} - K\frac{s_{12}}{t_{12}} = A^e{}_{12} + \frac{KA^e + (A^m)^2}{A^e - K\sum_1^4 \frac{s_{12}}{t_{12}}}\frac{s_{12}}{t_{12}}$$

or

$$K^2\sum_1^4 \frac{s_{12}}{t_{12}} - 2KA^e - (A^m)^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (213)$$

This equation for K does not depend on the particular wall chosen and thus justifies substitution (211).

Solving (213),

$$K = \frac{A^e - \left[(A^e)^2 + (A^m)^2\sum_1^4 \frac{s_{12}}{t_{12}}\right]^{1/2}}{\sum_1^4 \frac{s_{12}}{t_{12}}} \quad . \quad . \quad . \quad . \quad (214)$$

The positive sign of the root has no physical significance. Substituting (214) into (212a) it follows that,

$$A = \left[(A^e)^2 + (A^m)^2\sum_1^4 \frac{s_{12}}{t_{12}}\right]^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (215)$$

Equation (214) completes the problem of reducing the given multi-cell tube to a single-cell tube which behaves in an identical manner under self-equilibrating load systems. It only remains to prove that the values $h^o{}_{12}$ and $h^i{}_{12}$ are normalised. By virtue of the normalisation of h_{12} etc.

$$\sum_1^4 \frac{1}{B_1}(h_{12} - h_{41})^2 = \lambda^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (78a)$$

Now from equations (203) and (211),

$$\sum_1^4 \frac{1}{B_1}\{(h^o{}_{12} + h^i{}_{12}) - (h^o{}_{41} + h^i{}_{41})\}^2 = \lambda^2$$

which proves that $h^o{}_{12}$ etc. are normalised.

The detailed procedure to determine the eigenvalue λ and the h^o- and h^i-functions is recapitulated as follows:

(a) Find the length thickness ratio s/t of each equivalent wall from equation (206).

(b) Find the areas $A^e{}_{12}$ and A^m from (207) and (210).

(c) Find the value of K from (214).
(d) Find the equivalent areas A_{12} from (212).
(e) Find λ^2 and h_{12} etc. for the equivalent single-cell tube using the method of 4.3.1.
(f) Find the h'_{12} values from (211) and finally the h^{o}_{12} and h^{i}_{12} values of the actual multi-cell tube from (204).

In the particular case of Fig. 15 where there are no inner walls (1,2) and (3,4) the preceding formulæ still apply by writing,

$$t^{i}_{12}=t^{i}_{34}=0$$

The necessary modifications to the formulæ are then,

$$\left.\begin{aligned} &\frac{s_{12}}{t_{12}}=\frac{s^{o}_{12}}{t^{o}_{12}},\ \frac{s_{34}}{t_{34}}=\frac{s^{o}_{34}}{t^{o}_{34}} \\ &A^{e}_{12}=A^{o}_{12},\ A^{e}_{34}=A^{o}_{34} \\ &h'_{12}=h^{o}_{12},\ h'_{34}=h^{o}_{34} \\ &(A^{m})^2=\frac{(A^{oi}_{23})^2}{(s^{o}_{23}/t^{o}_{23})+(s^{i}_{23}/t^{i}_{23})}+\frac{(A^{oi}_{41})^2}{(s^{o}_{41}/t^{o}_{41})+(s^{i}_{41}/t^{i}_{41})} \end{aligned}\right\} \quad . \quad . \quad . \quad (216)$$

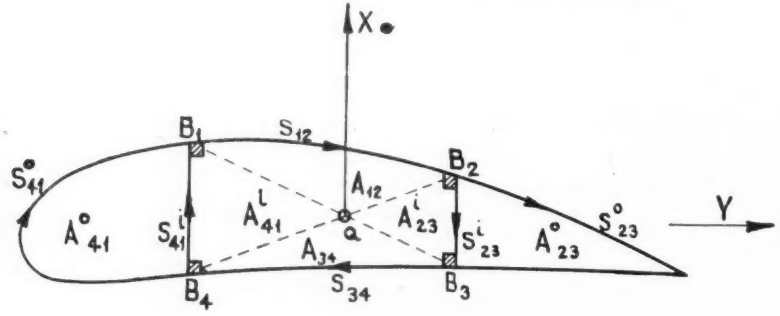

Fig. 15.
Four-boom wing section with nose and trailing cells.

It is interesting to note that the swept areas A_{12} and A_{34} of the equivalent tube are different from A^{o}_{12} and A^{o}_{34} (see equation (212)), while the corresponding length thickness ratios are the same.

The preceding analysis does not depend in any way on the number of booms provided the tube is of the type shown in Fig. 16, *i.e.*, all walls connect adjacent booms. This generalisation will prove very useful in discussing the six boom multi-cell tube in Part 5 of this paper.

4.3.3. *The generalised Bredt-Batho function h_B for the multi-cell tube of Fig.* 14.

For the completion of the analysis for the multi-cell tube one requires the generalised Bredt-Batho function h_B. The analysis given will be directly applicable to the tube shown in Fig. 14 but may be immediately generalised for any tube of the type shown in Fig. 16.

Physically the method will again amount to replacement of the actual tube by an equivalent single-cell tube. The latter tube is not in general identical with that of 4.3.2 but much of the notation there used will be applied here also.

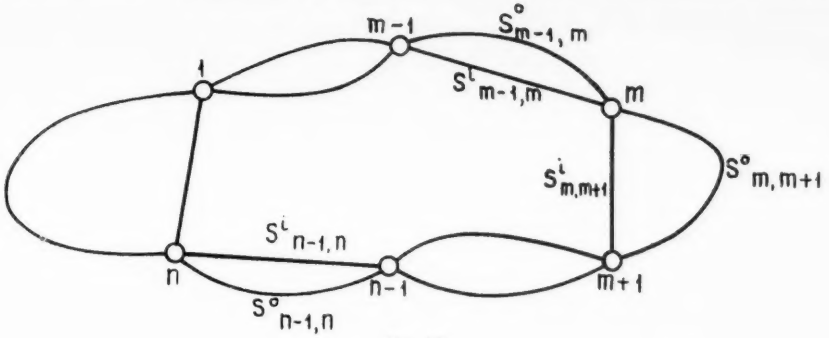

Fig. 16.
Multi-cell n-boom tube.

The basic theory for the h_B distribution has been given in 4.1.2. Rewriting equations (113) to (115) for the tube of Fig. 14 one obtains,

$$\sum_1^4 (A^o{}_{12} h^o{}_{12} + A^i{}_{12} h^i{}_{12}) = A^o \qquad . \quad . \quad . \quad . \quad (217)$$

$$\frac{1}{A^{oi}{}_{12}} \left[\frac{s^o{}_{12}}{t^o{}_{12}} h^o{}_{12} - \frac{s^i{}_{12}}{t^i{}_{12}} h^i{}_{12} \right] = \frac{1}{A^o} \sum_1^4 \frac{s^o{}_{12}}{t^o{}_{12}} h^o{}_{12} = C \qquad . \quad . \quad . \quad (218)$$

where C is an undetermined constant. There are three similar equations for the cells $A^{oi}{}_{23}$, $A^{oi}{}_{34}$ and $A^{oi}{}_{41}$.

$$\left.\begin{aligned} h^o{}_{12} + h^i{}_{12} - h^o{}_{41} - h^i{}_{41} &= 0 \\ h^o{}_{23} + h^i{}_{23} - h^o{}_{12} - h^i{}_{12} &= 0 \\ h^o{}_{34} + h^i{}_{34} - h^o{}_{23} - h^i{}_{23} &= 0 \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad . \quad (219)$$

In the above equations the suffix B has been omitted for clarity in printing.

Equations (219) are satisfied by putting,

$$h^o{}_{12} + h^i{}_{12} = \ldots\ldots = h^o{}_{41} + h^i{}_{41} = h' \qquad . \quad . \quad . \quad (220)$$

Solving (218) and (220) for $h^o{}_{12}$ and $h^i{}_{12}$ etc.,

$$\left.\begin{aligned} h^o{}_{12} &= \frac{(s^i{}_{12}/t^i{}_{12})}{(s^o{}_{12}/t^o{}_{12}) + (s^i{}_{12}/t^i{}_{12})} h' + \frac{A^{oi}{}_{12}}{(s^o{}_{12}/t^o{}_{12}) + (s^i{}_{12}/t^i{}_{12})} C \\ h^i{}_{12} &= \frac{(s^o{}_{12}/t^o{}_{12})}{(s^o{}_{12}/t^o{}_{12}) + (s^i{}_{12}/t^i{}_{12})} h' - \frac{A^{oi}{}_{12}}{(s^o{}_{12}/t^o{}_{12}) + (s^i{}_{12}/t^i{}_{12})} C \end{aligned}\right| \qquad . \quad . \quad (221)$$

Substituting the first of equations (221) into the second expression in (218) and using the relations (206) and (207),

$$\frac{1}{A^o} \left[\sum_1^4 \frac{s_{12}}{t_{12}} h' + (A^o - A^i) C \right] = C$$

or

$$\sum_1^4 \frac{s_{12}}{t_{12}} h' + (A^o - A^o - A^i) C = 0 \qquad . \quad . \quad . \quad . \quad . \quad (222)$$

Substituting equations (221) into (217) and using (207) and (210),

$$A^o h' + (A^m)^2 C = A^o \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (223)$$

Solving equations (222) and (223) for h' and C,

$$\left.\begin{aligned} h' &= \frac{A^o}{A^B} \\ C &= \frac{A^o \sum_1^4 \frac{s_{12}}{t_{12}}}{A^B [A^o + A^i - A^o]} \end{aligned}\right| \qquad . \quad . \quad . \quad . \quad . \quad . \quad (224)$$

where A^B is the area of the equivalent single-cell tube for the generalised Bredt-Batho distribution and is given by,

$$A^B = \frac{A^c\,[A^o + A^i - A^c] + (A^m)^2 \sum_1^4 \frac{s_{12}}{t_{12}}}{A^o + A^i - A^c} \quad . \quad . \quad . \quad . \quad (225)$$

The final formulæ for the h_B functions are,

$$\left.\begin{aligned} h^o{}_{12} &= \frac{A^o}{A^B}\,\frac{1}{(s^o{}_{12}/t^o{}_{12})}\left\{\frac{s_{12}}{t_{12}} + \frac{A^o{}_{12} - A^c{}_{12}}{A^o + A^i - A^c}\sum_1^4 \frac{s_{12}}{t_{12}}\right\} \\ h^i{}_{12} &= \frac{A^o}{A^B}\,\frac{1}{(s^i{}_{12}/t^i{}_{12})}\left\{\frac{s_{12}}{t_{12}} + \frac{A^i{}_{12} - A^c{}_{12}}{A^o + A^i - A^c}\sum_1^4 \frac{s_{12}}{t_{12}}\right\} \end{aligned}\right\} \quad . \quad . \quad . \quad (226)$$

and similar formulæ for $h^o{}_{23}$ etc.

For the case illustrated in Fig. 15 where the walls $s^i{}_{12}$ and $s^i{}_{34}$ are not present the formulæ for $h^o{}_{12}$ and $h^o{}_{34}$ become simply,

$$h^o{}_{12} = h^o{}_{34} = h' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (227)$$

4.3.4. *The differential equation for the boom loads in the four-boom tube.*

The four-boom tube has only one eigenvalue λ and thus from equation (143) the stress function must always take the form,

$$F = g_B h_B + gh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (228)$$

where the functions h and h_B are as determined in 4.3.2 and 4.3.3. All the results developed in 4.1 and 4.2 for the general case are directly applicable here by simply omitting the summation sign and the suffix i for the different eigenfunctions. Since the four-boom case, however, is of special importance for practical applications some of the previous results will be re-stated for convenience of reference.

In the special case when $t^* = t_s^* = \rho^o$ the differential equation (157) for the g-function is,

$$r^2\frac{d^2g}{dr^2} + (1-\alpha)\,r\,\frac{dg}{dr} - \mu^2 r_o{}^2 g = -\frac{r_o{}^2}{2A}\sum_S \frac{s_{12}}{t_{12}}\,h_{12}\left\{\frac{T(1+\alpha)}{r} - \frac{dT}{dr}\right\} \quad . \quad (157a)$$

For a multi-cell tube the value $\frac{1}{A}\sum_S \frac{s_{12}}{t_{12}}\,h_{12}$ is taken conveniently over the periphery of the single-cell equivalent tube and is given in equation (198). The load in the boom B_1 can be expressed as,

$$P_1 = g\,(h_{12} - h_{41}) \quad . \quad . \quad . \quad . \quad . \quad (229)$$

Note that this formula is also valid for the multi-cell tube when h_{12} and h_{41} are referred to the equivalent single-cell tube.

Using (199) equation (229) becomes,

$$P_1 = g\frac{A_{234}}{\left[\sum_1^4 \frac{s_{12}}{t_{12}}\,(\bar{A_{34}})^2\right]^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad (229a)$$

Multiplying equation (157a) by $(h_{12} - h_{41})$ and using equation (198) and (229a) one obtains

$$r^2\frac{d^2P_1}{dr^2} + (1-\alpha)\,r\,\frac{dP_1}{dr} - \mu^2 r_o{}^2 P_1 = -r_o{}^2\,\frac{A_{234}}{2A}\,H\left\{\frac{T(1+\alpha)}{r} - \frac{dT}{dr}\right\} \quad . \quad . \quad (230)$$

where,

$$H=\frac{1}{\sum_1^4 \frac{s_{12}}{t_{12}}(\bar{A}_{34})^2}\left\{\frac{s_{12}}{t_{12}}\bar{A}_{34}-\frac{s_{23}}{t_{23}}\bar{A}_{41}+\frac{s_{34}}{t_{34}}\bar{A}_{12}-\frac{s_{41}}{t_{41}}\bar{A}_{23}\right\} \quad . \quad . \quad (231)$$

Since,

$$\frac{P_1}{A_{234}}=-\frac{P_2}{A_{341}}=\frac{P_3}{A_{412}}=-\frac{P_4}{A_{123}}=\frac{P}{2A^*} \quad . \quad . \quad . \quad (232)$$

where,

$$P=P_1-P_2+P_3-P_4=g\frac{2A^*}{\left[\sum_1^4 \frac{s_{12}}{t_{12}}(\bar{A}_{34})^2\right]^{1/2}} \quad . \quad . \quad . \quad . \quad (233)$$

equation (230) can be written,

$$r^2\frac{d^2P}{dr^2}+(1-\alpha)r\frac{dP}{dr}-\mu^2r_o^2P=-r_o^2\frac{A^*}{A}H\left\{\frac{T(1+\alpha)}{r}-\frac{dT}{dr}\right\} \quad . \quad . \quad (234)$$

For the parallel tube (234) reduces to,

$$\frac{d^2P}{dz'^2}-\mu^2P=+\frac{A^*}{A}H\frac{dT}{dz'} \quad . \quad . \quad . \quad . \quad . \quad (234a)$$

Note that the quantities A and H must be referred to the equivalent single-cell tube of 4.3.2. when dealing with a multi-cell tube. Equation (234) may be used as the basis of an analysis of the four-boom tube, but since the results have already been obtained in the general case using the g-function method it will be more convenient to quote the results directly from the general case.

4.3.5. *The four-boom tube under concentrated torque.*

For a torque T at the section $r=r_1$ one finds from equations (136a), (136b),

For $r_c \leqslant r \leqslant r_1$,

$$\left.\begin{aligned}\sigma_1&=\frac{T}{2A}G_\sigma'(H_\sigma)_1\\ \tau^o_{12}&=\frac{T}{2A}G_\tau'(H_\tau)^o_{12}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (236a)$$

and for $r_1 \leqslant r \leqslant r_o$,

$$\left.\begin{aligned}\sigma_1&=\frac{T}{2A}G_\sigma(H_\sigma)_1\\ \tau^o_{12}&=\frac{T}{2A}\left[\frac{A}{A^o}\rho^{-(\alpha+2)}\frac{(h_B)^o_{12}}{t^o_{12}}+G_\tau(H_\tau)^o_{12}\right]\end{aligned}\right\} \quad . \quad . \quad (236b)$$

and similar expressions for the stresses σ_2, τ^o_{23}, τ^i_{12}, etc.

In the above formulæ the H-functions are obtained by substituting equations (198) and (199) into (138),

$$\left.\begin{aligned}(H_\sigma)_1&=\frac{A_{234}H}{\mu B_1}\ ;\ (H_\sigma)_2=-\frac{A_{341}H}{\mu B_2}\\ (H_\sigma)_3&=\frac{A_{412}H}{\mu B_3}\ ;\ (H_\sigma)_4=-\frac{A_{123}H}{\mu B_4}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (237)$$

$$(H_\tau)^o_{12}=\frac{h^o_{12}}{t^o_{12}}\left[\sum_1^4\frac{s_{12}}{t_{12}}(\bar{A}_{34})^2\right]^{1/2}H \quad . \quad . \quad . \quad . \quad (238)$$

where h^o_{12}, etc., have to be calculated from equations (204), (211), etc., in 4.3.2. For a single-

cell tube equations (238) reduce by applying equation (197) to,

$$\left.\begin{aligned}(H_\tau)_{12} &= \frac{\bar{A}_{34}H}{t_{12}} \quad ; \quad (H_\tau)_{23} = -\frac{\bar{A}_{41}H}{t_{23}} \\ (H_\tau)_{34} &= \frac{\bar{A}_{12}H}{t_{34}} \quad ; \quad (H_\tau)_{41} = -\frac{\bar{A}_{23}H}{t_{41}}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (239)$$

The G-functions are given by equations (137) for the conical tube and (142) for the cylindrical tube. Some typical plots of the G-functions for the case $\alpha = 0$ are given in Figs. 7 to 10.

Note that in equations (236a) and (236b) the area A refers to the equivalent tube. Since h_B is based on the total area A^0 (see 4.1.2) the factor $\frac{A}{A^0}$ occurs in the Bredt-Batho part of the shear stress in equation (236b). Note also that the integral $\oint_s \frac{h_1}{t}\,ds$ in equations (138) is taken round the walls of the equivalent tube.

4.3.6. *The four-boom conical tube under distributed torque* $T = T_0\rho^\gamma$.

From equations (165),

$$\left.\begin{aligned}\sigma_1 &= \frac{T_0}{2A}\,G_\tau\,(H_\tau)_1 \\ \tau^0{}_{12} &= \frac{T_0}{2A}\left[\frac{A}{A^0}\,\rho^{\gamma-(\alpha+2)}\,\frac{(h_B)^0{}_{12}}{t^0{}_{12}} + G_\tau\,(H_\tau)^0{}_{12}\right]\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (240)$$

The H-functions are the same as given in the previous section (equations (237), (238) and (239)). The G-functions are given in equations (166).

4.3.7. *The four-boom tube under uniformly distributed torque.*

For the conical tube from equations (169),

$$\left.\begin{aligned}\sigma_1 &= \frac{T_0}{2A}\,G_\tau\,(H_\tau)_1 \\ \tau^0{}_{12} &= \frac{T_0}{2A}\left[\frac{A}{A^0}\,\frac{\rho-\rho_c}{1-\rho_c}\,\rho^{-(\alpha+2)}\,\frac{(h_B)^0{}_{12}}{t^0{}_{12}} + G_\tau\,(H_\tau)^0{}_{12}\right]\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (241)$$

The H-functions are as given in 4.3.5. and the G-functions in equations (170) and (171).

For the cylindrical tube from equations (179)

$$\left.\begin{aligned}\sigma_1 &= \frac{T_0}{2A}\,G_\tau\,(H_\tau)_1 \\ \tau^0{}_{12} &= \frac{T_0}{2A}\left[\frac{A}{A^0}\,\frac{z'}{c}\,\frac{(h_B)^0{}_{12}}{t^0{}_{12}} + G_\tau\,(H_\tau)^0{}_{12}\right]\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (242)$$

where the G-functions are given in equation (180). Some typical plots of the G-functions for cylindrical and conical tubes with $\alpha = 0$ are shown in Figs. 11 and 12.

4.3.8. *The warping function* W^* *for the four-boom tube.*

The warping of a four-boom tube may be represented by a single function,

$$W^* = \begin{vmatrix} w_1^* & x_1' & y_1' & 1 \\ w_2^* & x_2' & y_2' & 1 \\ w_3^* & x_3' & y_3' & 1 \\ w_4^* & x_4' & y_4' & 1 \end{vmatrix} \quad . \quad . \quad . \quad . \quad . \quad (243)$$

Thus W^* is proportional to the volume of the tetrahedron formed by the boom-diaphragm intersections (*i.e.* points 1, 2, 3 and 4). Equation (243) may be written,

$$W^* = \rho^2 \begin{vmatrix} w_1^* & x_1 & y_1 & 1 \\ w_2^* & x_2 & y_2 & 1 \\ w_3^* & x_3 & y_3 & 1 \\ w_4^* & x_4 & y_4 & 1 \end{vmatrix} \quad . \quad . \quad . \quad (243a)$$

or expanding the determinant,

$$W^* = 2\rho^2 [w_1^* A_{234} - w_2^* A_{341} + w_3^* A_{412} - w_4^* A_{123}] \quad . \quad . \quad . \quad (244)$$

The warping W^* will be separated into two parts, the first W_P^* due to the self-equilibrating system and the second W_B^* due to the generalised Bredt-Batho system.

From equation (96) the warping due to the self-equilibrating system gh can be written,

$$w_1^* = \frac{1}{Gt^*} \rho \frac{1}{\lambda^2} \frac{dg}{dr} \frac{1}{B_1} [h_{12} - h_{41}] \quad . \quad . \quad . \quad . \quad . \quad (245)$$

and similar equations for w_2^*, w_3^* and w_4^*.

It follows from equations (203) and (211) that in the case of a multi-cell tube of the type shown in Fig. 14 the equivalent single-cell tube deduced in 4.3.2 may be used in equation (245).

Using in equation (245) the values of λ^2, $(h_{12} - h_{41})$ and $\frac{dg}{dr}$ as given by equations (193), (199) and (233) respectively and substituting in equation (244) one obtains for $t^* = \rho^n$,

$$W_P^* = \frac{1}{A^* G} \rho^{3-n} \frac{dP}{dr} \sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2 \quad . \quad . \quad . \quad . \quad . \quad (246)$$

For the parallel tube replace $\rho^{3-n} \frac{dP}{dr}$ by $\frac{dP}{dz'}$. Considering now the warping due to the generalised Bredt-Batho distribution $F = g_B h_B$ one obtains from equation (95*c*),

$$\frac{\partial w^*}{\partial s} = -\frac{1}{Gt^*} \rho \frac{dg_B}{dr} \left\{ \frac{h_B}{t} - \frac{\hat{p}}{2A} \oint_S \frac{h_B}{t} \, ds \right\}$$

and integrating between booms 1 and 2 over the outer wall and using equation (144),

$$w_2^* - w_1^* = \frac{1}{Gt^*} \frac{T}{2A^o \rho} \left\{ \frac{s^o_{12}}{t^o_{12}} (h_B)^o_{12} - \frac{A^o_{12}}{2A^o} \sum_1^4 \frac{s^o_{12}}{t^o_{12}} (h_B)^o_{12} \right\}$$

Substituting from equation (226) and using equation (206),

$$w_2^* - w_1^* = \frac{1}{Gt^*} \frac{T}{2\bar{A}^B \rho} \left\{ \frac{s_{12}}{t_{12}} - \frac{A^B_{12}}{2\bar{A}^B} \sum_1^4 \frac{s_{12}}{t_{12}} \right\} \quad . \quad . \quad . \quad . \quad (247)$$

where A^B is defined by equations (225) and,

$$\frac{A^B_{12}}{A^B} = \frac{A^o_{12} + A^i_{12} - A^c_{12}}{A^o + A^i - A^c} \quad . \quad . \quad . \quad . \quad . \quad (248)$$

For the single-cell tube A^B and A^B_{12} reduce to the true areas A and A_{12}.

Introducing notation for the B equivalent tube parallel to that used in 4.3.1 (equation (191a)) equation (244) can be transformed as follows:—

$$W_B^* = -2\rho^2 [(w_2^* - w_1^*) \bar{A}^B_{34} - (w_3^* - w_2^*) \bar{A}^B_{41} + (w_1^* - w_3^*) \bar{A}^B_{12} - (w_1^* - w_4^*) \bar{A}^B_{23}] \quad (249)$$

where,

$$\left.\begin{aligned} \bar{A}^B_{34} &= A^*_{34} + N^B, \quad \bar{A}^B_{41} = A^*_{41} - N^B \\ \bar{A}^B_{12} &= A^*_{12} + N^B, \quad \bar{A}^B_{23} = A^*_{23} - N^B \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (250)$$

$$\bar{N}^B = \frac{1}{A^B} [(A^*_{41} N^B_{23} + A^*_{23} N^B_{41}) - (A^*_{12} N^B_{34} + A^*_{34} N^B_{12})] \quad . \quad . \quad . \quad (251)$$

and,

$$N^{B}_{12} = A^{B}_{12} - A^{*}_{12} \text{ etc.} \quad . \quad . \quad . \quad . \quad . \quad (252)$$

Substituting the four equations of the form (247) into equation (249) and putting $t^{*} = \rho^{\alpha}$,

$$W_{B}^{*} = -\frac{1}{A^{B}G}\rho^{1-\alpha}T\left[\frac{s_{12}}{t_{12}}\bar{A}^{B}_{34} - \frac{s_{23}}{t_{23}}\bar{A}^{B}_{41} + \frac{s_{34}}{t_{34}}\bar{A}^{B}_{12} - \frac{s_{41}}{t_{41}}\bar{A}^{B}_{23}\right] \quad . \quad . \quad (253)$$

In a single-cell tube the index B is omitted from this equation and the areas A_{34} etc. are as defined in equations (191) and (192), For a cylindrical tube omit the term $\rho^{1-\alpha}$ from equation (253).

The total warping function W^{*} is given by the addition of equations (246b) and (253),

$$W^{*} = W_{P}^{*} + W_{B}^{*} \quad . \quad . \quad . \quad . \quad . \quad . \quad (254)$$

Note that the warping W_{P}^{*} is proportional to $\dfrac{dP}{dr}$ and W_{B}^{*} is proportional to T.

4.3.9. *Stress distribution and warping at end of tube under end load systems P_o and P_c.*

The important case of a conical or cylindrical tube under self-equilibrating end load systems at both ends defined by P_o and P_c, will be treated directly from the differential equations (234) and (234a).

Rewriting these equations with the R.H.S. equal to zero one obtains,

$$r^{2}\frac{d^{2}P}{dr^{2}} + (1-\alpha)\,r\,\frac{dP}{dr} - \mu^{2}r_{o}^{2}P = 0 \quad . \quad . \quad . \quad . \quad . \quad (255)$$

$$\frac{d^{2}P}{dz'^{2}} - \mu^{2}P = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (255a)$$

The solution of equation (255) is, from equation (86),

$$P = r^{\alpha/2}\left[D_{1}r^{\nu} + D_{2}r^{-\nu}\right] \quad . \quad . \quad . \quad . \quad . \quad (256)$$

where

$$\nu = \frac{1}{2}\left[\alpha^{2} + 4\mu^{2}r_{o}^{2}\right]^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (86a)$$

Obtaining the constants D_1 and D_2 from the boundary conditions,

for $r = r_o$, $P = P_o$,

and for $r = r_c$, $P = P_c$,

equation (256) becomes,

$$P = \frac{\rho^{\alpha/2}}{\rho_{c}^{-\nu} - \rho_{c}^{\nu}}\left\{P_{o}\left(\rho_{c}^{-\nu}\rho^{\nu} - \rho_{c}^{\nu}\rho^{-\nu}\right) + P_{c}\rho_{c}^{-\alpha/2}\left(\rho^{-\nu} - \rho^{\nu}\right)\right\} \quad . \quad . \quad . \quad (257)$$

The warping at the two ends is obtained from equation (246a) with $\dfrac{dP}{dr}$ determined from equation (257).

Thus,

$$(W_{P}^{*})_{o} = \frac{1}{\rho_{c}^{-\nu} - \rho_{c}^{\nu}}\left\{P_{o}\left[\left(\frac{\alpha}{2} + \nu\right)\rho_{c}^{-\nu} - \left(\frac{\alpha}{2} - \nu\right)\rho_{c}^{\nu}\right] - 2\nu P_{c}\rho_{c}^{-\alpha/2}\right\} \times \frac{1}{r_{o}A^{*}G}\sum_{1}^{4}\frac{s_{12}}{t_{12}}(\bar{A}_{34})^{2} \quad (258)$$

$$(W_{P}^{*})_{c} = \frac{\rho_{c}^{(4-\alpha)/2}}{\rho_{c}^{-\nu} - \rho_{c}^{\nu}}\left\{2\nu P_{o} + P_{c}\rho_{c}^{-\alpha/2}\left[\left(\frac{\alpha}{2} - \nu\right)\rho_{c}^{-\nu} - \left(\frac{\alpha}{2} + \nu\right)\rho_{c}^{\nu}\right]\right\} \times \frac{1}{r_{o}A^{*}G}\sum_{1}^{4}\frac{s_{12}}{t_{12}}(\bar{A}_{34})^{2} \quad (259)$$

The solution of equation (255a) with the boundary conditions,

for $z' = c$, $P = P_o$,

and for $z' = 0$, $P = P_c$,

is

$$P = P_{o}\frac{\sinh \mu z'}{\sinh \mu c} + P_{c}\frac{\sinh \mu\,(c - z')}{\sinh \mu c} \quad . \quad . \quad . \quad . \quad . \quad (260)$$

For the cylindrical tube the warping at one end only is necessary and is obtained from equation (246a) with $\frac{dP}{dz'}$ as determined from equation (260).

Thus,

$$(W_P^*)_o = \left[P_o \frac{\cosh \mu z'}{\sinh \mu c} - P_c \frac{\cosh \mu (c - z')}{\sinh \mu c} \right] \frac{\mu c}{cA^*G} \sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2 \quad . \quad . \quad (261)$$

The direct stresses are found immediately from the boom loads as determined from equations (232). For the shear stress in a wall equation (28) yields,

$$\tau = - \frac{1}{\rho^a} \frac{dg}{dr} \frac{h}{t}$$

Substituting for $\frac{dg}{dr}$ from equation (233),

$$\tau = - \frac{\frac{1}{\rho^a} \frac{dP}{dr} \left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2 \right]^{1,2}}{2A^*} \frac{h}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (262)$$

Thus for the external wall 12 the shear stress becomes,

$$\tau^o_{12} = - \frac{1}{\rho^a} \frac{dP}{dr} \frac{(H_\tau)^o_{12}}{2A^*H} \quad . \quad . \quad . \quad . \quad . \quad . \quad (263)$$

where $(H_\tau)^o_{12}$ is given by equation (238) and $\frac{dP}{dr}$ is obtained from equation (257).

For a single-cell tube equation (263) reduces to,

$$\tau_{12} = - \frac{1}{\rho^a} \frac{dP}{dr} \frac{\bar{A}_{34}}{2A^*t_{12}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (263a)$$

For the cylindrical tube substitute, in equations (263) and (263a), $\frac{dP}{dz'}$ in place of $\frac{1}{\rho^a} \frac{dP}{dr}$.

(TO BE CONTINUED.)

REVIEWS

RELAXATION METHODS IN THEORETICAL PHYSICS.

R. V. Southwell, M.A., Ll.D, F.R.S. Oxford University Press, London, 1946. 248 pp. 20s.

Although self-contained, this book is essentially a sequel to Professor Southwell's earlier work "Relaxation Methods in Engineering Science." The two together constitute an able exposition of the relaxation process and provide a thorough survey of the kind of problems which can be solved successfully by this particular method of attack.

In his earlier book, Professor Southwell dealt with systems of finite freedom and with equations in one independent variable; in the one under review, he has applied the same notions to partial differential equations involving two independent space-variables and to conditions which must be satisfied at a given "boundary," thus opening the door to consideration of a greatly enlarged range of problems.

"Relaxation Methods in Theoretical Physics" is excellently reproduced, lucidly written and should be invaluable to students and others who wish to understand and apply the latest developments of the relaxation process.

PROCEEDINGS FOR THE SOCIETY FOR EXPERIMENTAL STRESS ANALYSIS.

Volume III, Number 2. Edited by C. Lipson and William Murray, Addison-Wesley Press Inc., Massachusetts, 1946. $6.00.

The second number of Volume III of the "Proceedings for the Society for Experimental Stress Analysis" maintains the same high standard both as regards subject matter and its method of presentation, as the earlier numbers. The articles cover a particularly wide range, as instanced by two by no means trivial articles on the use of bicycle spokes in testing aircraft structures and the application of Stresscoat to the study of the mechanical behaviour of the skull under impact loads.

Included at the end of the number is a useful collection of short papers on fatigue, the proceedings of a panel discussion on fatigue failure of manufactured parts.

The reproduction is excellent and the photographs and drawings are outstanding.

The full list of articles is as follows:—

Improvements in Rosette Computer.

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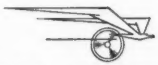

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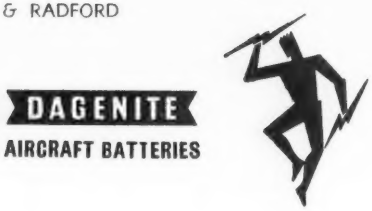

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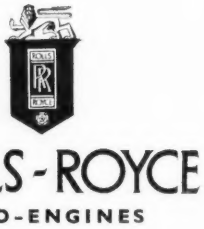

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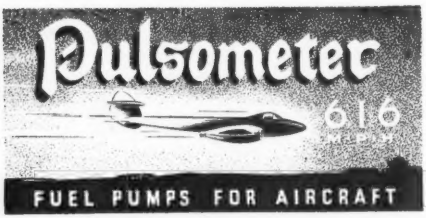

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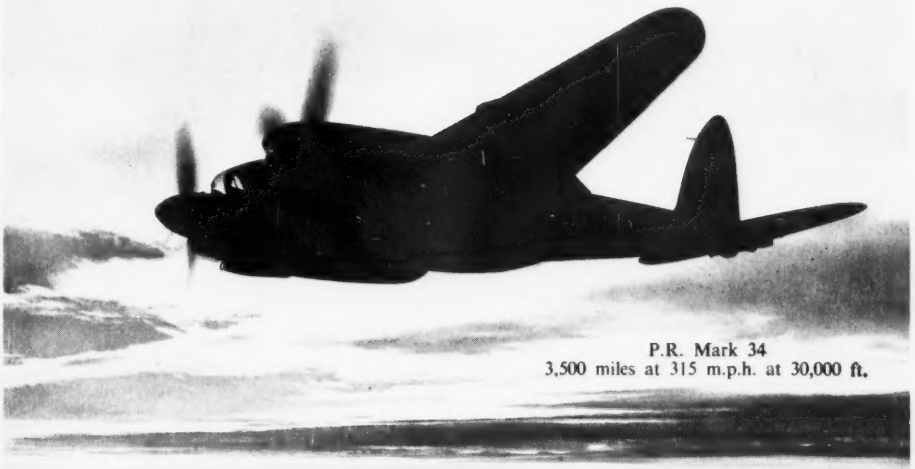

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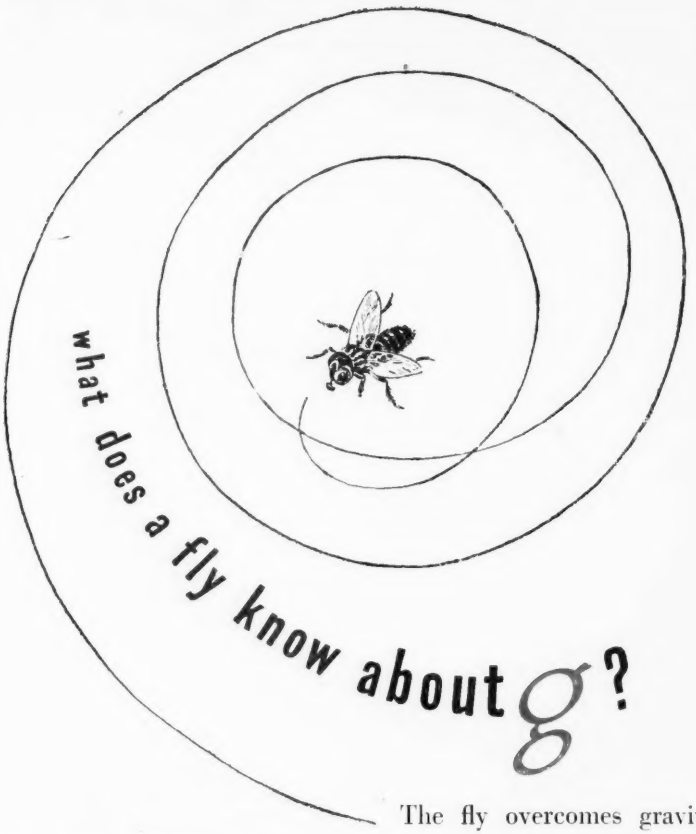

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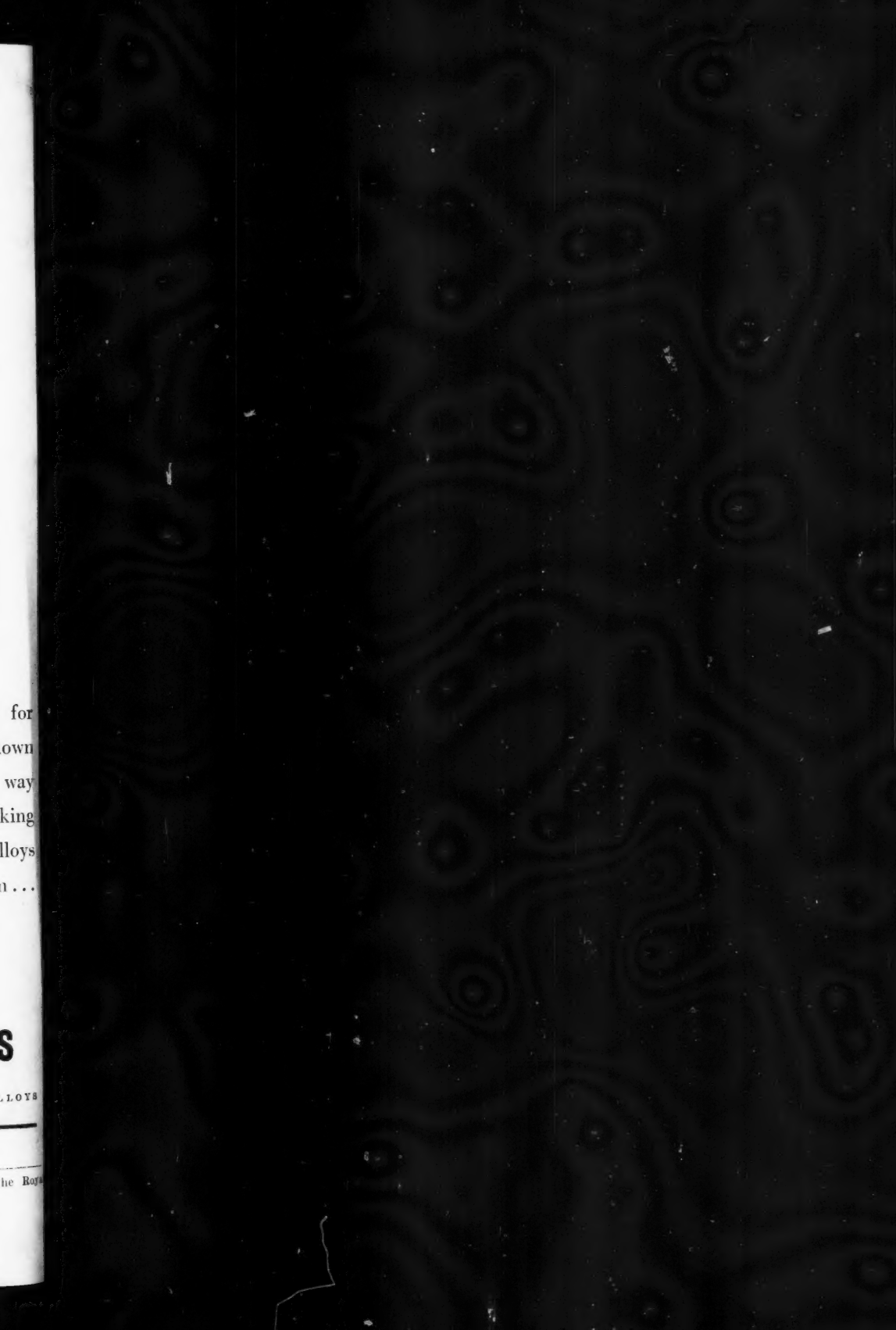
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